PUNJAB • PECTAA / PCTB • CLASS 10

10th Class Mathematics Notes

Full-Book Study Notes — Punjab Board

What this book contains: complete study notes for all 12 units of the new PECTAA Class 10 Mathematics book, with key definitions, important formulas, worked examples, short questions, long questions and multiple choice questions with an answer key.

How to use it: read the key notes of a unit, study the worked examples, then try the short and long questions yourself before reading the answers. Finish with the MCQs and check your score in the answer key.

Note: these are original study notes written to help students revise. They are not the textbook and do not reproduce textbook text. Always study the official PECTAA textbook alongside these notes, and confirm the paper pattern with your board.

Zahid Notes — Free Study Material for Students

Paper Pattern and How the Book Is Organised

Paper Pattern (third-party reproduced — not officially confirmed)

The Class 10 Mathematics paper for the Punjab boards is reported, from the PECTAA pairing scheme as reproduced by study-note sources, as follows. Confirm it with your board before the examination.

Section	Questions	Marks
Objective — Multiple Choice Questions	15 MCQs (1 mark each)	15
Subjective — Short Questions	9 questions (4 marks each)	36
Subjective — Long Questions	3 questions (8 marks each)	24
Total		**75**

Time allowed: 2 hours 30 minutes (20 minutes for the objective part, 2 hours 10 minutes for the subjective part), as reported in the same pairing-scheme reproductions.

One scheme for all boards: the nine Punjab boards (Lahore, Gujranwala, Faisalabad, Rawalpindi, Multan, Sargodha, Bahawalpur, DG Khan, Sahiwal) follow the same PECTAA scheme. There is no separate board-wise pattern.

Contents — 12 Units

1. Complex Numbers
2. Quadratic Equations and Inequalities
3. Matrices and Determinants
4. Functions and Graphs
5. Algebraic Fractions
6. Vectors in Plane
7. Trigonometry
8. Chords and Arcs of a Circle
9. Tangent and Angles of a Circle
10. Practical Geometry of Circles
11. Information Handling
12. Probability

Practice totals in this book: 120 multiple choice questions (10 per unit), 60 short questions (5 per unit) and 24 long questions (2 per unit), all with answers.

> **Revision tip:** Mathematics is learned by solving, not by reading alone. Cover the answer, solve each question on paper, and only then compare your working.

Unit 1: Complex Numbers

What you will learn: What a complex number is, the imaginary unit i, conjugates, modulus, and the four operations on complex numbers.

Key Notes

- **Imaginary unit:** The number i is defined by i squared = -1, so i = the square root of -1. Powers of i repeat in a cycle of four: i, -1, -i, 1, then i again.
- **Complex number:** A number of the form z = a + bi, where a and b are real numbers. Here a is the real part and b is the imaginary part. Every real number is also a complex number with b = 0.
- **Conjugate:** The conjugate of z = a + bi is z-bar = a - bi. A complex number multiplied by its conjugate gives a real number: (a + bi)(a - bi) = a squared + b squared.
- **Modulus:** The modulus (absolute value) of z = a + bi is |z| = the square root of (a squared + b squared). It is the distance of the point (a, b) from the origin.
- **Addition and subtraction:** Add or subtract the real parts together and the imaginary parts together: (a + bi) + (c + di) = (a + c) + (b + d)i.
- **Multiplication:** Multiply as with binomials and replace i squared by -1: (a + bi)(c + di) = (ac - bd) + (ad + bc)i.
- **Division:** Multiply the numerator and the denominator by the conjugate of the denominator, then simplify.

Important Formulas

> i squared = -1 ; i cubed = -i ; i to the power 4 = 1

> z = a + bi ; conjugate z-bar = a - bi ; |z| = sqrt(a squared + b squared)

> z times z-bar = a squared + b squared (always real)

Worked Examples

Example. Simplify (3 + 2i) + (1 - 4i).
Solution: (3 + 1) + (2 - 4)i = 4 - 2i.

Example. Multiply (2 + 3i)(1 - i).
Solution: 2 - 2i + 3i - 3i squared = 2 + i + 3 = 5 + i.

Example. Find i to the power 23.
Solution: 23 divided by 4 leaves remainder 3, so i to the power 23 = i cubed = -i.

Example. Find the modulus of 3 - 4i.
Solution: |z| = sqrt(9 + 16) = sqrt(25) = 5.

Unit 1 — Questions and Practice

Short Questions

Q1. Simplify i to the power 10.
Answer: 10 divided by 4 leaves remainder 2, so i to the power 10 = i squared = -1.

Q2. Write the conjugate of 5 - 2i.
Answer: The conjugate is 5 + 2i.

Q3. Simplify (4 + i) - (2 - 3i).
Answer: (4 - 2) + (1 + 3)i = 2 + 4i.

Q4. Find the modulus of -5 + 12i.
Answer: sqrt(25 + 144) = sqrt(169) = 13.

Q5. Simplify (1 + i) squared.
Answer: 1 + 2i + i squared = 1 + 2i - 1 = 2i.

Long Questions

Q1. Simplify (2 + 3i)(3 - 2i) divided by (1 + i).
Answer: Numerator: 6 - 4i + 9i - 6i squared = 6 + 5i + 6 = 12 + 5i. Divide by (1 + i): multiply top and bottom by (1 - i). (12 + 5i)(1 - i) = 12 - 12i + 5i - 5i squared = 17 - 7i. Denominator: (1 + i)(1 - i) = 2. Answer: (17 - 7i)/2 = 17/2 - (7/2)i.

Q2. Find the real numbers x and y if (x + 2i) + (3 - yi) = 7 + 5i.
Answer: Equate real parts: x + 3 = 7, so x = 4. Equate imaginary parts: 2 - y = 5, so y = -3.

Multiple Choice Questions

1. The value of i squared is
(A) 1 (B) -1 (C) i (D) 0

2. The real part of 4 - 7i is
(A) -7 (B) 7 (C) 4 (D) -4

3. The conjugate of 2 + 5i is
(A) -2 + 5i (B) 2 - 5i (C) -2 - 5i (D) 5 + 2i

4. The modulus of 5 - 12i is
(A) 7 (B) 17 (C) 13 (D) 12

5. The value of i to the power 4 is
(A) -1 (B) i (C) 1 (D) -i

6. (1 + i)(1 - i) equals
(A) 0 (B) 1 (C) 2 (D) 2i

7. The imaginary part of 3i is
(A) 0 (B) 3 (C) 1 (D) -3

8. i to the power 7 equals
(A) i (B) -i (C) 1 (D) -1

9. (2 + i) + (3 - 2i) equals
(A) 5 - i (B) 5 + i (C) 1 - i (D) 6 - 2i

10. The product of (a + bi) and its conjugate is
(A) a squared - b squared (B) a squared + b squared (C) 2ab (D) a + b

Unit 2: Quadratic Equations and Inequalities

What you will learn: Solving quadratic equations, the discriminant and the nature of roots, sum and product of roots, and quadratic inequalities.

Key Notes

- **Standard form:** A quadratic equation is ax squared + bx + c = 0, where a is not zero. a, b and c are real numbers.
- **Factorisation:** If the quadratic factorises, set each factor equal to zero. For example x squared - 5x + 6 = (x - 2)(x - 3) = 0 gives x = 2 or x = 3.
- **Quadratic formula:** x = (-b plus or minus the square root of (b squared - 4ac)) divided by 2a. It works for every quadratic equation.
- **Discriminant:** D = b squared - 4ac. If D is greater than 0 there are two distinct real roots; if D = 0 the roots are equal; if D is less than 0 the roots are complex (not real).
- **Sum and product of roots:** If the roots are alpha and beta, then alpha + beta = -b/a and alpha times beta = c/a.
- **Completing the square:** Rewrite x squared + bx as (x + b/2) squared - (b/2) squared. This method proves the quadratic formula and finds the turning point.
- **Quadratic inequality:** First solve the matching equation to find the boundary values, then test the sign of the expression in each interval. For x squared greater than 9, the solution is x less than -3 or x greater than 3.

Important Formulas

> x = (-b +/- sqrt(b squared - 4ac)) / 2a

> D = b squared - 4ac ; alpha + beta = -b/a ; alpha x beta = c/a

> x squared + bx = (x + b/2) squared - (b/2) squared

Worked Examples

Example. Solve x squared - 5x + 6 = 0.
Solution: Factorise: (x - 2)(x - 3) = 0, so x = 2 or x = 3.

Example. Find the discriminant of x squared + 4x + 4 = 0 and describe the roots.
Solution: D = 16 - 16 = 0, so the roots are real and equal: x = -2 (twice).

Example. Find the sum of the roots of 2x squared - 6x + 5 = 0.
Solution: Sum = -b/a = 6/2 = 3.

Example. Solve x squared - 4 is greater than 0.
Solution: Boundary values are x = 2 and x = -2. The expression is positive outside the roots, so x less than -2 or x greater than 2.

Unit 2 — Questions and Practice

Short Questions

Q1. Find the roots of x squared - 9 = 0.
Answer: x squared = 9, so x = 3 or x = -3.

Q2. Find the discriminant of x squared - 2x + 5 = 0 and describe the roots.
Answer: D = 4 - 20 = -16, which is less than 0, so the roots are complex, not real.

Q3. Find the product of the roots of 3x squared + 7x - 2 = 0.
Answer: Product = c/a = -2/3.

Q4. Solve x squared greater than 9.
Answer: x less than -3 or x greater than 3.

Q5. Form the quadratic equation whose roots are 4 and -1.
Answer: (x - 4)(x + 1) = 0, so x squared - 3x - 4 = 0.

Long Questions

Q1. Solve 2x squared - 7x + 3 = 0 by the quadratic formula.
Answer: a = 2, b = -7, c = 3. D = 49 - 24 = 25, sqrt(D) = 5. x = (7 plus or minus 5)/4, so x = 12/4 = 3 or x = 2/4 = 1/2.

Q2. Solve the inequality x squared - 3x - 10 is less than or equal to 0.
Answer: Factorise: (x - 5)(x + 2) is less than or equal to 0. The expression is negative or zero between the roots -2 and 5. Solution: -2 is less than or equal to x, and x is less than or equal to 5.

Multiple Choice Questions

1. The standard form of a quadratic equation is
(A) ax + b = 0 (B) ax squared + bx + c = 0 (C) ax cubed + bx = 0 (D) x squared + y squared = 1

2. The discriminant of ax squared + bx + c = 0 is
(A) b squared + 4ac (B) b squared - 4ac (C) 4ac - b (D) b - 4ac

3. If D = 0, the roots are
(A) distinct real (B) equal real (C) complex (D) no roots

4. The sum of the roots of x squared - 7x + 10 = 0 is
(A) -7 (B) 7 (C) 10 (D) -10

5. The product of the roots of x squared - 7x + 10 = 0 is
(A) 7 (B) -7 (C) 10 (D) -10

6. The roots of x squared - 25 = 0 are
(A) 5 only (B) -5 only (C) 5 and -5 (D) 0 and 5

7. The equation with roots 2 and 5 is
(A) x squared - 7x + 10 = 0 (B) x squared + 7x + 10 = 0 (C) x squared - 3x - 10 = 0 (D) x squared + 3x - 10 = 0

8. If D is less than 0, the roots are
(A) real and equal (B) real and distinct (C) complex (D) zero

9. The solution of x squared less than 4 is
(A) -2 less than x less than 2 (B) x greater than 2 (C) x less than -2 (D) x = 2 only

10. By the quadratic formula, the roots of x squared - 3x - 4 = 0 are
(A) 4 and -1 (B) -4 and 1 (C) 3 and 4 (D) 1 and 4

Unit 3: Matrices and Determinants

What you will learn: Types of matrices, addition and multiplication, transpose, determinants of order two, and the inverse of a 2 by 2 matrix.

Key Notes

- **Matrix:** A rectangular array of numbers in rows and columns. A matrix with m rows and n columns is of order m by n.
- **Types:** Row matrix (one row), column matrix (one column), square matrix (rows = columns), diagonal matrix (non-zero entries only on the diagonal), identity matrix I (ones on the diagonal, zeros elsewhere), zero matrix (all entries zero), symmetric matrix (equal to its transpose).
- **Addition and subtraction:** Two matrices can be added or subtracted only if they have the same order; add or subtract the matching entries.
- **Scalar multiplication:** Multiply every entry of the matrix by the number.
- **Matrix multiplication:** AB is defined only when the number of columns of A equals the number of rows of B. The (i, j) entry of AB is the dot product of row i of A and column j of B. In general AB is not equal to BA.
- **Transpose:** The transpose A-transpose is formed by turning rows into columns.
- **Determinant of order two:** For the matrix with rows (a, b) and (c, d), the determinant is ad - bc.
- **Inverse of a 2 by 2 matrix:** If det(A) is not zero, A-inverse = (1 / det A) times the matrix with rows (d, -b) and (-c, a). A matrix with determinant zero is singular and has no inverse.

Important Formulas

> det of [[a, b], [c, d]] = ad - bc

> A-inverse = (1 / det A) x [[d, -b], [-c, a]] (when det A is not 0)

> A x A-inverse = A-inverse x A = I

Worked Examples

Example. Find the determinant of the matrix with rows (3, 2) and (1, 4).
Solution: det = 3 x 4 - 2 x 1 = 12 - 2 = 10.

Example. Multiply the matrix with rows (1, 2) and (3, 4) by the column matrix (5, 6).
Solution: First entry: 1 x 5 + 2 x 6 = 17. Second entry: 3 x 5 + 4 x 6 = 39. Result: column matrix (17, 39).

Example. Find the inverse of the matrix with rows (2, 0) and (0, 2).
Solution: det = 4. Inverse = (1/4) times the matrix with rows (2, 0) and (0, 2), which is the matrix with rows (1/2, 0) and (0, 1/2).

Example. Is the matrix with rows (1, 2) and (2, 4) invertible?
Solution: det = 1 x 4 - 2 x 2 = 0, so it is singular and has no inverse.

Unit 3 — Questions and Practice

Short Questions

Q1. What is the order of a matrix with 3 rows and 2 columns?
Answer: It is of order 3 by 2.

Q2. Find the transpose of the matrix with rows (1, 2) and (3, 4).
Answer: The transpose has rows (1, 3) and (2, 4).

Q3. Find the determinant of the matrix with rows (5, 3) and (2, 1).
Answer: det = 5 x 1 - 3 x 2 = 5 - 6 = -1.

Q4. Add the matrices with rows (1, 2) and (3, 4), and with rows (5, 6) and (7, 8).
Answer: The sum has rows (6, 8) and (10, 12).

Q5. When is a square matrix singular?
Answer: When its determinant is zero.

Long Questions

Q1. Find the inverse of the matrix A with rows (4, 3) and (2, 5).
Answer: det(A) = 4 x 5 - 3 x 2 = 20 - 6 = 14. A-inverse = (1/14) times the matrix with rows (5, -3) and (-2, 4), that is rows (5/14, -3/14) and (-2/14, 4/14).

Q2. Solve the system 2x + y = 5 and x - y = 1 by matrix inversion.
Answer: Write A with rows (2, 1) and (1, -1), and B = column (5, 1). det(A) = -2 - 1 = -3. A-inverse = (-1/3) times the matrix with rows (-1, -1) and (-1, 2), which is rows (1/3, 1/3) and (1/3, -2/3). X = A-inverse B: x = (5 + 1)/3 = 2, y = (5 - 2)/3 = 1. So x = 2 and y = 1.

Multiple Choice Questions

1. A matrix with 2 rows and 3 columns is of order
(A) 3 by 2 (B) 2 by 3 (C) 2 by 2 (D) 6

2. The identity matrix of order 2 has diagonal entries
(A) 0, 0 (B) 1, 1 (C) 2, 2 (D) 1, 0

3. Two matrices can be added only if they have the same
(A) determinant (B) order (C) inverse (D) trace

4. The determinant of the matrix with rows (2, 3) and (1, 4) is
(A) 5 (B) 11 (C) 2 (D) -5

5. If det(A) = 0, then A is
(A) invertible (B) singular (C) identity (D) symmetric

6. The transpose of the matrix with rows (1, 2) and (3, 4) has first row
(A) 1, 2 (B) 1, 3 (C) 3, 4 (D) 2, 4

7. Matrix multiplication is in general
(A) commutative (B) not commutative (C) always zero (D) always identity

8. For AB to be defined, the number of columns of A must equal the number of
(A) columns of B (B) rows of B (C) rows of A (D) entries of B

9. The inverse of the matrix with rows (1, 0) and (0, 1) is
(A) itself (B) zero matrix (C) does not exist (D) the matrix with rows (0, 1) and (1, 0)

10. det of the matrix with rows (3, 1) and (2, 5) is
(A) 13 (B) 17 (C) 11 (D) 7

Unit 4: Functions and Graphs

What you will learn: Functions, domain and range, types of functions, graphs of linear and quadratic functions, inverse and composite functions.

Key Notes

- **Function:** A rule that assigns to each input x exactly one output y. We write y = f(x). The set of allowed inputs is the domain; the set of outputs is the range.
- **One-one and onto:** A function is one-one if different inputs give different outputs. It is onto if every value in the codomain is an output of some input.
- **Linear function:** f(x) = mx + c. Its graph is a straight line with slope m and y-intercept c.
- **Quadratic function:** f(x) = ax squared + bx + c with a not zero. Its graph is a parabola. The vertex is at x = -b/(2a). It opens upward if a is greater than 0 and downward if a is less than 0.
- **Inverse function:** If f is one-one, its inverse f-inverse reverses the mapping: f-inverse(f(x)) = x. To find it, write y = f(x), solve for x in terms of y, then swap x and y.
- **Composite function:** (f composed with g)(x) = f(g(x)): apply g first, then f.
- **Graph reading:** The x-intercepts of a graph are the solutions of f(x) = 0. The y-intercept is f(0).

Important Formulas

> f(x) = mx + c (line, slope m, intercept c)

> Vertex of y = ax squared + bx + c is at x = -b / 2a

> (f o g)(x) = f(g(x)) ; f-inverse(f(x)) = x

Worked Examples

Example. If f(x) = 2x + 3, find f(4).
Solution: f(4) = 2 x 4 + 3 = 11.

Example. Find the domain of f(x) = 1 / (x - 2).
Solution: The denominator cannot be zero, so x is not 2. Domain: all real numbers except 2.

Example. Find the vertex of y = x squared - 4x + 3.
Solution: x = -b/2a = 4/2 = 2. y = 4 - 8 + 3 = -1. Vertex is (2, -1).

Example. Find the inverse of f(x) = 2x - 4.
Solution: y = 2x - 4 gives x = (y + 4)/2, so f-inverse(x) = (x + 4)/2.

Unit 4 — Questions and Practice

Short Questions

Q1. If f(x) = x squared - 1, find f(3).
Answer: f(3) = 9 - 1 = 8.

Q2. Find the range of f(x) = 3x + 1 for x in {0, 1, 2}.
Answer: f(0) = 1, f(1) = 4, f(2) = 7, so the range is {1, 4, 7}.

Q3. Find the inverse of y = 2x - 4.
Answer: x = (y + 4)/2, so the inverse function is (x + 4)/2.

Q4. If f(x) = x + 1 and g(x) = 2x, find (f o g)(1).
Answer: g(1) = 2, then f(2) = 3. So (f o g)(1) = 3.

Q5. Is the relation y = x squared a function?
Answer: Yes: each input x gives exactly one output y.

Long Questions

Q1. Describe the graph of y = -x squared + 4x - 3: find its vertex and its x-intercepts.
Answer: a = -1, so the parabola opens downward. Vertex: x = -4/(2 x -1) = 2, y = -4 + 8 - 3 = 1, so the vertex is (2, 1). x-intercepts: solve -x squared + 4x - 3 = 0, that is x squared - 4x + 3 = 0, (x - 1) (x - 3) = 0, so x = 1 or x = 3. The y-intercept is -3.

Q2. Find the inverse of f(x) = (3x + 2)/5 and verify it.
Answer: y = (3x + 2)/5 gives 5y = 3x + 2, so x = (5y - 2)/3. Hence f-inverse(x) = (5x - 2)/3. Check: f(f-inverse(x)) = (3 x (5x - 2)/3 + 2)/5 = (5x - 2 + 2)/5 = x. Verified.

Multiple Choice Questions

1. A function assigns to each input
(A) many outputs (B) exactly one output (C) no output (D) two outputs

2. The domain of f(x) = 1/x is
(A) all real numbers (B) all real numbers except 0 (C) x greater than 0 only (D) x less than 0 only

3. The graph of y = 2x + 1 is a
(A) parabola (B) circle (C) straight line (D) hyperbola

4. The vertex of y = x squared - 2x + 1 is at
(A) (1, 0) (B) (0, 1) (C) (-1, 0) (D) (2, 1)

5. If f(x) = 3x - 2, then f(2) equals
(A) 4 (B) 6 (C) 8 (D) 2

6. The inverse of f(x) = x + 5 is
(A) x - 5 (B) x + 5 (C) 5 - x (D) 5x

7. If f(x) = x squared, then f(-3) equals
(A) -9 (B) 9 (C) 6 (D) -6

8. The slope of the line y = -4x + 7 is
(A) 7 (B) 4 (C) -4 (D) -7

9. The y-intercept of y = 5x - 3 is
(A) 5 (B) -3 (C) 3 (D) 0

10. A parabola y = ax squared + bx + c opens upward when
(A) a is less than 0 (B) a is greater than 0 (C) a = 0 (D) c is greater than 0

Unit 5: Algebraic Fractions

What you will learn: Simplifying algebraic fractions, HCF and LCM of algebraic expressions, addition, subtraction, multiplication and division of fractions.

Key Notes

- **Algebraic fraction:** A fraction whose numerator or denominator contains an algebraic expression, such as (x squared - 9)/(x + 3).
- **Simplifying:** Factorise the numerator and the denominator, then cancel the common factors. The denominator must never be zero.
- **HCF and LCM:** The HCF of algebraic expressions is the product of the common factors with the lowest powers. The LCM is the product of all factors with the highest powers.
- **Addition and subtraction:** Find the LCM of the denominators, rewrite each fraction with that denominator, then add or subtract the numerators.
- **Multiplication:** Multiply the numerators together and the denominators together, then simplify by cancelling common factors.
- **Division:** Multiply by the reciprocal of the second fraction: (a/b) divided by (c/d) = (a/b) x (d/c).
- **Difference of squares:** a squared - b squared = (a - b)(a + b). This factorisation is used constantly when simplifying fractions.

Important Formulas

> a squared - b squared = (a - b)(a + b)

> (a/b) + (c/d) = (ad + bc) / bd

> (a/b) divided by (c/d) = (ad) / (bc)

Worked Examples

Example. Simplify (x squared - 4)/(x - 2).
Solution: Numerator = (x - 2)(x + 2). Cancel (x - 2) to get x + 2, provided x is not 2.

Example. Add 1/(x + 1) + 1/(x - 1).
Solution: Common denominator (x squared - 1). Sum = (x - 1 + x + 1)/(x squared - 1) = 2x/(x squared - 1).

Example. Simplify (2x squared)/(4x).
Solution: Cancel 2x: (2x squared)/(4x) = x/2, provided x is not 0.

Example. Multiply (x + 2)/(x - 1) by (x - 1)/(x + 3).
Solution: Cancel (x - 1): result (x + 2)/(x + 3).

Unit 5 — Questions and Practice

Short Questions

Q1. Simplify (2x squared)/(4x).
Answer: x/2, for x not 0.

Q2. Simplify (x squared - 1)/(x - 1).
Answer: (x - 1)(x + 1)/(x - 1) = x + 1, for x not 1.

Q3. Add 1/x + 1/(2x).
Answer: Common denominator 2x: (2 + 1)/(2x) = 3/(2x).

Q4. Simplify (a squared - b squared)/(a - b).
Answer: (a - b)(a + b)/(a - b) = a + b, for a not b.

Q5. Simplify (x + 2)/(x squared + 4x + 4).
Answer: Denominator = (x + 2) squared, so the fraction equals 1/(x + 2), for x not -2.

Long Questions

Q1. Simplify (x squared - 5x + 6)/(x squared - 9) multiplied by (x + 3)/(x - 2).
Answer: First fraction = (x - 3)(x - 2)/((x - 3)(x + 3)) = (x - 2)/(x + 3). Multiply by (x + 3)/(x - 2): the factors cancel and the result is 1, provided x is not 3, -3 or 2.

Q2. Simplify 2/(x - 1) - 3/(x + 1).
Answer: Common denominator (x squared - 1). Numerator: 2(x + 1) - 3(x - 1) = 2x + 2 - 3x + 3 = 5 - x. Result: (5 - x)/(x squared - 1).

Multiple Choice Questions

1. (x squared - 9)/(x - 3) simplifies to
(A) x - 3 (B) x + 3 (C) x squared - 3 (D) 9

2. The value of (a squared - b squared)/(a + b) is
(A) a - b (B) a + b (C) ab (D) a squared + b squared

3. 1/x + 1/x equals
(A) 1/(2x) (B) 2/x (C) 1/x squared (D) 2x

4. (x + 1)/(x + 1) equals, for x not -1
(A) 0 (B) x (C) 1 (D) x + 1

5. The HCF of 6x and 9x squared is
(A) 3x (B) 18x squared (C) 9x (D) 6x

6. (2/3) divided by (4/5) equals
(A) 8/15 (B) 5/6 (C) 6/5 (D) 10/12

7. Simplify (x squared - 4)/(x + 2).
(A) x - 2 (B) x + 2 (C) x - 4 (D) x + 4

8. The LCM of x and x + 1 is
(A) x(x + 1) (B) x + 1 (C) x squared (D) 1

9. (3x)/(6x) simplifies to
(A) 3 (B) 2 (C) 1/2 (D) x

10. 1/(x - 1) - 1/(x + 1) equals
(A) 2/(x squared - 1) (B) 0 (C) 2x/(x squared - 1) (D) 1/(x squared - 1)

Unit 6: Vectors in Plane

What you will learn: Vectors, magnitude and unit vectors, addition and scalar multiplication, position vectors, and the dot product.

Key Notes

- **Vector:** A quantity with both magnitude and direction, written as a directed line segment. A vector in the plane can be written as (x, y) or as xi + yj.
- **Magnitude:** The magnitude (length) of vector (x, y) is the square root of (x squared + y squared).
- **Unit vector:** A vector of length 1. The unit vector in the direction of a is a divided by |a|. The standard unit vectors are i = (1, 0) and j = (0, 1).
- **Position vector:** The position vector of a point P(x, y) is the vector from the origin O to P, that is xi + yj.
- **Addition:** Add vectors component by component: (a, b) + (c, d) = (a + c, b + d). Geometrically this is the triangle law or the parallelogram law.
- **Scalar multiplication:** k times (x, y) = (kx, ky). The direction stays the same (or reverses if k is negative) and the length is multiplied by |k|.
- **Dot product:** For a = (a1, a2) and b = (b1, b2), a dot b = a1b1 + a2b2 = |a||b| cos(theta), where theta is the angle between them.
- **Parallel and perpendicular:** Two non-zero vectors are parallel if one is a scalar multiple of the other. They are perpendicular if their dot product is zero.

Important Formulas

> |(x, y)| = sqrt(x squared + y squared)

> Unit vector = a / |a|

> a . b = a1b1 + a2b2 = |a| |b| cos(theta) ; a . b = 0 means perpendicular

Worked Examples

Example. Find the magnitude of 3i + 4j.
Solution: |a| = sqrt(9 + 16) = 5.

Example. Find the unit vector in the direction of (6, 8).
Solution: Magnitude is 10, so the unit vector is (6/10, 8/10) = (3/5, 4/5).

Example. Find the dot product of (2, 3) and (4, -1).
Solution: 2 x 4 + 3 x (-1) = 8 - 3 = 5.

Example. Add the vectors (1, 2) and (3, -1).
Solution: (1 + 3, 2 - 1) = (4, 1).

Unit 6 — Questions and Practice

Short Questions

Q1. Find the magnitude of the vector (-12, 5).
Answer: sqrt(144 + 25) = sqrt(169) = 13.

Q2. Add a = (1, 2) and b = (3, -1).
Answer: a + b = (4, 1).

Q3. Find 3 times the vector (2, -1).
Answer: (6, -3).

Q4. Are the vectors (1, 0) and (0, 1) perpendicular?
Answer: Yes: their dot product is 1 x 0 + 0 x 1 = 0.

Q5. Write the position vector of the point P(4, -3).
Answer: 4i - 3j.

Long Questions

Q1. Find the angle between the vectors a = (3, 4) and b = (4, 3).
Answer: a . b = 12 + 12 = 24. |a| = 5 and |b| = 5. cos(theta) = 24/(5 x 5) = 24/25 = 0.96. So theta is about 16.3 degrees.

Q2. Show that the vectors (2, 1) and (4, 2) are parallel.
Answer: (4, 2) = 2 x (2, 1), so the second vector is a scalar multiple of the first. Hence they are parallel (collinear).

Multiple Choice Questions

1. The magnitude of the vector (6, 8) is
(A) 10 (B) 14 (C) 48 (D) 100

2. The unit vector along i is
(A) j (B) i (C) 0 (D) 2i

3. (2, 3) + (1, -1) equals
(A) (3, 2) (B) (1, 4) (C) (3, 4) (D) (2, 2)

4. The dot product of (1, 2) and (3, 4) is
(A) 11 (B) 7 (C) 10 (D) 5

5. Two vectors are perpendicular when their dot product is
(A) 1 (B) -1 (C) 0 (D) infinity

6. The magnitude of (0, -7) is
(A) 0 (B) -7 (C) 7 (D) 49

7. 5 times the vector (1, -2) is
(A) (5, -2) (B) (1, -10) (C) (5, -10) (D) (6, 3)

8. The position vector of (0, 5) is
(A) 5i (B) 5j (C) i + 5j (D) 0

9. If a . b = |a| |b|, the angle between a and b is
(A) 90 degrees (B) 0 degrees (C) 180 degrees (D) 45 degrees

10. The vector (3, 4) and the vector (6, 8) are
(A) perpendicular (B) parallel (C) equal (D) opposite

Unit 7: Trigonometry

What you will learn: Trigonometric ratios, standard angles, identities, and heights and distances using angles of elevation and depression.

Key Notes

- **Ratios:** In a right-angled triangle with angle theta: sin(theta) = opposite/hypotenuse, cos(theta) = adjacent/hypotenuse, tan(theta) = opposite/adjacent = sin(theta)/cos(theta).
- **Reciprocals:** cosec(theta) = 1/sin(theta), sec(theta) = 1/cos(theta), cot(theta) = 1/tan(theta).
- **Standard angles:** sin 0 = 0, sin 30 = 1/2, sin 45 = 1/sqrt(2), sin 60 = sqrt(3)/2, sin 90 = 1. cos 0 = 1, cos 30 = sqrt(3)/2, cos 45 = 1/sqrt(2), cos 60 = 1/2, cos 90 = 0. tan 0 = 0, tan 30 = 1/sqrt(3), tan 45 = 1, tan 60 = sqrt(3).
- **Pythagorean identity:** sin squared(theta) + cos squared(theta) = 1. From it: 1 + tan squared(theta) = sec squared(theta), and 1 + cot squared(theta) = cosec squared(theta).
- **Angle of elevation:** The angle between the horizontal line and the line of sight when looking up at an object.
- **Angle of depression:** The angle between the horizontal line and the line of sight when looking down at an object. It equals the angle of elevation from the object to the observer.
- **Heights and distances:** Draw the right-angled triangle, mark the known side and angle, and use the ratio that links them (tan for opposite/adjacent, sin or cos when the hypotenuse is involved).

Important Formulas

> sin squared(theta) + cos squared(theta) = 1

> 1 + tan squared(theta) = sec squared(theta) ; 1 + cot squared(theta) = cosec squared(theta)

> tan(theta) = sin(theta) / cos(theta) ; sin 30 = 1/2 ; cos 60 = 1/2 ; tan 45 = 1

Worked Examples

Example. Find sin 30 degrees.
Solution: sin 30 = 1/2.

Example. If sin(theta) = 3/5 and theta is acute, find cos(theta).
Solution: cos squared(theta) = 1 - 9/25 = 16/25, so cos(theta) = 4/5.

Example. Find tan 45 degrees.
Solution: tan 45 = 1.

Example. A pole casts a shadow 10 m long when the sun's angle of elevation is 60 degrees. Find the height.
Solution: tan 60 = height/10, so height = 10 x sqrt(3), about 17.32 m.

Unit 7 — Questions and Practice

Short Questions

Q1. Find cos 60 degrees.
Answer: cos 60 = 1/2.

Q2. Simplify sec squared(theta) - tan squared(theta).
Answer: It equals 1, from the identity 1 + tan squared = sec squared.

Q3. Find sin 90 degrees.
Answer: sin 90 = 1.

Q4. If tan(theta) = 1 and theta is acute, find theta.
Answer: theta = 45 degrees.

Q5. Find cosec 30 degrees.
Answer: cosec 30 = 1/sin 30 = 2.

Long Questions

Q1. Prove that (1 - cos squared(theta)) / sin(theta) = sin(theta).
Answer: 1 - cos squared(theta) = sin squared(theta) by the Pythagorean identity. So the left side is sin squared(theta)/sin(theta) = sin(theta). Proved.

Q2. A ladder leans against a wall making an angle of 60 degrees with the ground. Its foot is 5 m from the wall. Find the length of the ladder.
Answer: cos 60 = adjacent/hypotenuse = 5/length, so length = 5 / cos 60 = 5 / (1/2) = 10 m. The ladder is 10 m long.

Multiple Choice Questions

1. sin 30 degrees equals
(A) 1 (B) 1/2 (C) sqrt(3)/2 (D) 0

2. cos 90 degrees equals
(A) 1 (B) 1/2 (C) 0 (D) -1

3. tan 45 degrees equals
(A) 0 (B) 1/2 (C) 1 (D) sqrt(3)

4. sin squared(theta) + cos squared(theta) equals
(A) 0 (B) 2 (C) 1 (D) tan(theta)

5. cosec 45 degrees equals
(A) 1 (B) sqrt(2) (C) 2 (D) 1/sqrt(3)

6. 1 + tan squared(theta) equals
(A) sin squared(theta) (B) cos squared(theta) (C) sec squared(theta) (D) cot squared(theta)

7. If sin(theta) = 1/2 and theta is acute, theta is
(A) 30 degrees (B) 45 degrees (C) 60 degrees (D) 90 degrees

8. tan(theta) equals
(A) sin(theta) x cos(theta) (B) sin(theta)/cos(theta) (C) cos(theta)/sin(theta) (D) 1/sin(theta)

9. sec 60 degrees equals
(A) 1 (B) 2 (C) 1/2 (D) sqrt(3)

10. The angle of elevation is measured from the
(A) vertical (B) horizontal (C) line of sight only (D) ground level point

Unit 8: Chords and Arcs of a Circle

What you will learn: Chords, arcs, central and inscribed angles, and the key theorems about chords and angles in a circle.

Key Notes

- **Circle parts:** Radius: distance from centre to the circle. Diameter: the longest chord, equal to twice the radius. Chord: a line segment joining two points on the circle. Arc: a part of the circumference; a minor arc is less than a semicircle and a major arc is greater.
- **Central angle:** The angle at the centre subtended by an arc. The angle at the circumference subtended by the same arc is half the central angle.
- **Angle in a semicircle:** The angle in a semicircle is a right angle (90 degrees).
- **Perpendicular from centre:** The perpendicular from the centre of a circle to a chord bisects the chord. Conversely, the line joining the centre to the midpoint of a chord is perpendicular to the chord.
- **Equal chords:** Equal chords of a circle are equidistant from the centre, and chords equidistant from the centre are equal.
- **Arc length:** For radius r and central angle theta in degrees, arc length = (theta/360) x 2 x pi x r. In radians, arc length = r x theta.
- **Angles in the same segment:** Angles in the same segment of a circle are equal.

Important Formulas

> Arc length = (theta / 360) x 2 pi r (theta in degrees) ; arc length = r theta (theta in radians)

> Angle at centre = 2 x angle at circumference (same arc)

> Chord length = 2 x sqrt(r squared - d squared), where d is the distance of the chord from the centre

Worked Examples

Example. A chord is 6 cm from the centre of a circle of radius 10 cm. Find the chord length.
Solution: Half-chord = sqrt(100 - 36) = sqrt(64) = 8 cm, so the chord is 16 cm long.

Example. Find the arc length for radius 7 cm and central angle 90 degrees (use pi = 22/7).
Solution: Arc = (90/360) x 2 x (22/7) x 7 = (1/4) x 44 = 11 cm.

Example. The angle at the centre is 100 degrees. Find the angle at the circumference on the same arc.
Solution: The angle at the circumference is half of 100 degrees, that is 50 degrees.

Example. A chord of length 8 cm is at distance 3 cm from the centre. Find the radius.
Solution: Half-chord is 4 cm, so r = sqrt(16 + 9) = sqrt(25) = 5 cm.

Unit 8 — Questions and Practice

Short Questions

Q1. What is the longest chord of a circle called?
Answer: The diameter.

Q2. Find the arc length for radius 7 cm and central angle 90 degrees (pi = 22/7).
Answer: (90/360) x 2 x (22/7) x 7 = 11 cm.

Q3. What is the angle in a semicircle?
Answer: A right angle, 90 degrees.

Q4. Equal chords of a circle are equidistant from the what?
Answer: From the centre of the circle.

Q5. A chord is 6 cm from the centre of a circle of radius 10 cm. Find the chord length.
Answer: Half-chord = sqrt(100 - 36) = 8 cm, so the chord is 16 cm.

Long Questions

Q1. Prove that the perpendicular from the centre of a circle to a chord bisects the chord.
Answer: Let O be the centre, AB a chord, and OM perpendicular to AB. In triangles OMA and OMB: OA = OB (radii), OM is common, and both triangles are right-angled at M. By the RHS congruence rule the triangles are congruent, so AM = MB. Hence OM bisects AB.

Q2. A chord of length 24 cm is at a distance of 5 cm from the centre. Find the radius of the circle.
Answer: Half of the chord is 12 cm. By Pythagoras in the triangle formed by the radius, the half-chord and the distance: r squared = 12 squared + 5 squared = 144 + 25 = 169, so r = 13 cm.

Multiple Choice Questions

1. The longest chord of a circle is the
(A) radius (B) diameter (C) arc (D) tangent

2. The angle in a semicircle is
(A) 45 degrees (B) 60 degrees (C) 90 degrees (D) 180 degrees

3. The angle at the centre is how many times the angle at the circumference on the same arc?
(A) equal (B) half (C) double (D) triple

4. A chord 8 cm long is 3 cm from the centre. The radius is
(A) 4 cm (B) 5 cm (C) 6 cm (D) 7 cm

5. Equal chords are equidistant from the
(A) circumference (B) centre (C) diameter (D) tangent

6. The perpendicular from the centre to a chord
(A) trisects it (B) bisects it (C) does not meet it (D) doubles it

7. Angles in the same segment of a circle are
(A) supplementary (B) equal (C) complementary (D) different

8. The circumference of a circle of radius r is
(A) pi r (B) 2 pi r (C) pi r squared (D) 2 pi r squared

9. A minor arc is
(A) equal to a semicircle (B) less than a semicircle (C) greater than a semicircle (D) the whole circle

10. Arc length for r = 14 cm and central angle 90 degrees (pi = 22/7) is
(A) 11 cm (B) 22 cm (C) 44 cm (D) 7 cm

Unit 9: Tangent and Angles of a Circle

What you will learn: Tangents to a circle, the alternate segment theorem, and angles in cyclic quadrilaterals.

Key Notes

- **Tangent:** A straight line that touches a circle at exactly one point, called the point of contact.
- **Tangent and radius:** The tangent at a point of a circle is perpendicular to the radius through that point.
- **Tangents from an external point:** The two tangents drawn from an external point to a circle are equal in length.
- **Alternate segment theorem:** The angle between a tangent and a chord through the point of contact equals the angle in the alternate segment.
- **Cyclic quadrilateral:** A quadrilateral whose four vertices lie on a circle. Its opposite angles are supplementary (they add up to 180 degrees).
- **Angle at centre:** The angle subtended by an arc at the centre is double the angle subtended by the same arc at any point on the remaining part of the circle.
- **Exterior angle of a cyclic quadrilateral:** The exterior angle of a cyclic quadrilateral equals the interior opposite angle.

Important Formulas

> Tangent is perpendicular to the radius at the point of contact

> Tangents from one external point are equal in length

> Opposite angles of a cyclic quadrilateral add to 180 degrees

Worked Examples

Example. A point P is 13 cm from the centre O of a circle of radius 5 cm. Find the length of the tangent from P.
Solution: Tangent length = sqrt(OP squared - r squared) = sqrt(169 - 25) = sqrt(144) = 12 cm.

Example. In a cyclic quadrilateral, one angle is 70 degrees. Find the opposite angle.
Solution: Opposite angles add to 180 degrees, so the opposite angle is 180 - 70 = 110 degrees.

Example. The angle at the centre subtended by an arc is 120 degrees. Find the angle at the circumference.
Solution: Half of 120 degrees is 60 degrees.

Example. Two tangents from a point are 8 cm and x cm long. Find x.
Solution: Tangents from the same external point are equal, so x = 8 cm.

Unit 9 — Questions and Practice

Short Questions

Q1. How many points does a tangent share with a circle?
Answer: Exactly one point, the point of contact.

Q2. What is the angle between a tangent and the radius at the point of contact?
Answer: 90 degrees.

Q3. Tangents drawn from the same external point are what?
Answer: Equal in length.

Q4. The angle at the centre is double the angle at the what?
Answer: At the circumference (on the same arc).

Q5. In a cyclic quadrilateral, if one angle is 70 degrees, what is its opposite angle?
Answer: 180 - 70 = 110 degrees.

Long Questions

Q1. Prove that the two tangents drawn from an external point to a circle are equal.
Answer: Let P be the external point, O the centre, and A and B the points of contact. In triangles OAP and OBP: OA = OB (radii), OP is common, and both angles at A and B are right angles (radius perpendicular to tangent). By RHS congruence the triangles are congruent, so PA = PB.

Q2. A point P is 10 cm from the centre of a circle of radius 6 cm. Find the length of the tangent from P and the angle between OP and the tangent.
Answer: Tangent length = sqrt(100 - 36) = 8 cm. In triangle OAP, sin of the angle at P = 6/10 = 0.6, so the angle is about 36.9 degrees.

Multiple Choice Questions

1. A tangent touches a circle at
(A) two points (B) exactly one point (C) no point (D) three points

2. The angle between a tangent and the radius at the point of contact is
(A) 45 degrees (B) 60 degrees (C) 90 degrees (D) 180 degrees

3. Tangents from the same external point are
(A) unequal (B) equal (C) perpendicular (D) parallel

4. In a cyclic quadrilateral, opposite angles are
(A) equal (B) supplementary (C) complementary (D) 90 degrees each

5. If one angle of a cyclic quadrilateral is 80 degrees, its opposite angle is
(A) 80 degrees (B) 100 degrees (C) 90 degrees (D) 180 degrees

6. The angle at the centre is 140 degrees. The angle at the circumference on the same arc is
(A) 70 degrees (B) 140 degrees (C) 280 degrees (D) 40 degrees

7. The alternate segment theorem relates the angle between a tangent and a chord to the angle in the
(A) same segment (B) alternate segment (C) centre (D) diameter

8. A line that cuts a circle at two points is a
(A) tangent (B) secant (C) radius (D) diameter

9. The exterior angle of a cyclic quadrilateral equals the
(A) adjacent angle (B) interior opposite angle (C) sum of all angles (D) right angle

10. Point P is 25 cm from the centre of a circle of radius 7 cm. The tangent length is
(A) 24 cm (B) 18 cm (C) 25 cm (D) 32 cm

Unit 10: Practical Geometry of Circles

What you will learn: Constructions with ruler and compasses: circles through points, circumscribed and inscribed circles, and tangents to a circle.

Key Notes

- **Circumcentre:** The centre of the circumscribed circle of a triangle. It is the intersection of the perpendicular bisectors of the sides, and it is equidistant from the three vertices.
- **Incentre:** The centre of the inscribed circle of a triangle. It is the intersection of the angle bisectors, and it is equidistant from the three sides.
- **Circle through three points:** Three non-collinear points define exactly one circle. Its centre is the intersection of the perpendicular bisectors of the segments joining the points.
- **Tangent at a point on the circle:** Join the point to the centre, then draw the line perpendicular to this radius at the point. That line is the tangent.
- **Tangents from an external point:** Join the external point P to the centre O, find the midpoint M of OP, draw a circle with centre M and radius MP; it cuts the given circle at the points of contact. Join P to each point of contact.
- **Circumscribed circle:** Draw the triangle, construct the perpendicular bisectors of two sides, mark their intersection as the centre, and draw the circle through any vertex.
- **Inscribed circle:** Draw the triangle, construct the bisectors of two angles, mark their intersection as the centre, drop a perpendicular from it to a side, and draw the circle with that perpendicular as radius.

Important Formulas

> Circumcentre = intersection of perpendicular bisectors of the sides

> Incentre = intersection of angle bisectors

> Tangent at a point = line perpendicular to the radius at that point

Worked Examples

Example. How is the circumcentre of a triangle found?
Solution: Construct the perpendicular bisectors of any two sides; their intersection is the circumcentre.

Example. How is a tangent drawn at a point P on a circle with centre O?
Solution: Join OP and draw the line through P perpendicular to OP. That line is the tangent at P.

Example. How many circles pass through three non-collinear points?
Solution: Exactly one.

Example. What is the centre of the inscribed circle of a triangle called?
Solution: The incentre.

Unit 10 — Questions and Practice

Short Questions

Q1. The circumcentre is the intersection of the what?
Answer: The perpendicular bisectors of the sides of the triangle.

Q2. The incentre is the intersection of the what?
Answer: The angle bisectors of the triangle.

Q3. A tangent at a point on a circle is perpendicular to the what?
Answer: To the radius through that point.

Q4. How many circles pass through three non-collinear points?
Answer: Exactly one circle.

Q5. To draw tangents from an external point, what is the first step?
Answer: Join the external point to the centre of the circle.

Long Questions

Q1. Describe the steps to circumscribe a circle about a triangle ABC with sides 5 cm, 6 cm and 7 cm.
Answer: Draw triangle ABC with the given sides. Construct the perpendicular bisectors of AB and BC and mark their intersection as O. With centre O and radius OA, draw the circle. It passes through A, B and C.

Q2. Describe the steps to inscribe a circle in a triangle ABC.
Answer: Draw triangle ABC. Bisect angles A and B and mark their intersection as I. From I drop a perpendicular to side BC, meeting it at D. With centre I and radius ID, draw the circle. It touches all three sides of the triangle.

Multiple Choice Questions

1. The circumcentre is the intersection of the
(A) angle bisectors (B) perpendicular bisectors of the sides (C) medians (D) altitudes

2. The incentre is the intersection of the
(A) perpendicular bisectors (B) angle bisectors (C) medians (D) diagonals

3. A tangent at a point on a circle is drawn perpendicular to the
(A) chord (B) radius through the point (C) diameter only (D) arc

4. Three non-collinear points determine
(A) no circle (B) exactly one circle (C) two circles (D) infinitely many circles

5. The centre of the inscribed circle is equidistant from the
(A) vertices (B) sides (C) midpoints only (D) circumcentre

6. The centre of the circumscribed circle is equidistant from the
(A) sides (B) vertices (C) midpoints (D) angles

7. To draw a tangent from an external point, the helper circle is drawn with centre at the
(A) centre of the given circle (B) midpoint of OP (C) point of contact (D) external point

8. The perpendicular bisector of a chord passes through the
(A) vertex (B) centre of the circle (C) tangent (D) arc

9. An inscribed circle touches
(A) one side (B) two sides (C) all three sides of the triangle (D) no side

10. The circumcentre of a right-angled triangle lies at the
(A) vertex (B) midpoint of the hypotenuse (C) incentre (D) centroid

Unit 11: Information Handling

What you will learn: Collecting and organising data, frequency tables, mean, median and mode, and reading bar charts, histograms and pie charts.

Key Notes

- **Data:** Facts or numbers collected for a purpose. Raw data is unorganised; a frequency table organises it by counting how often each value occurs.
- **Mean:** The average: sum of all values divided by the number of values. For a frequency table, mean = (sum of frequency times value) divided by (sum of frequencies).
- **Median:** The middle value when the data is arranged in order. For n values, the median is the ((n + 1)/2)th value when n is odd; when n is even it is the average of the two middle values.
- **Mode:** The value that occurs most often in the data.
- **Range:** The difference between the greatest and the smallest value.
- **Bar chart:** Bars of equal width whose heights show frequencies. It is used for discrete or categorical data.
- **Histogram:** A graph for continuous grouped data, with bars joined together and areas proportional to frequencies.
- **Pie chart:** A circle divided into sectors; each sector angle = (frequency / total) x 360 degrees.
- **Cumulative frequency:** The running total of frequencies. Its graph (ogive) is used to find the median of grouped data.

Important Formulas

> Mean = (sum of values) / (number of values)

> Median position = (n + 1) / 2 th value (ordered data)

> Range = greatest value - smallest value ; Sector angle = (frequency / total) x 360 degrees

Worked Examples

Example. Find the mean of 4, 6, 8, 10, 12.
Solution: Sum = 40, count = 5, mean = 40/5 = 8.

Example. Find the median of 3, 5, 7, 9, 11.
Solution: The data is ordered; the middle (3rd) value is 7.

Example. Find the mode of 2, 3, 3, 5, 3, 7.
Solution: 3 occurs three times, more than any other value, so the mode is 3.

Example. Find the range of 5, 12, 8, 20, 3.
Solution: Greatest = 20, smallest = 3, range = 17.

Unit 11 — Questions and Practice

Short Questions

Q1. Find the range of 5, 12, 8, 20, 3.
Answer: 20 - 3 = 17.

Q2. Find the mean of the first five natural numbers.
Answer: (1 + 2 + 3 + 4 + 5)/5 = 15/5 = 3.

Q3. Find the median of 2, 4, 6, 8.
Answer: The two middle values are 4 and 6, so the median is (4 + 6)/2 = 5.

Q4. Find the mode of 1, 2, 2, 3, 3, 3, 4.
Answer: 3 occurs three times, so the mode is 3.

Q5. What does the sum of all frequencies equal?
Answer: The total number of observations.

Long Questions

Q1. Find the mean of the data: value 2 occurs 3 times, value 4 occurs 5 times, value 6 occurs 2 times.
Answer: Sum of frequency times value = 2 x 3 + 4 x 5 + 6 x 2 = 6 + 20 + 12 = 38. Total frequency = 3 + 5 + 2 = 10. Mean = 38/10 = 3.8.

Q2. The marks of 10 students are 12, 15, 18, 20, 22, 25, 28, 30, 32, 35. Find the mean, the median and the range.
Answer: Sum = 237, so mean = 237/10 = 23.7. The data is ordered; the two middle values are 22 and 25, so the median is (22 + 25)/2 = 23.5. Range = 35 - 12 = 23.

Multiple Choice Questions

1. The mean of 5, 10, 15 is
(A) 5 (B) 10 (C) 15 (D) 30

2. The median of 3, 7, 9, 12, 15 is
(A) 7 (B) 9 (C) 12 (D) 10

3. The mode of 4, 4, 6, 8, 8, 8 is
(A) 4 (B) 6 (C) 8 (D) 4 and 8

4. The range of 2, 9, 14, 20 is
(A) 20 (B) 18 (C) 14 (D) 2

5. In a pie chart, the total of all sector angles is
(A) 90 degrees (B) 180 degrees (C) 270 degrees (D) 360 degrees

6. The mean of the first 10 natural numbers is
(A) 5 (B) 5.5 (C) 10 (D) 11

7. A histogram is used for
(A) categorical data (B) continuous grouped data (C) two variables only (D) probability

8. The median of 6, 2, 9, 4 is
(A) 4 (B) 5 (C) 6 (D) 2

9. If a value occurs 6 times out of 24 observations, its pie-chart sector angle is
(A) 60 degrees (B) 90 degrees (C) 120 degrees (D) 45 degrees

10. The sum of frequencies in a table equals the
(A) mean (B) total number of observations (C) mode (D) range

Unit 12: Probability

What you will learn: Experiments and sample spaces, probability of an event, complement, mutually exclusive and independent events.

Key Notes

- **Experiment and sample space:** An experiment is an activity with uncertain outcomes. The sample space is the set of all possible outcomes. An event is a subset of the sample space.
- **Probability of an event:** If all outcomes are equally likely, P(E) = (number of favourable outcomes) / (total number of outcomes).
- **Range of probability:** For any event E, 0 is less than or equal to P(E), and P(E) is less than or equal to 1. P(E) = 0 for an impossible event and P(E) = 1 for a certain event.
- **Complement:** P(not E) = 1 - P(E).
- **Mutually exclusive events:** Two events that cannot happen at the same time. For such events, P(A or B) = P(A) + P(B).
- **Independent events:** Events where one does not affect the other. For independent events, P(A and B) = P(A) x P(B).
- **Addition rule:** In general, P(A or B) = P(A) + P(B) - P(A and B).
- **Without replacement:** When an item is taken out and not put back, the total number of outcomes drops by one for the next draw.

Important Formulas

> P(E) = favourable outcomes / total outcomes

> P(not E) = 1 - P(E) ; 0 <= P(E) <= 1

> Mutually exclusive: P(A or B) = P(A) + P(B) ; Independent: P(A and B) = P(A) x P(B)

Worked Examples

Example. A fair coin is tossed once. Find P(head).
Solution: One favourable outcome out of two, so P = 1/2.

Example. A die is rolled once. Find P(even number).
Solution: Favourable outcomes are 2, 4, 6: three out of six, so P = 3/6 = 1/2.

Example. A bag has 3 red and 2 blue balls. One ball is drawn. Find P(red).
Solution: 3 favourable out of 5, so P = 3/5.

Example. P(E) = 0.3. Find P(not E).
Solution: P(not E) = 1 - 0.3 = 0.7.

Unit 12 — Questions and Practice

Short Questions

Q1. What is the probability of a certain event?
Answer: 1.

Q2. What is the probability of an impossible event?
Answer: 0.

Q3. A bag has 3 red and 2 blue balls. One ball is drawn. Find P(red).
Answer: 3/5.

Q4. A die is rolled once. Find P(not 6).
Answer: 5/6.

Q5. Two coins are tossed. Write the sample space.
Answer: {HH, HT, TH, TT}.

Long Questions

Q1. Two fair dice are rolled. Find the probability that the sum is 7.
Answer: There are 36 equally likely outcomes. The pairs giving sum 7 are (1,6), (2,5), (3,4), (4,3), (5,2), (6,1): six outcomes. P = 6/36 = 1/6.

Q2. A bag contains 4 red and 6 black balls. Two balls are drawn one after another without replacement. Find the probability of drawing a red ball first and a black ball second.
Answer: P(red first) = 4/10. After removing one red ball, 9 balls remain with 6 black, so P(black second) = 6/9. Combined probability = (4/10) x (6/9) = 24/90 = 4/15.

Multiple Choice Questions

1. The probability of getting a head on a fair coin is
(A) 1 (B) 1/2 (C) 1/4 (D) 0

2. The probability of an impossible event is
(A) 1 (B) 1/2 (C) 0 (D) cannot be found

3. A die is rolled once. P(6) equals
(A) 1/2 (B) 1/3 (C) 1/6 (D) 1

4. If P(E) = 0.4, then P(not E) equals
(A) 0.4 (B) 0.6 (C) 1.4 (D) 0

5. The sample space of tossing two coins has how many outcomes?
(A) 2 (B) 3 (C) 4 (D) 6

6. A bag has 5 white and 5 black balls. P(white) is
(A) 1/5 (B) 1/2 (C) 1/10 (D) 5

7. Two events that cannot occur together are called
(A) independent (B) mutually exclusive (C) certain (D) complementary

8. For independent events A and B, P(A and B) equals
(A) P(A) + P(B) (B) P(A) - P(B) (C) P(A) x P(B) (D) P(A)/P(B)

9. A card is drawn from a pack of 52 cards. P(ace) equals
(A) 1/13 (B) 1/4 (C) 1/52 (D) 4/13

10. P(E) can never be
(A) 0.5 (B) 1 (C) 1.5 (D) 0

Answer Key — Multiple Choice Questions

Unit 1 (Complex Numbers):

1. B 2. C 3. B 4. C 5. C 6. C 7. B 8. B 9. A 10. B

Unit 2 (Quadratic Equations and Inequalities):

1. B 2. B 3. B 4. B 5. C 6. C 7. A 8. C 9. A 10. A

Unit 3 (Matrices and Determinants):

1. B 2. B 3. B 4. A 5. B 6. B 7. B 8. B 9. A 10. A

Unit 4 (Functions and Graphs):

1. B 2. B 3. C 4. A 5. A 6. A 7. B 8. C 9. B 10. B

Unit 5 (Algebraic Fractions):

1. B 2. A 3. B 4. C 5. A 6. B 7. A 8. A 9. C 10. A

Unit 6 (Vectors in Plane):

1. A 2. B 3. A 4. A 5. C 6. C 7. C 8. B 9. B 10. B

Unit 7 (Trigonometry):

1. B 2. C 3. C 4. C 5. B 6. C 7. A 8. B 9. B 10. B

Unit 8 (Chords and Arcs of a Circle):

1. B 2. C 3. C 4. B 5. B 6. B 7. B 8. B 9. B 10. B

Unit 9 (Tangent and Angles of a Circle):

1. B 2. C 3. B 4. B 5. B 6. A 7. B 8. B 9. B 10. A

Unit 10 (Practical Geometry of Circles):

1. B 2. B 3. B 4. B 5. B 6. B 7. B 8. B 9. C 10. B

Unit 11 (Information Handling):

1. B 2. B 3. C 4. B 5. D 6. B 7. B 8. B 9. B 10. B

Unit 12 (Probability):

1. B 2. C 3. C 4. B 5. C 6. B 7. B 8. C 9. A 10. C

Short and long questions are answered with full working in each unit.