10th Class Mathematics Notes
Federal Board (FBISE)
Full Book Notes

Complete chapter-wise study notes with worked examples and original practice questions for Class 10 Mathematics, Federal Board of Intermediate and Secondary Education (FBISE), Islamabad.

About these notes

These notes follow the chapters of the National Book Foundation (NBF) Mathematics book for Class 10, Federal Board. The twelve chapters covered, in order, are:

1.Complex Numbers

2.Quadratic Equations

3.Matrices and Determinants

4.Linear and Quadratic Inequalities

5.Algebraic Fractions

6.Functions and Graphs

7.Vectors in Plane

8.Application of Trigonometry

9.Chord and Arcs of a Circle

10.Tangents and Angles of a Circle

11.Practical Geometry of Circles

12.Basic Statistics

About Chapter 5: Some books and notes call Chapter 5 "Algebraic Functions" and others call it "Algebraic Fractions". The content is the same either way (rational expressions and partial fractions), and this book uses "Algebraic Fractions".

About the paper: The FBISE Class 10 Mathematics paper is SLO-based (student learning outcomes) as per the Federal Board. The paper is 75 marks in total with no practical, and the MCQs are answered in 20 minutes on a separate OMR sheet. Students should check the current model paper on the official FBISE website, fbise.edu.pk, before their examination, because the board may revise the paper pattern.

Published by Zahid Notes (zahidenotes.com) as free study material for students. The practice questions in this book are original questions written for revision; they are not taken from any past paper.

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How to Use These Notes

Read one chapter at a time. First read the study notes and learn the key formulae and key definitions. Then work through the worked examples with a pen and paper, covering the solution and trying it yourself. Then close the notes and try the multiple-choice questions, short questions, and long questions for that chapter from memory. Check your MCQ answers against the answer key at the end of the book.

Mathematics is learned by practice. Do every question yourself before looking at the answer, and repeat a chapter after a few days to make the methods stick.

CHAPTER 1

Complex Numbers

What is a Complex Number?

A complex number is a number of the form a + bi, where a and b are real numbers and i is the imaginary unit, defined by i squared = -1. The number a is called the real part and b is called the imaginary part of the complex number. Complex numbers were introduced because equations such as x squared + 1 = 0 have no solution among the real numbers. Every real number is also a complex number, with imaginary part zero.

Operations on Complex Numbers

Complex numbers are added and subtracted by adding or subtracting their real parts and their imaginary parts separately. To multiply two complex numbers, multiply them as binomials and replace i squared by -1. The conjugate of a + bi is a - bi; multiplying a complex number by its conjugate gives a real number, a squared + b squared. Division is done by multiplying the numerator and the denominator by the conjugate of the

denominator.

Modulus and the Argand Plane

The modulus of a complex number a + bi is the distance of the point from the origin in the Argand plane, and it equals the square root of a squared + b squared. In the Argand plane, the horizontal axis is the real axis and the vertical axis is the imaginary axis, so every complex number is shown by one point. Two complex numbers are equal when their real parts are equal and their imaginary parts are equal.

Key Formulae

> i squared = -1; i cubed = -i; i to the power 4 = 1.

> z = a + bi; conjugate of z = a - bi; z x conjugate(z) = a squared + b squared.

> Modulus: |z| = square root of (a squared + b squared).

Worked Example: Multiply (2 + 3i) by (1 - i).

(2 + 3i)(1 - i) = 2 - 2i + 3i - 3i squared = 2 + i + 3 = 5 + i.

Worked Example: Find the modulus of 3 + 4i.

|3 + 4i| = square root of (9 + 16) = square root of 25 = 5.

Key Definitions

Term	Definition
Complex number	A number of the form a + bi, where a and b are real numbers and i squared = -1.
Imaginary unit (i)	The number defined by i squared = -1.
Real part	In a + bi, the number a.
Imaginary part	In a + bi, the number b.
Conjugate	The conjugate of a + bi is a - bi.

Key Terms to Remember

Term	Meaning
Purely imaginary number	A complex number whose real part is zero, for example 4i.
Argand plane	A plane in which complex numbers are shown as points.
Real axis	The horizontal axis of the Argand plane.
Imaginary axis	The vertical axis of the Argand plane.
Equal complex numbers	Two complex numbers with equal real parts and equal imaginary parts.
Additive inverse	The additive inverse of a + bi is -a - bi.
Multiplicative inverse	The number which, multiplied by a complex number, gives 1.
Rationalising	Removing a complex number from a denominator by multiplying by its conjugate.

Multiple-Choice Questions

1. i squared equals

(A) 1(B) 0(C) i(D) -1

2. The real part of 5 + 3i is

(A) 3(B) 5(C) 8(D) -5

3. The conjugate of 2 - 3i is

(A) 2 + 3i(B) -2 + 3i(C) -2 - 3i(D) 3 - 2i

4. The modulus of 3 + 4i is

(A) 5(B) 7(C) 25(D) 12

5. (2 + i) + (3 - 2i) equals

(A) 5 - i(B) 5 + i(C) 1 + i(D) 6 - 2i

6. i cubed equals

(A) i(B) -i(C) 1(D) -1

7. (1 + i)(1 - i) equals

(A) 0(B) 1(C) 2(D) 1 - i

8. The imaginary part of 7 - 2i is

(A) 7(B) -2(C) 2(D) -7

9. If (x + 3) + (2y - 1)i = 5 + i, then x equals

(A) 2(B) 5(C) 8(D) -2

10. The modulus of 1 + i is

(A) square root of 2(B) 2(C) 1(D) square root of 3

Short Questions

1. What is a complex number?
   A complex number is a number of the form a + bi, where a and b are real numbers and i squared = -1.
2. What is the imaginary unit i?
   The imaginary unit i is the number defined by i squared = -1.
3. What is the conjugate of a complex number?
   The conjugate of a + bi is a - bi.
4. How is the modulus of a complex number found?
   The modulus of a + bi is the square root of (a squared + b squared).
5. How are two complex numbers added?
   Add their real parts together and their imaginary parts together.
6. How are two complex numbers multiplied?
   Multiply them as binomials and replace i squared by -1.
7. When are two complex numbers equal?
   When their real parts are equal and their imaginary parts are equal.
8. What is a purely imaginary number?
   A complex number whose real part is zero, for example 4i.
9. What is the Argand plane?
   A plane in which complex numbers are shown as points, with the real axis horizontal and the imaginary axis vertical.
10. What is the real part of 6 - 5i?
    The real part is 6.

Long Questions

1. Explain complex numbers and the operations of addition, subtraction and

multiplication on them, with examples.

2. Explain the conjugate of a complex number and show how division of complex numbers is carried out.
3. Explain the modulus of a complex number and the Argand plane, with examples.

CHAPTER 2

Quadratic Equations

Standard Form and Roots

A quadratic equation is an equation of the form a x squared + b x + c = 0, where a, b and c are real numbers and a is not zero. A root of the equation is a value of x that makes the equation true. A quadratic equation has at most two roots, and a root is also called a solution of the equation.

Solving by Factorisation and Completing the Square

If the left side of a quadratic equation can be factorised, the equation is solved by setting each factor equal to zero, because a product is zero only when at least one factor is zero. Completing the square rewrites x squared + b x as (x + b/2) squared minus (b/2) squared, which turns the equation into a perfect square that can be solved by taking square roots.

The Quadratic Formula and the Nature of Roots

Every quadratic equation can be solved by the quadratic formula, x = (-b plus or minus the square root of (b squared - 4ac)) divided by 2a. The expression b squared - 4ac is called the discriminant. If the discriminant is positive, the roots are real and different; if it is zero, the roots are real and equal; if it is negative, the roots are not real. The sum of the roots is -b/a and the product of the roots is c/a.

Key Formulae

> Quadratic formula: x = (-b +/- square root of (b squared - 4ac)) / 2a.

> Discriminant: D = b squared - 4ac. D > 0: real and different roots; D = 0: real and equal roots; D < 0: no real roots.

> Sum of roots = -b/a; product of roots = c/a.

Worked Example: Solve x squared - 5x + 6 = 0 by factorisation.

x squared - 5x + 6 = (x - 2)(x - 3) = 0, so x = 2 or x = 3.

Worked Example: Solve x squared + 4x + 3 = 0 by the quadratic formula.

x = (-4 +/- square root of (16 - 12)) / 2 = (-4 +/- 2) / 2, so x = -1 or x = -3.

Key Definitions

Term	Definition
Quadratic equation	An equation of the form a x squared + b x + c = 0, with a not equal to zero.
Root	A value of x that makes the equation true.
Discriminant	The expression b squared - 4ac, which decides the nature of the roots.
Quadratic formula	x = (-b plus or minus the square root of (b squared - 4ac)) divided by 2a.
Completing the square	Rewriting a quadratic as a perfect square plus a constant, so it can be solved by taking square roots.

Key Terms to Remember

Term	Meaning
Coefficient	The number multiplying a power of x, for example a and b.
Constant term	The number c, which has no x with it.
Factorisation	Writing an expression as a product of factors.
Perfect square	An expression of the form (x + k) squared.
Nature of roots	Whether the roots are real and different, real and equal, or not real.

Sum of roots	The sum of the two roots, equal to -b/a.
Product of roots	The product of the two roots, equal to c/a.
Solution set	The set of all values of x that satisfy the equation.

Multiple-Choice Questions

1. The roots of x squared - 9 = 0 are

(A) plus or minus 3(B) 3 only(C) plus or minus 9(D) 0

2. The discriminant of x squared - 5x + 6 = 0 is

(A) 1(B) 13(C) 25(D) 7

3. The sum of the roots of x squared + 3x - 4 = 0 is

(A) 3(B) -3(C) 4(D) -4

4. The product of the roots of 2x squared - 6x + 4 = 0 is

(A) 2(B) 4(C) 3(D) -3

5. If the discriminant is zero, the roots are

(A) real and equal(B) real and different(C) not real(D) zero

6. x squared + 2x + 1 = 0 has roots

(A) -1, -1(B) 1, 1(C) -1, 1(D) 0, 0

7. The quadratic formula is x =

(A) (-b +/- square root of (b squared - 4ac)) / 2a(B) (-b +/- square root of (b squared + 4ac)) / 2a(C) (-b +/- square root of (b squared - 4ac)) / a(D) (b +/- square root of (b squared - 4ac)) / 2a

8. The roots of x squared - 5x + 6 = 0 are

(A) 2 and 3(B) 1 and 6(C) -2 and -3(D) 5 and 1

9. If the roots of a quadratic equation are 2 and 5, the equation is

(A) x squared - 7x + 10 = 0(B) x squared + 7x + 10 = 0(C) x squared - 7x - 10 = 0(D) x squared + 7x - 10 = 0

10. If b squared - 4ac < 0, the roots are

(A) real(B) equal(C) not real (complex)(D) zero

Short Questions

1. What is a quadratic equation?
   A quadratic equation is an equation of the form a x squared + b x + c = 0, where a is not zero.
2. What is a root of an equation?
   A root is a value of x that makes the equation true.
3. What is the discriminant?
   The discriminant is b squared - 4ac, which decides whether the roots are real and different, real and equal, or not real.
4. How many roots can a quadratic equation have?
   At most two roots.
5. What is the quadratic formula?
   x = (-b plus or minus the square root of (b squared - 4ac)) divided by 2a.
6. What is the sum of the roots of a x squared + b x + c = 0?
   The sum of the roots is -b/a.
7. What is the product of the roots?
   The product of the roots is c/a.
8. When are the roots real and equal?
   When the discriminant is zero.
9. Solve x squared - 4 = 0.
   x squared = 4, so x = 2 or x = -2.
10. How is an equation solved by factorisation?
    Factorise the left side and set each factor equal to zero.

Long Questions

1. Solve a quadratic equation by completing the square, explaining each step.
2. Derive the quadratic formula and use it to solve a quadratic equation.
3. Explain how the discriminant decides the nature of the roots, and find the sum and product of the roots of a given quadratic equation.

CHAPTER 3

Matrices and Determinants

What is a Matrix?

A matrix is a rectangular arrangement of numbers in rows and columns, written inside square brackets. The order of a matrix is written as m by n, where m is the number of rows and n is the number of columns. A square matrix has the same number of rows and columns. Special matrices are the row matrix, column matrix, zero matrix, diagonal matrix and identity matrix, which has ones on the main diagonal and zeros elsewhere.

Addition, Subtraction and Multiplication

Two matrices can be added or subtracted only if they have the same order, by adding or subtracting the numbers in the same positions. A matrix can be multiplied by a number by multiplying every entry by that number. Two matrices A and B can be multiplied, in the order A times B, only if the number of columns of A equals the number of rows of B. The transpose of a matrix is got by turning its rows into columns.

Determinant, Inverse and Cramer's Rule

The determinant of a 2 by 2 matrix [[a, b], [c, d]] is ad - bc. A square matrix has an inverse only if its determinant is not zero; a matrix whose determinant is zero is called singular. The inverse of a 2 by 2 matrix is 1/(ad - bc) times [[d, -b], [-c, a]]. A pair of simultaneous linear equations can be solved by Cramer's rule, using determinants, or by multiplying both sides by the inverse of the coefficient matrix.

Key Formulae

> Determinant of [[a, b], [c, d]] = ad - bc.

> Inverse of [[a, b], [c, d]] = 1 / (ad - bc) x [[d, -b], [-c, a]], provided ad - bc is not zero.

> (A B) transpose = B transpose times A transpose; A inverse times A = I.

Worked Example: Multiply [[1, 2], [3, 4]] by [[2, 0], [1, 3]].

[[1x2 + 2x1, 1x0 + 2x3], [3x2 + 4x1, 3x0 + 4x3]] = [[4, 6], [10, 12]].

Worked Example: Find the determinant of [[2, 3], [1, 4]].

Determinant = 2x4 - 3x1 = 8 - 3 = 5.

Key Definitions

Term	Definition
Matrix	A rectangular arrangement of numbers in rows and columns.
Order	The size of a matrix, written as (number of rows) by (number of columns).
Square matrix	A matrix with the same number of rows and columns.
Identity matrix	A square matrix with ones on the main diagonal and zeros elsewhere.
Determinant	A number associated with a square matrix; for a 2 by 2 matrix [[a, b], [c, d]] it is ad - bc.
Inverse	A matrix A inverse such that A times A inverse = A inverse times A = the identity matrix.

Key Terms to Remember

Term	Meaning
Element (entry)	A single number in a matrix.
Row	A horizontal line of entries in a matrix.
Column	A vertical line of entries in a matrix.
Transpose	The matrix got by turning the rows of a matrix into columns.
Symmetric matrix	A square matrix that equals its own transpose.
Singular matrix	A square matrix whose determinant is zero; it has no inverse.
Non-singular matrix	A square matrix whose determinant is not zero; it has an inverse.
Cramer's rule	A method of solving simultaneous equations using determinants.

Multiple-Choice Questions

1. The order of a matrix with 3 rows and 4 columns is

(A) 4 by 3(B) 3 by 4(C) 12(D) 7

2. The determinant of [[2, 3], [1, 4]] is

(A) 5(B) 8(C) 11(D) 2

3. The identity matrix of order 2 is

(A) [[1, 0], [0, 1]](B) [[0, 1], [1, 0]](C) [[1, 1], [1, 1]](D) [[0, 0], [0, 0]]

4. The product A B is defined when

(A) the rows of A equal the columns of B(B) the columns of A equal the rows of B(C) A and B have the same order(D) A is square

5. The transpose of [[1, 2], [3, 4]] is

(A) [[1, 2], [3, 4]](B) [[1, 3], [2, 4]](C) [[4, 3], [2, 1]](D) [[2, 1], [4, 3]]

6. A matrix is singular if its determinant is

(A) 1(B) 0(C) -1(D) 2

7. (-1) times [[2, 3]] equals

(A) [[-2, -3]](B) [[2, 3]](C) [[2, -3]](D) [[-2, 3]]

8. The determinant of [[5, 0], [0, 5]] is

(A) 0(B) 10(C) 25(D) 5

9. The inverse of a matrix exists when its determinant is

(A) zero(B) non-zero(C) equal to 1(D) negative

10. A + A transpose is always

(A) symmetric(B) skew-symmetric(C) the identity matrix(D) the zero matrix

Short Questions

1. What is a matrix?
   A matrix is a rectangular arrangement of numbers in rows and columns, written inside square brackets.
2. What is the order of a matrix?
   The order is written as m by n, where m is the number of rows and n is the number of columns.
3. When can two matrices be added?
   Only when they have the same order.

4. When can two matrices be multiplied?
   When the number of columns of the first equals the number of rows of the second.
5. What is the transpose of a matrix?
   The transpose is got by turning the rows of the matrix into columns.
6. What is a singular matrix?
   A square matrix whose determinant is zero; it has no inverse.
7. What is the determinant of [[a, b], [c, d]]?
   It is ad - bc.
8. What is an identity matrix?
   A square matrix with ones on the main diagonal and zeros elsewhere.
9. What is the inverse of a matrix?
   A matrix A inverse such that A times A inverse = A inverse times A = the identity matrix.
10. What is Cramer's rule used for?
    It is a method of solving simultaneous linear equations using determinants.

Long Questions

1. Explain matrix addition, subtraction and multiplication, with the conditions under which each is possible.
2. Explain the determinant and the inverse of a 2 by 2 matrix, and find the inverse of a given matrix.
3. Solve a pair of simultaneous linear equations by Cramer's rule (or by the inverse-matrix method).

CHAPTER 4

Linear and Quadratic Inequalities

Inequalities and the Number Line

An inequality is a statement that one quantity is greater than, less than, greater than or equal to, or less than or equal to another. The solution of an inequality is a set of values, usually shown on a number line. On a number line, a closed (filled) circle means the endpoint is included, as in x greater than or equal to 5, and an open circle means it is not included, as in $x < 5$. A solution set can also be written in interval notation.

Solving Linear Inequalities

A linear inequality is solved in the same way as a linear equation, with one important rule:

when both sides are multiplied or divided by a negative number, the inequality sign is reversed. Adding or subtracting the same number from both sides does not change the sign. The solution of a linear inequality is usually an interval, such as x < 2.

Quadratic Inequalities

A quadratic inequality, such as x squared - 5x + 6 > 0, is solved by first factorising the quadratic and finding its critical values, the points where it equals zero. These values divide the number line into intervals, and the sign of the expression is tested in each interval. For (x - 2)(x - 3) > 0, the solution is x < 2 or x > 3. A quadratic inequality can also be solved by looking at the graph of the parabola.

Key Formulae

> If a < b then a + c < b + c and a - c < b - c.

> If a < b and c < 0 then ac > bc (the sign reverses).

> |x| < a means -a < x < a; |x| > a means x < -a or x > a.

Worked Example: Solve 2x + 3 < 7.

2x < 4, so x < 2.

Worked Example: Solve x squared - 5x + 6 > 0.

Factorising gives (x - 2)(x - 3) > 0, which is true when both factors are positive or

both negative, so x < 2 or x > 3.

Key Definitions

Term	Definition
Inequality	A statement that one quantity is greater than, less than, or equal to another in a non-equal sense.
Solution set	The set of all values that satisfy an inequality.
Interval	A set of numbers between two endpoints, written for example as (2, 5)

	or [2, 5].
Number line	A line on which numbers are shown as points, used to show solution sets.
Absolute value	The distance of a number from zero; |x| is x if x is not negative, and -x if x is negative.

Key Terms to Remember

Term	Meaning
Strict inequality	An inequality using < or >, which does not include the endpoint.
Non-strict inequality	An inequality using less than or equal to, or greater than or equal to, which includes the endpoint.
Interval notation	Writing a solution set with brackets, for example (2, 5) or [2, 5].
Open interval	An interval that does not include its endpoints, written with round brackets.
Closed interval	An interval that includes its endpoints, written with square brackets.
Critical value	A value where an expression equals zero; it divides the number line into intervals.
Boundary point	An endpoint of a solution interval.
Solution region	The part of the number line, or plane, that satisfies an inequality.

Multiple-Choice Questions

1. The solution of x + 5 < 8 is

(A) x < 3(B) x > 3(C) x < 13(D) x > 13

2. If x < 3 then -2x is

(A) greater than -6(B) less than -6(C) greater than 6(D) less than 6

3. The solution of 3x greater than or equal to 12 is

(A) x greater than or equal to 4(B) x less than or equal to 4(C) x greater than or equal to 36(D) x less than or equal to 36

4. |x| < 2 means

(A) -2 < x < 2(B) x < 2(C) x > -2(D) x = plus or minus 2

5. |x| > 3 means

(A) x > 3 or x < -3(B) -3 < x < 3(C) x = 3(D) x < 3

6. The inequality x greater than or equal to 5 is shown on the number line with

(A) an open circle at 5(B) a closed circle at 5(C) no circle(D) two circles

7. x squared - 9 > 0 has solution

(A) -3 < x < 3(B) x < -3 or x > 3(C) x > 3 only(D) x < 3

8. The solution of 2x - 1 less than or equal to 5 is

(A) x less than or equal to 3(B) x less than or equal to 2(C) x greater than or equal to 3(D) x less than or equal to 6

9. If a < b and c > 0 then ac is

(A) less than bc(B) greater than bc(C) equal to bc(D) less than or equal to bc

10. The solution set of (x - 1)(x - 2) < 0 is

(A) 1 < x < 2(B) x < 1 or x > 2(C) x = 1(D) x = 2

Short Questions

1. What is an inequality?
   An inequality is a statement that one quantity is greater than, less than, greater than or equal to, or less than or equal to another.
2. What does a closed circle on a number line mean?
   A closed circle means the endpoint is included in the solution set.
3. What happens to the inequality sign when both sides are multiplied by a negative number?
   The inequality sign is reversed.
4. Solve x - 4 > 1.
   x > 5.
5. What is a solution set?
   The solution set is the set of all values that satisfy an inequality.
6. What is |x| < a equivalent to?
   |x| < a means -a < x < a.
7. What is |x| > a equivalent to?
   |x| > a means x < -a or x > a.
8. What is a critical value?
   A critical value is a value where an expression equals zero; it divides the number line

into intervals.

9. Solve $(x - 2)(x - 3) < 0$.
   The product is negative between the critical values, so $2 < x < 3$.

10. What is interval notation?
    Interval notation writes a solution set with brackets, for example (2, 5) means all numbers greater than 2 and less than 5.

Long Questions

1. Explain how linear inequalities are solved, and solve two linear inequalities, showing the solution on a number line.
2. Explain how a quadratic inequality is solved by factorisation and the sign method, with an example.
3. Explain absolute-value inequalities and solve two examples.

CHAPTER 5

Algebraic Fractions

Rational Expressions and Simplification

An algebraic fraction is a fraction whose numerator and denominator are algebraic expressions, also called a rational expression. To simplify one, factorise the numerator and the denominator and cancel any common factors. A rational expression is in lowest terms when its numerator and denominator have no common factor except 1. An algebraic fraction is undefined where its denominator is zero. (Some books call this chapter Algebraic Functions; the content is the same.)

Operations on Algebraic Fractions

Algebraic fractions are added or subtracted by first finding a common denominator, usually the lowest common multiple of the denominators, then adding or subtracting the numerators. To multiply algebraic fractions, multiply the numerators together and the denominators together, then simplify. To divide, multiply by the reciprocal of the divisor.

Partial Fractions and Rational Equations

A complicated algebraic fraction can sometimes be written as a sum of simpler fractions, called partial fractions. For example, a fraction with denominator $(x - a)(x - b)$ can be written as $A/(x - a) + B/(x - b)$, where A and B are found by comparing numerators. A rational equation is an equation containing algebraic fractions; it is solved by multiplying both sides by the lowest common multiple of the denominators, and the answer must be

checked because a value that makes a denominator zero is not allowed.

Key Formulae

- a/b divided by c/d = a/b x d/c.
- Partial fractions: (px + q) / ((x - a)(x - b)) = A/(x - a) + B/(x - b).
- 1/x + 1/y = (x + y) / xy.

Worked Example: Simplify (x squared - 4) / (x + 2).

x squared - 4 = (x - 2)(x + 2), so the fraction simplifies to x - 2, for x not equal to -2.

Worked Example: Add 1/(x - 1) + 1/(x + 1).

The common denominator is x squared - 1, so the sum is ((x + 1) + (x - 1)) / (x

squared - 1) = 2x / (x squared - 1).

Key Definitions

Term	Definition
Rational expression	A fraction whose numerator and denominator are algebraic expressions.
Lowest terms	A fraction is in lowest terms when its numerator and denominator have no common factor except 1.
Partial fraction	One of the simpler fractions into which a complicated fraction is split.
LCM	The lowest common multiple of two or more expressions.
HCF	The highest common factor of two or more expressions.

Key Terms to Remember

Term	Meaning

Numerator	The top part of a fraction.
Denominator	The bottom part of a fraction.
Factor	An expression that divides another expression exactly.
Common factor	A factor shared by two or more expressions.
Lowest terms	A fraction with no common factor in numerator and denominator except 1.
Complex fraction	A fraction that itself contains fractions in its numerator or denominator.
Rational equation	An equation containing algebraic fractions.
Extraneous solution	A value found by solving that makes a denominator zero and must be rejected.

Multiple-Choice Questions

1. (x squared - 9) / (x - 3) simplifies to

(A) x + 3(B) x - 3(C) x + 9(D) 3

2. (2x) / (x squared) simplifies to

(A) 2/x(B) 2x(C) x/2(D) 2

3. 1/x + 1/y equals

(A) (x + y) / xy(B) 2 / (x + y)(C) 1 / xy(D) (x - y) / xy

4. The LCM of x and x + 1 is

(A) x(x + 1)(B) x + 1(C) x squared(D) 1

5. (x squared - 1) / (x + 1) equals

(A) x - 1(B) x + 1(C) x squared(D) 1

6. (3/4) divided by (1/2) equals

(A) 3/2(B) 3/8(C) 2/3(D) 6

7. (a squared - b squared) / (a - b) equals

(A) a + b(B) a - b(C) ab(D) a squared + b squared

8. The expression (x + 2) / (x - 2) is undefined at x =

(A) 2(B) -2(C) 0(D) 1

9. (x squared + 5x + 6) / (x + 2) equals

(A) x + 3(B) x + 2(C) x + 6(D) x + 1

10. 1/(x - 1) - 1/(x + 1) equals

(A) 2 / (x squared - 1)(B) 1 / (x squared - 1)(C) 2x / (x squared - 1)(D) 0

Short Questions

1. What is a rational expression?
   A rational expression is a fraction whose numerator and denominator are algebraic expressions.
2. How is an algebraic fraction simplified?
   Factorise the numerator and the denominator and cancel any common factors.
3. When is an algebraic fraction undefined?
   Where its denominator is zero.
4. How are algebraic fractions added?
   Find a common denominator, usually the LCM, then add the numerators.
5. How are algebraic fractions divided?
   Multiply by the reciprocal of the divisor.
6. What are partial fractions?
   Partial fractions are the simpler fractions into which a complicated fraction is split.
7. What is the LCM of two expressions?
   The lowest common multiple: the smallest expression divisible by both.
8. Simplify (x squared - 1) / (x - 1).
   x squared - 1 = (x - 1)(x + 1), so the fraction simplifies to x + 1, for x not equal to 1.
9. What is a rational equation?
   An equation containing algebraic fractions.
10. Why must answers to a rational equation be checked?
    Because a value that makes a denominator zero is not allowed and must be rejected.

Long Questions

1. Explain how algebraic fractions are simplified, added and subtracted, with examples.
2. Explain partial fractions and split a given fraction into partial fractions.
3. Solve a rational equation, showing the checking step.

CHAPTER 6

Functions and Graphs

What is a Function?

A function is a rule that assigns to each input exactly one output. The set of all inputs is called the domain and the set of all outputs is called the range. A function can be shown as a set of ordered pairs, a table, a formula, or a graph. A graph shows a function if every vertical line cuts it at most once; this is the vertical line test.

Types of Functions and Inverse Functions

A function is one-one (injective) if different inputs always give different outputs. It is onto (surjective) if every element of the codomain is an output. A function that is both one-one and onto is called bijective. The inverse of a bijective function f, written f inverse, undoes what f does: if f(a) = b then f inverse (b) = a.

Composite Functions and Graphs

The composite function (f of g)(x) is f(g(x)): first apply g, then apply f. The graph of a linear function y = mx + c is a straight line with slope m and y-intercept c. The graph of a quadratic function y = a x squared + b x + c is a parabola, which opens upward when a is positive and downward when a is negative.

Key Formulae

- (f of g)(x) = f(g(x)).
- If f(x) = ax + b then f inverse (x) = (x - b) / a.
- Linear function: y = mx + c (m = slope, c = y-intercept).

Worked Example: If f(x) = 2x + 3 and g(x) = x - 1, find (f of g)(x).

(f of g)(x) = f(g(x)) = f(x - 1) = 2(x - 1) + 3 = 2x + 1.

Worked Example: Find the inverse of f(x) = x + 5.

Let y = x + 5, then x = y - 5, so f inverse (x) = x - 5.

Key Definitions

Term	Definition
Function	A rule that assigns to each input exactly one output.
Domain	The set of all inputs of a function.
Range	The set of all outputs of a function.
One-one (injective)	A function in which different inputs always give different outputs.
Onto (surjective)	A function whose range equals its codomain.
Inverse function	The function that undoes another function; if f(a) = b then f inverse (b) = a.
Composite function	The function (f of g)(x) = f(g(x)).

Key Terms to Remember

Term	Meaning
Mapping	A rule that sends each element of one set to an element of another set.
Ordered pair	A pair (x, y) in which the order matters.
Codomain	The set from which the outputs of a function are taken.
Constant function	A function that always gives the same output.
Identity function	The function f(x) = x.
Linear function	A function of the form f(x) = mx + c, whose graph is a straight line.
Quadratic function	A function of the form f(x) = a x squared + b x + c, whose graph is a parabola.
Graph	The set of points (x, f(x)) drawn on a plane.

Multiple-Choice Questions

1. A relation is a function if

(A) each input has exactly one output(B) each output has one input(C) it has two

outputs(D) it is a straight line

2. The domain of f(x) = x + 1 is

(A) all real numbers(B) x > 0(C) x not equal to 0(D) x greater than or equal to 1

3. The range of f(x) = x squared is

(A) all real numbers(B) y greater than or equal to 0(C) y less than or equal to 0(D) y > 1

4. If f(x) = 3x - 2, then f(2) equals

(A) 4(B) 6(C) 8(D) 2

5. The inverse of f(x) = x + 5 is

(A) x - 5(B) 5 - x(C) 5x(D) x/5

6. A one-one function is also called

(A) injective(B) surjective(C) constant(D) identity

7. (f of g)(x) means

(A) f(x) + g(x)(B) f(g(x))(C) g(f(x))(D) f(x) times g(x)

8. If f(x) = 2x and g(x) = x + 1, then (g of f)(2) equals

(A) 5(B) 6(C) 4(D) 3

9. The graph of y = x squared is a

(A) line(B) parabola(C) circle(D) hyperbola

10. A function whose range equals its codomain is

(A) one-one(B) onto(C) into(D) constant

Short Questions

1. What is a function?
   A function is a rule that assigns to each input exactly one output.
2. What is the domain of a function?
   The domain is the set of all inputs of the function.
3. What is the range of a function?
   The range is the set of all outputs of the function.
4. What is the vertical line test?
   A graph shows a function if every vertical line cuts it at most once.
5. What is a one-one function?
   A function in which different inputs always give different outputs.

6. What is an onto function?
   A function whose range equals its codomain.
7. What is an inverse function?
   The inverse of f undoes what f does: if f(a) = b then f inverse (b) = a.
8. What is a composite function?
   The composite function (f of g)(x) = f(g(x)): first apply g, then apply f.
9. What is the graph of a linear function?
   The graph of y = mx + c is a straight line with slope m and y-intercept c.
10. What is the graph of a quadratic function?
    The graph of y = a x squared + b x + c is a parabola.

Long Questions

1. Explain what a function is, with the domain and range, and test whether given relations are functions.
2. Explain one-one, onto and bijective functions, and find the inverse of a given function.
3. Explain composite functions and sketch the graphs of a linear function and a quadratic function.

CHAPTER 7

Vectors in Plane

What is a Vector?

A vector is a quantity that has both magnitude (size) and direction, such as displacement, velocity or force. A quantity with magnitude only, such as speed or mass, is a scalar. A vector is shown by an arrow, whose length shows the magnitude and whose direction shows the direction of the vector. In the plane, a vector is written as a pair (x, y) or as x i + y j, where i and j are unit vectors along the x and y axes.

Addition, Subtraction and Scalar Multiplication

Vectors are added by the triangle law or the parallelogram law: place the tail of the second vector at the head of the first, and the sum is the vector from the start to the end. Subtraction is adding the negative of a vector. Multiplying a vector by a scalar multiplies its magnitude and keeps the direction if the scalar is positive, or reverses it if the scalar is negative.

Components, Unit Vectors and the Dot Product

The components of a vector (x, y) are its projections on the x and y axes, and its magnitude is the square root of (x squared + y squared). A unit vector has magnitude 1; the unit vector in the direction of a vector a is a divided by |a|. The dot product (scalar product) of two vectors a and b is |a| |b| cos theta, where theta is the angle between them; it is zero when the vectors are perpendicular.

Key Formulae

- For a = (x, y): |a| = square root of (x squared + y squared).
- Unit vector in the direction of a: a hat = a / |a|.
- Dot product: a . b = |a| |b| cos theta; a . b = 0 when a and b are perpendicular.

Worked Example: Find the magnitude and unit vector of a = (3, 4).

|a| = square root of (9 + 16) = 5. The unit vector is (3/5, 4/5).

Worked Example: Add a = (1, 2) and b = (3, 4).

a + b = (1 + 3, 2 + 4) = (4, 6).

Key Definitions

Term	Definition
Vector	A quantity that has both magnitude and direction.
Scalar	A quantity that has magnitude only.
Magnitude	The size or length of a vector.
Direction	The line and sense along which a vector acts.
Unit vector	A vector of magnitude 1.
Position vector	The vector from the origin to a point.

Key Terms to Remember

Term	Meaning
Equal vectors	Vectors with the same magnitude and the same direction.
Negative vector	A vector with the same magnitude but opposite direction.
Zero vector	A vector of magnitude zero; it has no direction.
Parallel vectors	Vectors that act along the same line or parallel lines.
Collinear vectors	Vectors that lie on the same line.
Resultant	The vector got by adding two or more vectors.
Component	The projection of a vector on an axis.
Dot product	The scalar product of two vectors, equal to |a| |b| cos theta.

Multiple-Choice Questions

1. The magnitude of the vector (3, 4) is

(A) 5(B) 7(C) 25(D) 12

2. A vector has

(A) magnitude only(B) direction only(C) magnitude and direction(D) neither

3. The unit vector in the direction of (3, 4) is

(A) (3/5, 4/5)(B) (3, 4)(C) (5, 5)(D) (1, 1)

4. If a = (1, 2) and b = (3, 4), then a + b equals

(A) (4, 6)(B) (2, 2)(C) (3, 8)(D) (4, 2)

5. The zero vector has magnitude

(A) 1(B) 0(C) -1(D) infinite

6. Scalar multiplication of 2 by (3, -1) gives

(A) (6, -2)(B) (5, 1)(C) (3, -2)(D) (6, 1)

7. Two vectors are equal if they have the same

(A) magnitude only(B) direction only(C) magnitude and direction(D) origin

8. The dot product of (1, 0) and (0, 1) is

(A) 1(B) 0(C) -1(D) 2

9. The magnitude of (6, 8) is

(A) 10(B) 14(C) 48(D) 100

10. The position vector of the point (2, 3) is

(A) 2i + 3j(B) 3i + 2j(C) 2i - 3j(D) 5

Short Questions

1. What is a vector?
   A vector is a quantity that has both magnitude and direction.
2. What is a scalar?
   A scalar is a quantity that has magnitude only, such as speed or mass.
3. How is a vector shown?
   A vector is shown by an arrow, whose length shows the magnitude and whose direction shows the direction.
4. What is the magnitude of (x, y)?
   The magnitude is the square root of (x squared + y squared).
5. What is a unit vector?
   A unit vector is a vector of magnitude 1.
6. How are vectors added?
   By the triangle law or the parallelogram law: place the tail of the second at the head of the first and join the start to the end.
7. What is a position vector?
   The position vector of a point is the vector from the origin to that point.
8. What is the dot product of two vectors?
   The dot product of a and b is |a| |b| cos theta, where theta is the angle between them.
9. When is the dot product zero?
   When the two vectors are perpendicular.
10. What is a zero vector?
    A vector of magnitude zero; it has no direction.

Long Questions

1. Explain vectors, scalars and the magnitude of a vector, with examples from daily life.
2. Explain the addition and subtraction of vectors by the triangle law and the parallelogram law.
3. Explain components, unit vectors and the dot product of two vectors.

CHAPTER 8

Application of Trigonometry

Trigonometric Ratios and Identities

In a right-angled triangle, the sine of an angle is the opposite side divided by the hypotenuse, the cosine is the adjacent side divided by the hypotenuse, and the tangent is the opposite side divided by the adjacent side. The main trigonometric identities are sin squared theta + cos squared theta = 1, tan theta = sin theta / cos theta, and 1 + tan squared theta = sec squared theta. The ratios of the special angles 30, 45 and 60 degrees are used in most problems.

Angle of Elevation and Angle of Depression

The angle of elevation is the angle between the horizontal and the line of sight to an object above the observer. The angle of depression is the angle between the horizontal and the line of sight to an object below the observer. The angle of elevation of an object from the ground equals the angle of depression of the observer from the object, because they are alternate angles.

Heights, Distances and Bearings

Trigonometry is used to find heights and distances that cannot be measured directly, such as the height of a tower or the width of a river. The method is to draw a right-angled triangle from the given information, choose the ratio that links the known and unknown sides, and solve. A bearing is the direction of one point from another, measured clockwise from north, and it is also found with trigonometry.

Key Formulae

> sin squared theta + cos squared theta = 1.

> tan theta = sin theta / cos theta; 1 + tan squared theta = sec squared theta.

> tan 45 degrees = 1; sin 30 degrees = 1/2; cos 60 degrees = 1/2; sin 60 degrees = square root of 3 / 2.

Worked Example: A ladder 5 m long leans against a wall at 60 degrees to the ground. How high does it reach?

Height = 5 x sin 60 degrees = 5 x (square root of 3 / 2), about 4.33 m.

Worked Example: The shadow of a tower is 10 m when the angle of elevation of the sun is 45 degrees. Find the height of the tower.

tan 45 degrees = height / 10, and tan 45 degrees = 1, so the height is 10 m.

Key Definitions

Term	Definition
Angle of elevation	The angle between the horizontal and the line of sight to an object above the observer.
Angle of depression	The angle between the horizontal and the line of sight to an object below the observer.
Bearing	The direction of one point from another, measured clockwise from north.
Trigonometric ratio	A ratio of two sides of a right-angled triangle, such as sin, cos or tan.
Identity	An equation that is true for all values of the angle, such as sin squared theta + cos squared theta = 1.

Key Terms to Remember

Term	Meaning
Hypotenuse	The side opposite the right angle in a right-angled triangle.
Opposite side	The side opposite a given angle.

Adjacent side	The side next to a given angle, other than the hypotenuse.
Elevation	The height of an object above the horizontal.
Depression	The depth of an object below the horizontal.
Bearing	A direction measured clockwise from north.
Complementary angles	Two angles that add up to 90 degrees.
Reference angle	The acute angle a line makes with the x-axis.

Multiple-Choice Questions

1. sin squared theta + cos squared theta equals

(A) 0(B) 1(C) 2(D) tan theta

2. tan 45 degrees equals

(A) 0(B) 1/2(C) 1(D) square root of 3

3. sin 30 degrees equals

(A) 1/2(B) square root of 3 / 2(C) 1(D) square root of 2 / 2

4. The angle of elevation is measured from the

(A) vertical(B) horizontal(C) diagonal(D) circle

5. If tan theta = 1 then theta equals

(A) 30 degrees(B) 45 degrees(C) 60 degrees(D) 90 degrees

6. cos 60 degrees equals

(A) 1/2(B) square root of 3 / 2(C) 1(D) 0

7. In a right triangle, sin theta equals

(A) opposite / hypotenuse(B) adjacent / hypotenuse(C) opposite / adjacent(D) hypotenuse / opposite

8. The angle of depression of an object equals the angle of

(A) elevation of the observer from the object(B) 90 degrees(C) 0 degrees(D) depression of the observer

9. If the height of a tower is 10 m and its shadow is 10 m, the angle of elevation of the sun is

(A) 30 degrees(B) 45 degrees(C) 60 degrees(D) 90 degrees

10. sec squared theta - tan squared theta equals

(A) 0(B) 1(C) 2(D) sin theta

Short Questions

1. What is sin theta in a right-angled triangle?
   sin theta is the opposite side divided by the hypotenuse.
2. What is cos theta?
   cos theta is the adjacent side divided by the hypotenuse.
3. What is tan theta?
   tan theta is the opposite side divided by the adjacent side.
4. What is the identity sin squared theta + cos squared theta = 1?
   It is a trigonometric identity, true for all values of theta.
5. What is the angle of elevation?
   The angle between the horizontal and the line of sight to an object above the observer.
6. What is the angle of depression?
   The angle between the horizontal and the line of sight to an object below the observer.
7. What is tan 45 degrees?
   tan 45 degrees = 1.
8. What is sin 30 degrees?
   sin 30 degrees = 1/2.
9. What is a bearing?
   A bearing is the direction of one point from another, measured clockwise from north.
10. How is the height of a tower found by trigonometry?
    Draw a right-angled triangle from the given information, choose the ratio linking the known and unknown sides, and solve.

Long Questions

1. Explain the trigonometric ratios and identities, and prove sin squared theta + cos squared theta = 1.
2. Explain the angles of elevation and depression and use them to find the height of a tower.
3. Solve a problem on heights and distances using trigonometry, with a diagram

described in words.

CHAPTER 9

Chord and Arcs of a Circle

Parts of a Circle

A circle is the set of all points at the same distance, the radius, from a fixed point, the centre. A chord is a line segment joining two points on a circle; the longest chord is the diameter, which passes through the centre. An arc is a part of the circumference; a minor arc is less than a semicircle and a major arc is more than a semicircle. A sector is the region between two radii and an arc, and a segment is the region between a chord and its arc.

Properties of Chords

The perpendicular from the centre of a circle to a chord bisects the chord. Equal chords of a circle are equidistant from the centre, and chords that are equidistant from the centre are equal. The perpendicular bisector of a chord passes through the centre of the circle. These properties are used to find the centre of a circle and to solve problems about chords.

Arc Length, Sector Area and the Angle at the Centre

The angle at the centre of a circle is twice the angle at the circumference standing on the same arc. The length of an arc with central angle theta is (theta / 360) times the circumference, 2 pi r. The area of the sector with central angle theta is (theta / 360) times the area of the circle, pi r squared.

Key Formulae

> Arc length = (theta / 360 degrees) x 2 pi r.

> Sector area = (theta / 360 degrees) x pi r squared.

> Circumference = 2 pi r; area of a circle = pi r squared.

Worked Example: A circle has radius 7 cm and a sector with central angle 90 degrees. Find the arc length.

Arc length = (90/360) x 2 x (22/7) x 7 = 11 cm.

Worked Example: Find the circumference of a circle of radius 7 cm (take pi = 22/7).

Circumference = 2 x (22/7) x 7 = 44 cm.

Key Definitions

Term	Definition
Chord	A line segment joining two points on a circle.
Arc	A part of the circumference of a circle.
Sector	The region between two radii and the arc joining them.
Segment	The region between a chord and its arc.
Central angle	The angle at the centre of a circle, between two radii.
Radius	The distance from the centre of a circle to any point on it.
Diameter	A chord through the centre; it is twice the radius.

Key Terms to Remember

Term	Meaning
Minor arc	An arc less than a semicircle.
Major arc	An arc more than a semicircle.

Semicircle	Half of a circle, cut by a diameter.
Concentric circles	Circles with the same centre but different radii.
Secant	A line that cuts a circle at two points.
Tangent	A line that touches a circle at exactly one point.
Centre	The fixed point from which all points of a circle are at the same distance.
Circumference	The length of the boundary of a circle.

Multiple-Choice Questions

1. A chord is a line segment joining

(A) two points on a circle(B) the centre to a point on the circle(C) two centres(D) two tangents

2. The longest chord of a circle is the

(A) radius(B) diameter(C) arc(D) sector

3. The length of an arc with central angle theta is

(A) (theta / 360) x 2 pi r(B) theta x r(C) 2 pi r(D) pi r squared

4. The perpendicular from the centre to a chord

(A) bisects the chord(B) is the longest line(C) is a tangent(D) makes 45 degrees

5. The area of a sector with central angle theta is

(A) (theta / 360) x pi r squared(B) (theta / 360) x 2 pi r(C) pi r squared(D) 2 pi r squared

6. A minor arc is

(A) less than a semicircle(B) more than a semicircle(C) a full circle(D) a diameter

7. The circumference of a circle of radius 7 cm (pi = 22/7) is

(A) 44 cm(B) 22 cm(C) 154 square cm(D) 14 cm

8. Equal chords of a circle are equidistant from the

(A) circumference(B) centre(C) tangent(D) diameter

9. The angle at the centre is

(A) half(B) equal to(C) twice(D) a quarter of

10. The area of a circle of radius 7 cm is

(A) 154 square cm(B) 44 cm(C) 22 square cm(D) 49 cm

Short Questions

1. What is a chord?
   A chord is a line segment joining two points on a circle.
2. What is the longest chord of a circle?
   The diameter, which passes through the centre.
3. What is an arc?
   An arc is a part of the circumference of a circle.
4. What is a sector?
   A sector is the region between two radii and the arc joining them.
5. What is a segment?
   A segment is the region between a chord and its arc.
6. What does the perpendicular from the centre to a chord do?
   It bisects the chord.
7. How is arc length found?
   Arc length = (theta / 360) x 2 pi r, where theta is the central angle.
8. How is sector area found?
   Sector area = (theta / 360) x pi r squared.
9. How are the angle at the centre and the angle at the circumference related?
   The angle at the centre is twice the angle at the circumference on the same arc.
10. What is the circumference of a circle?
    The circumference is 2 pi r.

Long Questions

1. Explain the parts of a circle: chord, arc, sector, segment, radius and diameter.
2. Prove that the perpendicular from the centre of a circle to a chord bisects the chord.
3. Find the arc length and sector area of a circle for a given radius and central angle.

CHAPTER 10

Tangents and Angles of a Circle

Tangent to a Circle

A tangent is a line that touches a circle at exactly one point, called the point of contact.

The tangent is perpendicular to the radius at the point of contact. The two tangents drawn from an external point to a circle are equal in length. If the radius is r and the distance from the centre to the external point is d, the length of the tangent is the square root of (d squared - r squared).

Angles in a Circle

The angle in a semicircle is a right angle. Angles in the same segment of a circle are equal. In a cyclic quadrilateral, a quadrilateral whose vertices lie on a circle, the opposite angles are supplementary, meaning they add up to 180 degrees. The angle at the centre is twice the angle at the circumference on the same arc.

The Alternate Segment Theorem

The alternate segment theorem says that the angle between a tangent and a chord at the point of contact is equal to the angle in the alternate segment, that is, the angle in the segment opposite to the angle. This theorem links tangents with angles in a circle and is used to solve many geometry problems.

Key Formulae

> Tangent is perpendicular to the radius at the point of contact.

> The two tangents from an external point are equal in length.

> Angle in a semicircle = 90 degrees; opposite angles of a cyclic quadrilateral add up to 180 degrees.

Worked Example: The radius of a circle is 3 cm and the distance from the centre to an external point P is 5 cm. Find the length of the tangent from P.

Tangent length = square root of (25 - 9) = square root of 16 = 4 cm.

Worked Example: An angle at the centre of a circle is 80 degrees. What is the angle at the circumference on the same arc?

The angle at the centre is twice the angle at the circumference, so the angle at the

circumference is 40 degrees.

Key Definitions

Term	Definition
Tangent	A line that touches a circle at exactly one point.
Point of contact	The point at which a tangent touches a circle.
Cyclic quadrilateral	A quadrilateral whose four vertices lie on a circle.
Alternate segment	The segment of a circle opposite to the angle between a tangent and a chord.
Semicircle	Half of a circle, cut by a diameter.

Key Terms to Remember

Term	Meaning
Secant	A line that cuts a circle at two points.
Tangent length	The length of the segment of a tangent from an external point to the point of contact.
Inscribed angle	An angle whose vertex lies on the circle.
Central angle	An angle whose vertex is the centre of the circle.
Opposite angles	Angles facing each other in a quadrilateral.
Supplementary angles	Two angles that add up to 180 degrees.
Segment	A region of a circle cut off by a chord.
Chord	A line segment joining two points on a circle.

Multiple-Choice Questions

1. A tangent to a circle is perpendicular to the radius at the

(A) centre(B) point of contact(C) chord(D) arc

2. The angle in a semicircle is

(A) 45 degrees(B) 60 degrees(C) 90 degrees(D) 180 degrees

3. Two tangents drawn from an external point to a circle are

(A) unequal(B) equal(C) parallel(D) perpendicular

4. Angles in the same segment of a circle are

(A) equal(B) supplementary(C) complementary(D) different

5. In a cyclic quadrilateral, opposite angles are

(A) equal(B) supplementary(C) complementary(D) 90 degrees

6. If the radius is 3 cm and the distance from the centre to an external point is 5 cm, the tangent length is

(A) 4 cm(B) 8 cm(C) 2 cm(D) square root of 34 cm

7. The alternate segment theorem relates the angle between a tangent and a chord to the angle in the

(A) same segment(B) alternate segment(C) centre(D) semicircle

8. A line that touches a circle at exactly one point is a

(A) secant(B) chord(C) tangent(D) diameter

9. The maximum number of tangents from a point outside a circle is

(A) 1(B) 2(C) 3(D) 4

10. If an angle at the centre is 80 degrees, the angle at the circumference on the same arc is

(A) 80 degrees(B) 40 degrees(C) 160 degrees(D) 20 degrees

Short Questions

1. What is a tangent?
   A tangent is a line that touches a circle at exactly one point.
2. What is the point of contact?
   The point at which a tangent touches a circle.
3. What is the angle in a semicircle?
   The angle in a semicircle is a right angle.
4. What are angles in the same segment?
   Angles in the same segment of a circle are equal.
5. What is a cyclic quadrilateral?
   A quadrilateral whose four vertices lie on a circle.

6. What is true of the opposite angles of a cyclic quadrilateral?
   They are supplementary: they add up to 180 degrees.
7. What is the alternate segment theorem?
   The angle between a tangent and a chord at the point of contact equals the angle in the alternate segment.
8. How many tangents can be drawn from an external point?
   Two, and they are equal in length.
9. What is the relation between the angle at the centre and the angle at the circumference?
   The angle at the centre is twice the angle at the circumference on the same arc.
10. If the radius is 5 cm and the tangent length is 12 cm, what is the distance from the centre to the external point?
    By Pythagoras: square root of (25 + 144) = 13 cm.

Long Questions

1. Explain the tangent to a circle and its properties, including the two tangents from an external point.
2. Prove that the angle in a semicircle is a right angle, and explain angles in the same segment.
3. State and explain the alternate segment theorem, with a diagram described in words.

CHAPTER 11

Practical Geometry of Circles

Constructions with Compass and Ruler

Practical geometry is the drawing of geometric figures with a compass and a ruler (straightedge) only. A compass draws circles and arcs, and a ruler draws straight lines. Every construction is built from a few basic steps: bisecting a line segment, bisecting an angle, and drawing a perpendicular. Constructions must be drawn neatly, with construction arcs left visible.

Tangents from a Point

To draw a tangent at a point P on a circle, join the centre O to P, then draw a line through P perpendicular to OP; this line is the tangent at P. To draw tangents from an external point P, join P to the centre O, draw a circle with OP as diameter, and the points where this circle cuts the given circle are the points of contact; join P to these points for the two

tangents.

Circumscribed and Inscribed Circles

The circumscribed circle of a triangle passes through all three vertices, and its centre, the circumcentre, is the point where the perpendicular bisectors of the sides meet. The inscribed circle of a triangle touches all three sides, and its centre, the incentre, is the point where the angle bisectors meet. The radius of the inscribed circle is called the inradius, and the radius of the circumscribed circle is called the circumradius.

Key Formulae

> Circumcentre = intersection of the perpendicular bisectors of the sides of the triangle.

> Incentre = intersection of the angle bisectors of the triangle.

> Tangent at a point P on a circle: the line through P perpendicular to the radius OP.

Worked Example: Draw a tangent at a point P on a circle with centre O.

Join O to P. With centre P, draw a line perpendicular to OP. This line is the tangent at P.

Worked Example: Find the circumcentre of a triangle.

Draw the perpendicular bisectors of two sides of the triangle; their meeting point is the circumcentre. A circle with this centre and radius equal to the distance to a vertex passes through all three vertices.

Key Definitions

Term	Definition
Construction	The drawing of a geometric figure using only a compass and a ruler.

Circumcentre	The centre of the circumscribed circle of a triangle; the intersection of the perpendicular bisectors of the sides.
Incentre	The centre of the inscribed circle of a triangle; the intersection of the angle bisectors.
Circumscribed circle	A circle that passes through all the vertices of a figure.
Inscribed circle	A circle inside a figure that touches all its sides.
Perpendicular bisector	A line that cuts a segment into two equal parts at a right angle.
Angle bisector	A line that divides an angle into two equal angles.

Key Terms to Remember

Term	Meaning
Compass	An instrument used to draw circles and arcs.
Straightedge	A ruler used only for drawing straight lines, not for measuring.
Bisector	A line that divides something into two equal parts.
Locus	The set of all points that satisfy a given condition.
Equilateral triangle	A triangle with all sides equal.
Perpendicular	A line that meets another at a right angle.
Arc	A part of the circumference of a circle.
Radius	The distance from the centre of a circle to a point on it.

Multiple-Choice Questions

1. The circumcentre of a triangle is the intersection of the

(A) angle bisectors(B) perpendicular bisectors of the sides(C) medians(D) altitudes

2. The incentre is the intersection of the

(A) perpendicular bisectors(B) angle bisectors(C) medians(D) altitudes

3. To draw a tangent at a point on a circle, the line through the point must be perpendicular to the

(A) chord(B) radius through that point(C) diameter(D) tangent

4. The inscribed circle of a triangle touches

(A) all three sides(B) one side(C) the vertices(D) the centre

5. The circumscribed circle of a triangle passes through

(A) the sides(B) all three vertices(C) the centre(D) one vertex

6. A perpendicular bisector of a chord passes through the

(A) tangent(B) centre(C) arc(D) vertex

7. The radius of the inscribed circle is called the

(A) circumradius(B) inradius(C) diameter(D) chord

8. How many tangents can be drawn from an external point?

(A) 1(B) 2(C) 3(D) 0

9. The centre of the circumscribed circle is equidistant from the

(A) sides(B) vertices(C) tangents(D) arcs

10. In construction, a compass is used to draw

(A) straight lines(B) circles and arcs(C) angles only(D) squares

Short Questions

1. What is practical geometry?
   Practical geometry is the drawing of geometric figures using only a compass and a ruler.
2. What is the circumcentre of a triangle?
   The circumcentre is the centre of the circumscribed circle; it is where the perpendicular bisectors of the sides meet.
3. What is the incentre of a triangle?
   The incentre is the centre of the inscribed circle; it is where the angle bisectors meet.
4. How is a tangent drawn at a point on a circle?
   Join the centre to the point, then draw a line through the point perpendicular to the radius.
5. What is a circumscribed circle?
   A circle that passes through all the vertices of a figure.
6. What is an inscribed circle?
   A circle inside a figure that touches all its sides.
7. What is a perpendicular bisector?

A line that cuts a segment into two equal parts at a right angle.

8. What is an angle bisector?
   A line that divides an angle into two equal angles.

9. How are tangents drawn from an external point?
   Join the point to the centre, draw a circle with that line as diameter, and join the point to the points where the circles cut each other.

10. What instruments are used in constructions?
    A compass and a ruler (straightedge).

Long Questions

1. Describe the steps to draw a tangent at a point on a circle, and to draw tangents from an external point.
2. Describe the steps to draw the circumscribed circle of a triangle.
3. Describe the steps to draw the inscribed circle of a triangle.

CHAPTER 12

Basic Statistics

Data and Frequency Distribution

Statistics is the study of collecting, organising and interpreting data. Data collected directly from the source is primary data; data taken from data already collected by others is secondary data. Raw data is organised into a frequency distribution: a table that shows how often each value, or each group of values (a class interval), occurs. The middle value of a class interval is called its class mark. Cumulative frequency is the running total of frequencies up to a class.

Measures of Central Tendency

The three measures of central tendency are the mean, the median and the mode. The mean is the sum of the values divided by the number of values; for grouped data it is the sum of (frequency times class mark) divided by the total frequency. The median is the middle value when the data is arranged in order. The mode is the value that occurs most often.

Measures of Dispersion and Graphs

Measures of dispersion show how spread out the data is. The range is the highest value minus the lowest value. The variance is the average of the squared differences from the mean, and the standard deviation is the square root of the variance. Data is shown in

graphs: a histogram shows grouped data as adjacent bars, a frequency polygon joins the midpoints of the bars, and an ogive is a graph of cumulative frequency.

Key Formulae

> Mean (ungrouped) = sum of all values / number of values.

> Mean (grouped) = sum of (frequency x class mark) / total frequency.

> Range = highest value - lowest value; variance = sum of (x - mean) squared / n; standard deviation = square root of variance.

Worked Example: Find the mean, range and variance of 2, 4, 6, 8.

Mean = (2 + 4 + 6 + 8) / 4 = 5. Range = 8 - 2 = 6. Variance = ((2-5) squared + (4-5) squared + (6-5) squared + (8-5) squared) / 4 = (9 + 1 + 1 + 9) / 4 = 5, so the standard deviation is the square root of 5, about 2.24.

Worked Example: Find the class mark of the interval 10-20.

Class mark = (10 + 20) / 2 = 15.

Key Definitions

Term	Definition
Statistics	The study of collecting, organising and interpreting data.
Frequency	How often a value occurs in a set of data.
Class interval	A group of values in a frequency distribution, such as 10-20.
Cumulative frequency	The running total of frequencies up to a class.
Histogram	A graph that shows grouped data as adjacent bars.

Key Terms to Remember

Term	Meaning
Raw data	Data as it is collected, before it is organised.
Grouped data	Data organised into class intervals.
Ungrouped data	Data listed value by value, without class intervals.
Ogive	A graph of cumulative frequency.
Frequency polygon	A graph that joins the midpoints of the tops of the bars of a histogram.
Tally	A mark used to count how often a value occurs.
Mid-point (class mark)	The middle value of a class interval.
Class mark	The middle value of a class interval.

Multiple-Choice Questions

1. The mean of 5, 10, 15 is

(A) 10(B) 15(C) 5(D) 30

2. The mode of 2, 3, 3, 4, 5 is

(A) 2(B) 3(C) 4(D) 5

3. The median of 4, 6, 8, 10, 12 is

(A) 6(B) 8(C) 10(D) 4

4. The range of 3, 7, 11, 15 is

(A) 3(B) 12(C) 15(D) 8

5. A histogram is used for

(A) ungrouped data(B) grouped (continuous) data(C) qualitative data(D) pie data

6. Cumulative frequency is the

(A) sum of frequencies up to a class(B) total of all data(C) mean(D) mode

7. The class mark of 10-20 is

(A) 10(B) 20(C) 15(D) 30

8. Standard deviation is the square root of the

(A) mean(B) variance(C) range(D) mode

9. The mean of 2, 4, 6, 8 is

(A) 4(B) 5(C) 6(D) 20

10. An ogive is a graph of

(A) frequency(B) cumulative frequency(C) mean(D) mode

Short Questions

1. What is statistics?
   Statistics is the study of collecting, organising and interpreting data.
2. What is the mean?
   The mean is the sum of the values divided by the number of values.
3. What is the median?
   The median is the middle value of a set of values arranged in order.
4. What is the mode?
   The mode is the value that occurs most often in a set of data.
5. How is the range calculated?
   The range is the highest value minus the lowest value.
6. What is a frequency distribution?
   A frequency distribution is a table that shows how often each value, or each group of values, occurs.
7. What is cumulative frequency?
   Cumulative frequency is the running total of frequencies up to a class.
8. What is a histogram?
   A histogram is a graph that shows grouped data as adjacent bars.
9. What is the difference between primary and secondary data?
   Primary data is collected directly from the source, while secondary data is taken from data already collected by others.
10. What is standard deviation?
    Standard deviation is the square root of the variance; it measures how spread out the data is.

Long Questions

1. Explain the mean, median and mode, and find them for a given set of data.
2. Describe how to make a frequency distribution table from raw data, and draw a histogram.

3. Explain the range, variance and standard deviation, and find them for a given set of data.

MCQ Answer Key (all chapters)

Chapter 1 — Complex Numbers

1. D 2. B 3. A 4. A 5. A 6. B 7. C 8. B 9. A 10. A

Chapter 2 — Quadratic Equations

1. A 2. A 3. B 4. A 5. A 6. A 7. A 8. A 9. A 10. C

Chapter 3 — Matrices and Determinants

1. B 2. A 3. A 4. B 5. B 6. B 7. A 8. C 9. B 10. A

Chapter 4 — Linear and Quadratic Inequalities

1. A 2. A 3. A 4. A 5. A 6. B 7. B 8. A 9. A 10. A

Chapter 5 — Algebraic Fractions

1. A 2. A 3. A 4. A 5. A 6. A 7. A 8. A 9. A 10. A

Chapter 6 — Functions and Graphs

1. A 2. A 3. B 4. A 5. A 6. A 7. B 8. A 9. B 10. B

Chapter 7 — Vectors in Plane

1. A 2. C 3. A 4. A 5. B 6. A 7. C 8. B 9. A 10. A

Chapter 8 — Application of Trigonometry

1. B 2. C 3. A 4. B 5. B 6. A 7. A 8. A 9. B 10. B

Chapter 9 — Chord and Arcs of a Circle

1. A 2. B 3. A 4. A 5. A 6. A 7. A 8. B 9. C 10. A

Chapter 10 — Tangents and Angles of a Circle

1. B 2. C 3. B 4. A 5. B 6. A 7. B 8. C 9. B 10. B

Chapter 11 — Practical Geometry of Circles

1. B 2. B 3. B 4. A 5. B 6. B 7. B 8. B 9. B 10. B

Chapter 12 — Basic Statistics

1. A 2. B 3. B 4. B 5. B 6. A 7. C 8. B 9. B 10. B

End of book. Good luck with your studies.