9th Class Mathematics Notes - Complete Chapter-Wise Study Notes for Punjab Students (2026-27)

Introduction

These mathematics class 9 notes cover the full book chapter by chapter, in simple words, for Punjab students preparing for the 2026-27 session. Each chapter gives a summary of the main ideas, the key formulas and rules to remember, worked examples with full solutions, important short questions with brief answers, important long questions with fuller answers, and a set of practice MCQs with an answer key. The notes explain ideas in the student's own study language and do not reproduce the textbook.

How to use these mathematics class 9 notes: read the summary of a chapter first and memorise its formulas, then study the worked examples step by step, then cover the answers and try the short questions on your own, and finally test yourself with the MCQs. Mathematics becomes easier with daily practice, so solve a few problems every day instead of reading the whole book in one sitting. These notes are Punjab-wide only: no board-specific claims are made anywhere in these notes.

Note: These notes follow the chapter order of the new PCTB Mathematics book for class 9 (2025 curriculum, session 2026-27), as listed by several study sources. Chapter titles and order may differ slightly in some printed editions, so students should check chapter titles against their own issued book. No paper pattern or marks distribution is claimed anywhere in these notes.

Chapter 1: Real Numbers

Summary

A number system is a set of numbers with rules for working with them. Natural numbers (N) are the counting numbers 1, 2, 3, and so on. Whole numbers (W) add zero to the natural numbers: 0, 1, 2, 3, and so on. Integers (Z) include all whole numbers and their negatives: ..., -3, -2, -1, 0, 1, 2, 3, ... Rational numbers (Q) are numbers that can be written as p/q, where p and q are integers and q is not zero; their decimal forms either terminate (like 0.5) or repeat (like 0.333...). Irrational numbers cannot be written as p/q; their decimals never terminate and never repeat, for example sqrt(2) and pi. Real numbers (R) are all rational and irrational numbers together, and every real number has a place on the number line.

Real numbers follow useful properties. Closure means the sum, difference, product and (except division by zero) quotient of two real numbers is again a real number. The commutative laws say a + b = b + a and ab = ba. The associative laws say (a + b) + c = a + (b + c) and (ab)c = a(bc). The distributive law says a(b + c) = ab + ac. Zero is the additive identity (a + 0 = a) and 1 is the multiplicative identity (a x 1 = a). Every number a has an additive inverse -a and, except zero, a multiplicative inverse 1/a.

A radical is a root of a number, written with the radical sign. In sqrt(n), n is the radicand. Radicals follow rules: sqrt(ab) = sqrt(a) x sqrt(b), sqrt(a/b) = sqrt(a)/sqrt(b), and sqrt(a^2) = a for a greater than or equal to zero. To rationalise a denominator means to remove radicals from the denominator by multiplying the top and bottom by a suitable factor, for example 1/sqrt(2) = sqrt(2)/2. Exponents follow laws: a^m x a^n = a^(m+n), a^m / a^n = a^(m-n), (a^m)^n = a^(mn), (ab)^n = a^n b^n, a^0 = 1 and a^(-n) = 1/a^n. The chapter also introduces complex numbers: numbers of the form a + bi, where i is the imaginary unit with i^2 = -1. The conjugate of a + bi is a - bi, and conjugates are used to simplify quotients of complex numbers.

Key formulas and rules

- Rational number: p/q, where p and q are integers and q is not zero.
- Closure: a + b, a - b, ab and a/b (b not zero) are real numbers for real a, b.
- Commutative: a + b = b + a and ab = ba.
- Associative: (a + b) + c = a + (b + c) and (ab)c = a(bc).
- Distributive: a(b + c) = ab + ac.
- Additive identity: a + 0 = a. Multiplicative identity: a x 1 = a.
- Additive inverse of a: -a. Multiplicative inverse of a (a not zero): 1/a.
- sqrt(ab) = sqrt(a) x sqrt(b) and sqrt(a/b) = sqrt(a)/sqrt(b) for suitable a, b.
- a^m x a^n = a^(m+n), a^m / a^n = a^(m-n), (a^m)^n = a^(mn), a^0 = 1, a^(-n) = 1/a^n.
- Complex number: a + bi, with i^2 = -1. Conjugate of a + bi is a - bi.

Worked examples

1. Simplify sqrt(72). Write 72 = 36 x 2, so sqrt(72) = sqrt(36) x sqrt(2) = 6 sqrt(2).

2. Rationalise the denominator of 3/sqrt(5). Multiply the top and bottom by sqrt(5): 3/sqrt(5) = (3 sqrt(5))/(sqrt(5) x sqrt(5)) = (3 sqrt(5))/5.

Short questions

1. What is the difference between a rational and an irrational number?

A rational number can be written as p/q with integers p, q and q not zero, and its decimal either terminates or repeats. An irrational number cannot be written as p/q, and its decimal neither terminates nor repeats, for example sqrt(2).

2. State the distributive law with an example.

The distributive law is a(b + c) = ab + ac. For example, 2(3 + 4) = 2 x 3 + 2 x 4 = 6 + 8 = 14.

3. Simplify: 2^3 x 2^4.

By the law a^m x a^n = a^(m+n), 2^3 x 2^4 = 2^(3+4) = 2^7 = 128.

4. What is the conjugate of 3 + 4i, and why is it useful?

The conjugate of 3 + 4i is 3 - 4i. It is useful because multiplying a complex number by its conjugate gives a real number: (3 + 4i)(3 - 4i) = 9 + 16 = 25.

Long questions

1. Explain the sets N, W, Z, Q and R and how they are related, with one example of each.

Natural numbers N = {1, 2, 3, ...} are the counting numbers. Whole numbers W = {0, 1, 2, ...} add zero. Integers Z = {..., -2, -1, 0, 1, 2, ...} add negatives. Rational numbers Q are all numbers of the form p/q with q not zero, for example 3/4. Irrational numbers, like sqrt(3), cannot be written as p/q. Real numbers R contain all rationals and all irrationals together. The sets sit inside each other: N is inside W, W is inside Z, Z is inside Q, and Q is inside R.

2. Simplify (sqrt(50) + sqrt(8)) and explain each step.

First simplify each radical: sqrt(50) = sqrt(25 x 2) = 5 sqrt(2), and sqrt(8) = sqrt(4 x 2) = 2 sqrt(2). Then add the like radicals: 5 sqrt(2) + 2 sqrt(2) = 7 sqrt(2). Like radicals can be added just like like terms, by adding their coefficients.

MCQs

1. Which of the following is an irrational number?

(a) 3/4

(b) 0.5

(c) sqrt(2)

(d) -7

2. The multiplicative identity of real numbers is:

(a) 0

(b) 1

(c) -1

(d) 2

3. sqrt(48) in simplest form is:

(a) 4 sqrt(3)

(b) 3 sqrt(4)

(c) 6 sqrt(2)

(d) 8 sqrt(3)

4. The value of 5^0 is:

(a) 0

(b) 5

(c) 1

(d) 25

5. The conjugate of 2 - 5i is:

(a) -2 + 5i

(b) 2 + 5i

(c) 5 - 2i

(d) -2 - 5i

6. Which property does a(b + c) = ab + ac show?

(a) Commutative law

(b) Associative law

(c) Distributive law

(d) Closure law

Answer key: 1. (c) 2. (b) 3. (a) 4. (c) 5. (b) 6. (c)

Chapter 2: Logarithms

Summary

Scientific notation writes very large or very small numbers as a x 10^n, where 1 is less than or equal to a and a is less than 10, and n is an integer. For example, 45000 = 4.5 x 10^4 and 0.00032 = 3.2 x 10^(-4). A logarithm is the inverse of an exponent: if a^x = y (with a greater than 0 and a not equal to 1), then log base a of y = x. So log base 2 of 8 = 3 because 2^3 = 8. The common logarithm uses base 10 and is written log, while the natural logarithm uses base e (about 2.718) and is written ln.

Logarithms follow laws that turn multiplication into addition and powers into multiplication. The product law says log(xy) = log x + log y. The quotient law says log(x/y) = log x - log y. The power law says log(x^n) = n log x. Two special values are log 1 = 0 (any base) and log a = 1 when the base is a. Logarithms are used to simplify long calculations: instead of multiplying large numbers, we add their logarithms and then find the antilogarithm of the result. Log tables give the mantissa (the decimal part) of common logarithms, while the characteristic (the integer part) is found from the position of the decimal point.

Key formulas and rules

- Scientific notation: a x 10^n, where 1 is less than or equal to a and a is less than 10.
- Definition: if a^x = y, then log base a of y = x (a greater than 0, a not equal to 1).
- Product law: log(xy) = log x + log y.
- Quotient law: log(x/y) = log x - log y.
- Power law: log(x^n) = n log x.
- log 1 = 0 and log base a of a = 1.
- Change of base: log base a of b = log b / log a (common logs).
- Antilog: if log x = y, then x = antilog(y).

Worked examples

1. Find log base 3 of 81. We need the power of 3 that gives 81. Since 3^4 = 81, log base 3 of 81 = 4.

2. Simplify log 20 - log 4 using the laws of logarithms. By the quotient law, log 20 - log 4 = log(20/4) = log 5.

Short questions

1. What is a logarithm?

A logarithm is the inverse of an exponent. If a^x = y, then log base a of y = x. It answers the question: to what power must the base be raised to get the number?

2. Write 0.00056 in scientific notation.

Move the decimal point 4 places to the right: 0.00056 = 5.6 x 10^(-4).

3. State the product and quotient laws of logarithms.

Product law: log(xy) = log x + log y. Quotient law: log(x/y) = log x - log y.

4. Find the value of log base 5 of 125.

Since 5^3 = 125, log base 5 of 125 = 3.

Long questions

1. Explain how logarithms simplify multiplication, with an example.

Logarithms turn multiplication into addition through the product law log(xy) = log x + log y. To multiply two large numbers, we look up their logarithms, add them, and then find the antilogarithm of the sum, which gives the product. For example, to find 25 x 40, we note log 25 + log 40 = log(25 x 40) = log 1000 = 3, so the product is 1000. This method was used for long calculations before calculators.

2. Simplify log(8) + log(125) - log(10) and explain each step.

First apply the product law: log(8) + log(125) = log(8 x 125) = log(1000). Then apply the quotient law: log(1000) - log(10) = log(1000/10) = log(100) = 2, because 10^2 = 100.

MCQs

1. log base 2 of 32 is:

(a) 4

(b) 5

(c) 6

(d) 8

2. In scientific notation, 72000 is written as:

(a) 72 x 10^3

(b) 7.2 x 10^4

(c) 0.72 x 10^5

(d) 7.2 x 10^3

3. log(xy) is equal to:

(a) log x - log y

(b) log x + log y

(c) log x x log y

(d) log x / log y

4. The value of log 1 (any base) is:

(a) 1

(b) 0

(c) 10

(d) Not defined

5. log(x^5) is equal to:

(a) 5 + log x

(b) 5 log x

(c) log x / 5

(d) log 5x

6. The natural logarithm has base:

(a) 10

(b) 2

(c) e

(d) 5

Answer key: 1. (b) 2. (b) 3. (b) 4. (b) 5. (b) 6. (c)

Chapter 3: Set and Functions

Summary

A set is a well-defined collection of distinct objects, called elements or members. Sets are usually written with capital letters, and elements are listed inside braces, for example A = {1, 2, 3}. A set can also be described by a property, like {x | x is an even number less than 10}. The empty set, written {} or phi, has no elements. A set with a countable number of elements is finite, and one that goes on without end is infinite. Set A is a subset of set B, written A subset B, if every element of A is also in B. Two sets are equal if they have exactly the same elements.

Operations on sets combine them in useful ways. The union A union B contains all elements in A or in B (or both). The intersection A intersection B contains only elements common to both. The difference A - B contains elements in A but not in B. The complement of A contains all elements of the universal set U that are not in A. Venn diagrams draw sets as overlapping circles inside a rectangle (the universal set) and make these operations easy to see. An ordered pair (a, b) has an order: (a, b) equals (c, d) only when a = c and b = d. The Cartesian product A x B is the set of all ordered pairs (a, b) with a in A and b in B.

A binary relation from A to B is a subset of A x B. A function is a special relation in which every element of the domain (the first set) is related to exactly one element of the range (the second set). For a function f, we write f(x) = y, where x is the input and y is the output. The domain is the set of all inputs and the range is the set of all outputs actually produced. A function is one-one if different inputs always give different outputs, and onto if every element of the second set is an output of some input. Linear functions have the form f(x) = ax + b and their graphs are straight lines, while quadratic functions have the form f(x) = ax^2 + bx + c and their graphs are curves called parabolas.

Key formulas and rules

- Union: A union B = {x | x in A or x in B}.
- Intersection: A intersection B = {x | x in A and x in B}.
- Difference: A - B = {x | x in A and x not in B}.
- Complement of A = {x | x in U and x not in A}.
- Number of elements: n(A union B) = n(A) + n(B) - n(A intersection B).
- Ordered pair equality: (a, b) = (c, d) means a = c and b = d.
- Cartesian product: A x B = {(a, b) | a in A, b in B}. If A has m elements and B has n, A x B has mn elements.
- Function rule: every element of the domain has exactly one image.
- Linear function: f(x) = ax + b. Quadratic function: f(x) = ax^2 + bx + c.

Worked examples

1. If A = {1, 2, 3, 4} and B = {3, 4, 5, 6}, find A union B and A intersection B. A union B = {1, 2, 3, 4, 5, 6} (all elements of both, written once). A intersection B = {3, 4} (only the common elements).

2. If f(x) = 2x + 3, find f(4). Replace x by 4: f(4) = 2(4) + 3 = 8 + 3 = 11.

Short questions

1. What is the difference between a finite and an infinite set?

A finite set has a countable number of elements, like {1, 2, 3}. An infinite set goes on without end, like the set of all natural numbers.

2. Define a function.

A function is a relation in which every element of the domain is related to exactly one element of the range. Each input has one and only one output.

3. If A = {a, b} and B = {1, 2, 3}, how many elements does A x B have?

A has 2 elements and B has 3, so A x B has 2 x 3 = 6 elements.

4. What are the domain and range of f(x) = x + 5 for x in {1, 2, 3}?

The domain is {1, 2, 3} (the inputs). The outputs are 6, 7, 8, so the range is {6, 7, 8}.

Long questions

1. Explain union, intersection and difference of sets with an example and Venn diagrams.

Take A = {1, 2, 3} and B = {3, 4, 5}. The union A union B = {1, 2, 3, 4, 5} collects every element in either set. The intersection A intersection B = {3} collects only the common element. The difference A - B = {1, 2} collects elements in A but not in B. In a Venn diagram, two overlapping circles are drawn inside a rectangle: the overlapping part shows the intersection, the whole area covered by both circles shows the union, and the part of circle A outside circle B shows the difference.

2. Show that the relation {(1, 2), (2, 4), (3, 6)} is a function, and find its domain and range.

Each first element (input) appears once: 1 maps to 2, 2 maps to 4, 3 maps to 6. Since every input has exactly one output, the relation is a function. Its domain is {1, 2, 3} and its range is {2, 4, 6}.

MCQs

1. The empty set is written as:

(a) {0}

(b) {}

(c) {phi}

(d) {1}

2. If A = {1, 2} and B = {2, 3}, then A intersection B is:

(a) {1, 2, 3}

(b) {2}

(c) {1}

(d) {3}

3. A function must give each input:

(a) Two outputs

(b) Exactly one output

(c) No output

(d) Many outputs

4. If f(x) = 3x - 1, then f(2) is:

(a) 5

(b) 6

(c) 7

(d) 4

5. The number of elements in A x B, where A has 4 elements and B has 3, is:

(a) 7

(b) 12

(c) 9

(d) 6

6. The complement of set A is taken with respect to:

(a) The empty set

(b) The universal set

(c) Any set

(d) Set A itself

Answer key: 1. (b) 2. (b) 3. (b) 4. (a) 5. (b) 6. (b)

Chapter 4: Factorization and Algebraic Manipulation

Summary

Factorization means writing an expression as a product of its factors. The first step is usually to take out the common factor: for example, 6x + 9 = 3(2x + 3). When there is no single common factor, grouping can help: ax + ay + bx + by = a(x + y) + b(x + y) = (x + y)(a + b). Special identities factorise quickly. The difference of two squares is a^2 - b^2 = (a - b)(a + b). The perfect squares are a^2 + 2ab + b^2 = (a + b)^2 and a^2 - 2ab + b^2 = (a - b)^2. The sums and differences of cubes are a^3 + b^3 = (a + b)(a^2 - ab + b^2) and a^3 - b^3 = (a - b)(a^2 + ab + b^2).

A quadratic trinomial x^2 + px + q is factorised by finding two numbers whose sum is p and whose product is q; then x^2 + px + q = (x + m)(x + n). For ax^2 + bx + c with a not equal to 1, find two numbers whose product is ac and whose sum is b, split the middle term, and factorise by grouping. Factorization also gives the HCF (highest common factor) and LCM (lowest common multiple) of algebraic expressions: the HCF is the product of the common factors with the lowest powers, and the LCM is the product of all factors with the highest powers. Algebraic manipulation means simplifying expressions using these ideas, including cancelling common factors in fractions and combining fractions over a common denominator.

Key formulas and rules

- Common factor: ab + ac = a(b + c).
- Difference of squares: a^2 - b^2 = (a - b)(a + b).
- Perfect square: a^2 + 2ab + b^2 = (a + b)^2 and a^2 - 2ab + b^2 = (a - b)^2.
- Sum of cubes: a^3 + b^3 = (a + b)(a^2 - ab + b^2).
- Difference of cubes: a^3 - b^3 = (a - b)(a^2 + ab + b^2).
- Quadratic: x^2 + px + q = (x + m)(x + n), where m + n = p and mn = q.
- HCF: product of common factors with lowest powers. LCM: product of all factors with highest powers.

Worked examples

1. Factorise x^2 + 7x + 12. Find two numbers with sum 7 and product 12: 3 and 4. So x^2 + 7x + 12 = (x + 3)(x + 4).

2. Factorise 2x^2 + 7x + 3. We need two numbers with product 2 x 3 = 6 and sum 7: 6 and 1. Split the middle term: 2x^2 + 6x + x + 3 = 2x(x + 3) + 1(x + 3) = (x + 3)(2x + 1).

Short questions

1. Factorise: 5x - 15.

The common factor is 5, so 5x - 15 = 5(x - 3).

2. Factorise: x^2 - 25.

This is a difference of squares: x^2 - 5^2 = (x - 5)(x + 5).

3. What is the HCF of 6x^2 and 9x?

The common factors are 3 and x, so the HCF is 3x.

4. Factorise: x^2 - 9x + 20.

Two numbers with sum -9 and product 20 are -4 and -5, so x^2 - 9x + 20 = (x - 4)(x - 5).

Long questions

1. Factorise fully: x^3 - 8.

This is a difference of cubes: x^3 - 2^3 = (x - 2)(x^2 + 2x + 4), using a^3 - b^3 = (a - b)(a^2 + ab + b^2) with a = x and b = 2. The quadratic factor x^2 + 2x + 4 cannot be factorised further over real numbers.

2. Find the HCF and LCM of x^2 - 4 and x^2 + 4x + 4.

Factorise: x^2 - 4 = (x - 2)(x + 2) and x^2 + 4x + 4 = (x + 2)^2. The common factor is (x + 2), so the HCF is (x + 2). The LCM takes all factors with highest powers: (x - 2)(x + 2)^2.

MCQs

1. The factorization of x^2 - 16 is:

(a) (x - 4)(x + 4)

(b) (x - 8)(x + 8)

(c) (x - 4)^2

(d) (x + 4)^2

2. a^2 + 2ab + b^2 equals:

(a) (a - b)^2

(b) (a + b)^2

(c) (a + b)(a - b)

(d) a^2 - b^2

3. The factors of x^2 + 5x + 6 are:

(a) (x + 1)(x + 6)

(b) (x + 2)(x + 3)

(c) (x - 2)(x - 3)

(d) (x + 5)(x + 1)

4. The HCF of 12a^2 and 18a is:

(a) 6a

(b) 6a^2

(c) 36a

(d) 3a

5. x^3 + 27 factorises as:

(a) (x + 3)(x^2 - 3x + 9)

(b) (x - 3)(x^2 + 3x + 9)

(c) (x + 3)^3

(d) (x + 3)(x^2 + 3x + 9)

6. The common factor in 4x^2 + 8x is:

(a) 2x

(b) 4x

(c) 8

(d) x^2

Answer key: 1. (a) 2. (b) 3. (b) 4. (a) 5. (a) 6. (b)

Chapter 5: Linear Equations and Inequalities

Summary

A linear equation in one variable has the form ax + b = 0, where a is not zero, and its solution is x = -b/a. To solve an equation, we use inverse operations: add or subtract the same number on both sides, or multiply or divide both sides by the same non-zero number, until the variable stands alone. Equations with fractions are cleared by multiplying both sides by the common denominator. A linear equation in two variables has the form ax + by = c and its graph is a straight line; each solution is an ordered pair (x, y) that satisfies the equation.

Two linear equations together form a simultaneous system. The elimination method adds or subtracts the equations to remove one variable, while the substitution method solves one equation for one variable and puts that expression into the other equation. Both methods give the same answer: the pair (x, y) that satisfies both equations, which is also the point where the two lines cross.

An inequality uses signs greater than, less than, greater than or equal to, or less than or equal to. Inequalities are solved like equations, with one important difference: multiplying or dividing both sides by a negative number reverses the inequality sign. For example, -2x greater than 6 gives x less than -3 after dividing by -2. The solution of an inequality is a range of values, often shown on a number line with an open circle for strict inequality and a closed circle when the endpoint is included.

Key formulas and rules

- Linear equation in one variable: ax + b = 0 gives x = -b/a (a not zero).
- Linear equation in two variables: ax + by = c.
- Solving rule: the same operation on both sides keeps the equation balanced.
- Elimination: add or subtract equations to remove one variable.
- Substitution: solve one equation for one variable, substitute into the other.
- Inequality rule: multiplying or dividing by a negative number reverses the sign.
- Absolute value: |x| = x if x is greater than or equal to 0, and -x if x is less than 0.

Worked examples

1. Solve 3x - 7 = 11. Add 7 to both sides: 3x = 18. Divide by 3: x = 6.

2. Solve the system x + y = 10 and x - y = 2 by elimination. Add the equations: (x + y) + (x - y) = 10 + 2, so 2x = 12 and x = 6. Put x = 6 in the first equation: 6 + y = 10, so y = 4. The solution is (6, 4).

Short questions

1. Solve: 5x + 3 = 23.

Subtract 3 from both sides: 5x = 20. Divide by 5: x = 4.

2. What happens to an inequality sign when both sides are multiplied by -3?

The sign reverses: greater than becomes less than, and less than becomes greater than.

3. Solve: 2x - 5 greater than 9.

Add 5 to both sides: 2x greater than 14. Divide by 2: x greater than 7.

4. Is (3, 2) a solution of x + 2y = 7?

Put x = 3 and y = 2: 3 + 2(2) = 3 + 4 = 7. Yes, it satisfies the equation.

Long questions

1. Solve the system 2x + 3y = 12 and x - y = 1 by substitution, and check the answer.

From the second equation, x = 1 + y. Substitute into the first: 2(1 + y) + 3y = 12, so 2 + 2y + 3y = 12, giving 5y = 10 and y = 2. Then x = 1 + 2 = 3. Check: 2(3) + 3(2) = 6 + 6 = 12, and 3 - 2 = 1. The solution (3, 2) is correct.

2. Solve 4 - 3x less than or equal to 10 and show the solution on a number line in words.

Subtract 4 from both sides: -3x less than or equal to 6. Divide by -3 and reverse the sign: x greater than or equal to -2. On a number line, this is a closed circle at -2 (because -2 is included) with shading to the right, covering all numbers from -2 upwards.

MCQs

1. The solution of 2x + 6 = 14 is:

(a) 2

(b) 4

(c) 6

(d) 8

2. Which pair satisfies x + y = 9 and x - y = 3?

(a) (6, 3)

(b) (5, 4)

(c) (3, 6)

(d) (4, 5)

3. Solving -x greater than 5 gives:

(a) x greater than -5

(b) x less than -5

(c) x greater than 5

(d) x less than 5

4. The equation 3x - 2 = 10 has solution:

(a) 2

(b) 3

(c) 4

(d) 6

5. A linear equation in two variables has a graph that is a:

(a) Circle

(b) Parabola

(c) Straight line

(d) Curve

6. In the inequality x less than or equal to 4, the endpoint 4 is:

(a) Not included

(b) Included

(c) Ignored

(d) Doubled

Answer key: 1. (b) 2. (a) 3. (b) 4. (c) 5. (c) 6. (b)

Chapter 6: Trigonometry

Summary

Trigonometry studies the relations between the angles and sides of triangles. In a right-angled triangle, with reference to an acute angle theta, the side opposite the right angle is the hypotenuse, the side opposite theta is the perpendicular, and the side next to theta (other than the hypotenuse) is the base. The six trigonometric ratios are: sin theta = perpendicular/hypotenuse, cos theta = base/hypotenuse, tan theta = perpendicular/base, cosec theta = hypotenuse/perpendicular, sec theta = hypotenuse/base, and cot theta = base/perpendicular. Note that cosec, sec and cot are the reciprocals of sin, cos and tan.

Some angles have exact ratios worth memorising. For 30 degrees: sin = 1/2, cos = sqrt(3)/2, tan = 1/sqrt(3). For 45 degrees: sin = 1/sqrt(2), cos = 1/sqrt(2), tan = 1. For 60 degrees: sin = sqrt(3)/2, cos = 1/2, tan = sqrt(3). The fundamental identities are: sin^2 theta + cos^2 theta = 1, 1 + tan^2 theta = sec^2 theta, and 1 + cot^2 theta = cosec^2 theta. These identities prove many other results and simplify expressions.

Trigonometry solves real-life height and distance problems. The angle of elevation is the angle between the horizontal line and the line of sight when looking up at an object, and the angle of depression is the angle when looking down. If the height of a tower and the angle of elevation are known, the distance can be found using tan theta = height/distance. Problems are solved by drawing a right-angled triangle, marking the known side and angle, choosing the ratio that links them to the unknown, and solving.

Key formulas and rules

- sin theta = perpendicular/hypotenuse, cos theta = base/hypotenuse, tan theta = perpendicular/base.
- cosec theta = 1/sin theta, sec theta = 1/cos theta, cot theta = 1/tan theta.
- tan theta = sin theta / cos theta.
- sin^2 theta + cos^2 theta = 1.
- 1 + tan^2 theta = sec^2 theta and 1 + cot^2 theta = cosec^2 theta.
- sin 30 = 1/2, cos 30 = sqrt(3)/2, tan 30 = 1/sqrt(3).
- sin 45 = 1/sqrt(2), cos 45 = 1/sqrt(2), tan 45 = 1.
- sin 60 = sqrt(3)/2, cos 60 = 1/2, tan 60 = sqrt(3).
- Angle of elevation: looking up from the horizontal. Angle of depression: looking down from the horizontal.

Worked examples

1. In a right triangle, the perpendicular is 3 and the hypotenuse is 5. Find sin theta and cos theta. sin theta = perpendicular/hypotenuse = 3/5. The base is sqrt(5^2 - 3^2) = sqrt(25 - 9) = sqrt(16) = 4, so cos theta = base/hypotenuse = 4/5.

2. A tower casts a shadow 20 m long when the angle of elevation of the sun is 60 degrees. Find the height of the tower. tan 60 = height/20, so height = 20 x tan 60 = 20 x sqrt(3) = 20 sqrt(3) m, about 34.6 m.

Short questions

1. Define sin theta and cos theta in a right triangle.

sin theta = perpendicular/hypotenuse and cos theta = base/hypotenuse, where the perpendicular and base are taken with reference to angle theta.

2. What is the value of tan 45 degrees?

tan 45 = 1, because in a 45-degree triangle the perpendicular and base are equal.

3. State the identity connecting sin and cos.

sin^2 theta + cos^2 theta = 1 for any angle theta.

4. What is the angle of depression?

It is the angle between the horizontal line and the line of sight when an observer looks down at an object below.

Long questions

1. Prove that 1 + tan^2 theta = sec^2 theta.

Start from sin^2 theta + cos^2 theta = 1. Divide every term by cos^2 theta: (sin^2 theta)/(cos^2 theta) + (cos^2 theta)/(cos^2 theta) = 1/(cos^2 theta). This gives tan^2 theta + 1 = sec^2 theta, since sin/cos = tan and 1/cos = sec.

2. A ladder 10 m long leans against a wall and makes an angle of 60 degrees with the ground. How high does it reach?

The ladder is the hypotenuse (10 m). Height = hypotenuse x sin 60 = 10 x sqrt(3)/2 = 5 sqrt(3) m, about 8.66 m. The ladder reaches about 8.66 m up the wall.

MCQs

1. sin theta is defined as:

(a) base/hypotenuse

(b) perpendicular/hypotenuse

(c) perpendicular/base

(d) hypotenuse/perpendicular

2. The value of cos 60 degrees is:

(a) sqrt(3)/2

(b) 1/2

(c) 1/sqrt(2)

(d) sqrt(3)

3. tan theta equals:

(a) cos theta / sin theta

(b) sin theta / cos theta

(c) 1/sin theta

(d) 1/cos theta

4. sin^2 theta + cos^2 theta equals:

(a) 0

(b) 2

(c) 1

(d) tan theta

5. sec theta is the reciprocal of:

(a) sin theta

(b) cos theta

(c) tan theta

(d) cot theta

6. The angle of elevation is measured from:

(a) The vertical

(b) The horizontal

(c) The hypotenuse

(d) The ground line only

Answer key: 1. (b) 2. (b) 3. (b) 4. (c) 5. (b) 6. (b)

Chapter 7: Coordinate Geometry

Summary

Coordinate geometry joins algebra and geometry using the coordinate plane. A point is written as an ordered pair (x, y), where x is the horizontal distance from the origin (0, 0) and y is the vertical distance. The plane is divided into four quadrants: in the first, both coordinates are positive; in the second, x is negative and y is positive; in the third, both are negative; in the fourth, x is positive and y is negative. Points on the x-axis have y = 0, and points on the y-axis have x = 0.

The distance between two points P(x1, y1) and Q(x2, y2) is found by the distance formula: d = sqrt((x2 - x1)^2 + (y2 - y1)^2), which comes from the Pythagoras theorem. The midpoint of PQ is ((x1 + x2)/2, (y1 + y2)/2). The section formula gives the point dividing PQ in the ratio m:n as ((mx2 + nx1)/(m + n), (my2 + ny1)/(m + n)). The slope (or gradient) of a line through P and Q is m = (y2 - y1)/(x2 - x1), which measures steepness: a positive slope rises to the right, a negative slope falls, zero slope is horizontal, and an undefined slope is vertical.

A straight line can be written in several forms. The slope-intercept form is y = mx + c, where m is the slope and c is the y-intercept. The point-slope form is y - y1 = m(x - x1). The two-point form uses two points to find the slope first and then the equation. Two lines are parallel if their slopes are equal, and perpendicular if the product of their slopes is -1 (m1 x m2 = -1). The area of a triangle with vertices (x1, y1), (x2, y2), (x3, y3) is (1/2)|x1(y2 - y3) + x2(y3 - y1) + x3(y1 - y2)|.

Key formulas and rules

- Distance: d = sqrt((x2 - x1)^2 + (y2 - y1)^2).
- Midpoint: ((x1 + x2)/2, (y1 + y2)/2).
- Section formula (m:n): ((mx2 + nx1)/(m + n), (my2 + ny1)/(m + n)).
- Slope: m = (y2 - y1)/(x2 - x1).
- Slope-intercept form: y = mx + c.
- Point-slope form: y - y1 = m(x - x1).
- Parallel lines: m1 = m2. Perpendicular lines: m1 x m2 = -1.
- Area of triangle: (1/2)|x1(y2 - y3) + x2(y3 - y1) + x3(y1 - y2)|.

Worked examples

1. Find the distance between A(1, 2) and B(4, 6). d = sqrt((4 - 1)^2 + (6 - 2)^2) = sqrt(3^2 + 4^2) = sqrt(9 + 16) = sqrt(25) = 5.

2. Find the equation of the line with slope 2 passing through (1, 3). Use point-slope form: y - 3 = 2(x - 1), so y - 3 = 2x - 2, giving y = 2x + 1.

Short questions

1. What is the slope of a horizontal line?

Zero, because y2 - y1 = 0, so the slope (y2 - y1)/(x2 - x1) = 0.

2. Find the midpoint of (2, 3) and (6, 7).

Midpoint = ((2 + 6)/2, (3 + 7)/2) = (4, 5).

3. When are two lines perpendicular?

When the product of their slopes is -1, that is, m1 x m2 = -1.

4. Write the slope-intercept form and name its parts.

y = mx + c, where m is the slope of the line and c is the y-intercept (where the line crosses the y-axis).

Long questions

1. Show that the points A(0, 0), B(3, 0) and C(0, 4) form a right-angled triangle.

Find the side lengths: AB = 3, AC = 4, and BC = sqrt((3 - 0)^2 + (0 - 4)^2) = sqrt(9 + 16) = 5. Since 3^2 + 4^2 = 9 + 16 = 25 = 5^2, the sides satisfy the Pythagoras theorem, so the triangle is right-angled at A.

2. Find the equation of the line joining (1, 2) and (3, 6).

First find the slope: m = (6 - 2)/(3 - 1) = 4/2 = 2. Use point-slope form with (1, 2): y - 2 = 2(x - 1), so y - 2 = 2x - 2, giving y = 2x.

MCQs

1. The distance between (0, 0) and (3, 4) is:

(a) 5

(b) 7

(c) 3

(d) 4

2. The midpoint of (1, 5) and (5, 1) is:

(a) (3, 3)

(b) (2, 2)

(c) (4, 4)

(d) (3, 2)

3. The slope of the line joining (2, 3) and (4, 7) is:

(a) 1

(b) 2

(c) 3

(d) 4

4. In y = mx + c, c represents:

(a) The slope

(b) The y-intercept

(c) The x-intercept

(d) The origin

5. Two lines with slopes 2 and -1/2 are:

(a) Parallel

(b) Perpendicular

(c) The same line

(d) Intersecting at 45 degrees

6. A point on the x-axis always has:

(a) x = 0

(b) y = 0

(c) x = y

(d) x = 1

Answer key: 1. (a) 2. (a) 3. (b) 4. (b) 5. (b) 6. (b)

Chapter 8: Logic

Summary

Logic is the study of correct reasoning. A statement (or proposition) is a sentence that is either true or false, but not both. For example, "2 + 2 = 4" is a true statement, and "5 is less than 3" is a false statement. Questions, commands and exclamations are not statements because they cannot be assigned true or false. Statements are combined with connectives: negation (not), conjunction (and), disjunction (or), implication (if... then) and biconditional (if and only if).

The truth value of a compound statement is found from a truth table. The negation of p, written ~p, is true exactly when p is false. The conjunction p and q is true only when both p and q are true. The disjunction p or q is false only when both p and q are false. The implication p implies q is false only when p is true and q is false; in all other cases it is true. The biconditional p if and only if q is true when p and q have the same truth value.

From an implication p implies q, we form three related statements: the converse (q implies p), the inverse (~p implies ~q), and the contrapositive (~q implies ~p). The contrapositive always has the same truth value as the original implication, but the converse and inverse do not. A statement that is always true is a tautology, one that is always false is a contradiction, and one that is sometimes true and sometimes false is a contingency. Logical reasoning is used in mathematics proofs and in computer programming, where conditions control what a program does.

Key formulas and rules

- Statement: a sentence that is either true or false, but not both.
- Negation: ~p is true exactly when p is false.
- Conjunction: p and q is true only when both p and q are true.
- Disjunction: p or q is false only when both p and q are false.
- Implication: p implies q is false only when p is true and q is false.
- Biconditional: p if and only if q is true when p and q have the same truth value.
- Converse of p implies q: q implies p. Inverse: ~p implies ~q. Contrapositive: ~q implies ~p.
- Contrapositive always matches the original implication in truth value.
- Tautology: always true. Contradiction: always false. Contingency: sometimes true.

Worked examples

1. Write the truth table for p and q (conjunction). When p is true and q is true, p and q is true. In the other three cases (true-false, false-true, false-false), p and q is false. So the conjunction is true in only one of the four rows.

2. Write the converse and contrapositive of: "If it rains, then the ground is wet." Converse: "If the ground is wet, then it rains." Contrapositive: "If the ground is not wet, then it does not rain." The contrapositive is always equivalent to the original, but the converse is not (the ground could be wet for another reason).

Short questions

1. What is a statement? Give one example that is not a statement.

A statement is a sentence that is either true or false, but not both. "Sit down" is not a statement, because a command cannot be called true or false.

2. When is p or q false?

p or q is false only when both p and q are false. In all other cases it is true.

3. What is the contrapositive of p implies q?

The contrapositive is ~q implies ~p, and it always has the same truth value as p implies q.

4. Define a tautology.

A tautology is a compound statement that is true for every combination of truth values of its parts, for example p or ~p.

Long questions

1. Build the truth table for p implies q and explain why it is false in only one row.

The four rows are: p true, q true gives true, because a true statement implying a true statement holds. p true, q false gives false, because a true statement cannot imply a false one. p false, q true gives true, and p false, q false gives true, because an implication makes no claim when its condition is false. So the implication is false only in the true-false row.

2. Explain the difference between converse, inverse and contrapositive with an example.

Take "If a number is even, then it is divisible by 2." The converse swaps the parts: "If a number is divisible by 2, then it is even." The inverse negates both parts: "If a number is not even, then it is not divisible by 2." The contrapositive both swaps and negates: "If a number is not divisible by 2, then it is not even." Only the contrapositive is always equivalent to the original statement.

MCQs

1. Which of the following is a statement?

(a) What is your name?

(b) Close the door.

(c) 3 + 4 = 7

(d) Wow!

2. The negation of a true statement is:

(a) True

(b) False

(c) Both

(d) Neither

3. p and q is true only when:

(a) p is true

(b) q is true

(c) Both p and q are true

(d) Either is true

4. p implies q is false when:

(a) p is false, q is true

(b) p is true, q is false

(c) Both are true

(d) Both are false

5. The contrapositive of p implies q is:

(a) q implies p

(b) ~p implies ~q

(c) ~q implies ~p

(d) ~p implies q

6. A statement that is always true is called a:

(a) Contradiction

(b) Contingency

(c) Tautology

(d) Fallacy

Answer key: 1. (c) 2. (b) 3. (c) 4. (b) 5. (c) 6. (c)

Chapter 9: Similar Figures

Summary

Ratio compares two quantities of the same kind by division: the ratio of a to b is written a:b or a/b. A proportion is an equality of two ratios, a:b = c:d, where a and d are the extremes and b and c are the means. In a proportion, the product of the extremes equals the product of the means: ad = bc. Ratio and proportion are used to scale figures, maps and models.

Two figures are similar if they have the same shape but may differ in size. Their corresponding angles are equal and their corresponding sides are in the same ratio. Triangles are similar by three tests: AAA (all three corresponding angles equal), SSS (all three corresponding sides in proportion), and SAS (two sides in proportion and the included angles equal). Similar triangles are the key to indirect measurement: the height of a tall object can be found from its shadow by comparing with a smaller object of known height.

The Pythagoras theorem states that in a right-angled triangle, the square of the hypotenuse equals the sum of the squares of the other two sides: c^2 = a^2 + b^2. It is used to find a missing side and to test whether a triangle is right-angled. When two figures are similar with linear scale factor k, the ratio of their areas is k^2. So if one triangle's sides are twice another's, its area is four times as large. The converse of Pythagoras also holds: if c^2 = a^2 + b^2 for the sides of a triangle, the triangle is right-angled.

Key formulas and rules

- Ratio: a:b = a/b (a and b of the same kind).
- Proportion: a:b = c:d gives ad = bc (product of extremes = product of means).
- Similar figures: corresponding angles equal, corresponding sides in proportion.
- Similarity tests for triangles: AAA, SSS, SAS.
- Pythagoras theorem: c^2 = a^2 + b^2 (c is the hypotenuse).
- Converse: if c^2 = a^2 + b^2, the triangle is right-angled.
- Area ratio of similar figures = (linear scale factor)^2.

Worked examples

1. A pole 6 m tall casts a shadow 4 m long. At the same time, a tower casts a shadow 20 m long. Find the tower's height. The pole and tower form similar triangles with their shadows, so height/shadow is the same: 6/4 = h/20. Cross-multiplying: 4h = 120, so h = 30 m.

2. Check whether sides 5, 12, 13 form a right-angled triangle. The longest side is 13. Check: 5^2 + 12^2 = 25 + 144 = 169 = 13^2. The Pythagoras relation holds, so the triangle is right-angled.

Short questions

1. What does it mean for two figures to be similar?

They have the same shape: corresponding angles are equal and corresponding sides are in the same ratio, though their sizes may differ.

2. State the SSS similarity test.

Two triangles are similar if all three pairs of corresponding sides are in the same proportion.

3. In a proportion a:b = c:d, which terms are the means?

b and c are the means, and a and d are the extremes. The rule is ad = bc.

4. If two similar triangles have sides in the ratio 3:5, what is the ratio of their areas?

The area ratio is the square of the linear ratio: 3^2 : 5^2 = 9:25.

Long questions

1. Prove the Pythagoras theorem statement and use it to find the diagonal of a rectangle 8 cm by 6 cm.

In a right triangle with legs a and b and hypotenuse c, c^2 = a^2 + b^2. A rectangle's diagonal divides it into two right triangles with legs 8 cm and 6 cm, so diagonal^2 = 8^2 + 6^2 = 64 + 36 = 100, giving diagonal = 10 cm.

2. Two triangles ABC and DEF are similar with AB = 4, DE = 6 and BC = 8. Find EF and explain the working.

Since the triangles are similar, corresponding sides are in proportion: AB/DE = BC/EF. So 4/6 = 8/EF. Cross-multiplying: 4 x EF = 48, so EF = 12. The scale factor from the first triangle to the second is 6/4 = 1.5, and 8 x 1.5 = 12 confirms the answer.

MCQs

1. In the proportion 2:3 = 4:6, the product of the means is:

(a) 8

(b) 12

(c) 18

(d) 24

2. Two triangles are similar by AAA if:

(a) All sides are equal

(b) All corresponding angles are equal

(c) One angle is equal

(d) They have the same area

3. In a right triangle with legs 9 and 12, the hypotenuse is:

(a) 13

(b) 14

(c) 15

(d) 21

4. If the sides of two similar figures are in ratio 2:3, their areas are in ratio:

(a) 2:3

(b) 4:9

(c) 8:27

(d) 1:2

5. The Pythagoras theorem applies to:

(a) All triangles

(b) Right-angled triangles only

(c) Isosceles triangles only

(d) Equilateral triangles only

6. If 3:5 = x:20, then x is:

(a) 10

(b) 12

(c) 15

(d) 8

Answer key: 1. (b) 2. (b) 3. (c) 4. (b) 5. (b) 6. (b)

Chapter 10: Graphs of Functions

Summary

A graph draws a function on the coordinate plane. The input values go on the x-axis and the output values on the y-axis. To graph a function, we make a table of values: choose several x values, compute the matching y values, plot the points (x, y), and join them. The graph of a linear function f(x) = ax + b is a straight line. Its slope a tells how steep it is, and its y-intercept b is where it crosses the y-axis. Two points are enough to draw a line, but a third point is a useful check.

The graph of a quadratic function f(x) = ax^2 + bx + c is a parabola, a U-shaped curve. If a is greater than zero, the parabola opens upward and has a lowest point (minimum); if a is less than zero, it opens downward and has a highest point (maximum). The turning point is called the vertex, and the parabola is symmetric about the vertical line through the vertex, called the axis of symmetry. The points where the graph crosses the x-axis are the x-intercepts (roots of the equation), and the point where it crosses the y-axis is the y-intercept, found by putting x = 0.

Reading a graph gives information without calculation: the domain is the set of x values covered, the range is the set of y values reached, and the function increases where the graph rises to the right and decreases where it falls. Graphs also solve equations: the solutions of f(x) = 0 are the x-intercepts, and the solutions of f(x) = g(x) are the x-coordinates of the intersection points of the two graphs.

Key formulas and rules

- Linear graph: f(x) = ax + b is a straight line with slope a and y-intercept b.
- Quadratic graph: f(x) = ax^2 + bx + c is a parabola.
- a greater than 0: parabola opens upward (minimum). a less than 0: opens downward (maximum).
- Vertex: the turning point of the parabola. Axis of symmetry: the vertical line through the vertex.
- y-intercept: value of f(0). x-intercepts: solutions of f(x) = 0.
- Increasing: graph rises to the right. Decreasing: graph falls to the right.
- f(x) = g(x) is solved by the intersection points of the two graphs.

Worked examples

1. Draw the graph of f(x) = 2x - 1 using a table of values. Take x = 0, 1, 2: f(0) = -1, f(1) = 1, f(2) = 3. Plot (0, -1), (1, 1), (2, 3) and join them with a straight line. The line crosses the y-axis at -1 and has slope 2.

2. For f(x) = x^2 - 4, find the intercepts. y-intercept: f(0) = -4, so (0, -4). x-intercepts: solve x^2 - 4 = 0, giving x^2 = 4, so x = 2 or x = -2. The graph is an upward parabola with vertex (0, -4) crossing the x-axis at -2 and 2.

Short questions

1. What shape is the graph of a quadratic function?

A parabola, a U-shaped curve that opens upward if the coefficient of x^2 is positive and downward if it is negative.

2. How do you find the y-intercept from a graph or a function?

From the function, put x = 0 and find f(0). On the graph, it is the point where the curve crosses the y-axis.

3. What is the axis of symmetry of a parabola?

The vertical line through the vertex, about which the parabola is symmetric: the left and right sides mirror each other.

4. How can a graph solve f(x) = 0?

The solutions are the x-intercepts, the points where the graph crosses the x-axis.

Long questions

1. Describe how to graph f(x) = x^2 + 2x - 3 and state its vertex, axis of symmetry and intercepts.

Make a table: x = -3 gives 0, x = -2 gives -3, x = -1 gives -4, x = 0 gives -3, x = 1 gives 0. Plot these points and join them in a smooth U-shaped curve opening upward. The lowest point (vertex) is (-1, -4), so the axis of symmetry is x = -1. The x-intercepts are -3 and 1, and the y-intercept is -3.

2. Explain how the graph of a linear function shows its slope and intercepts.

For f(x) = ax + b, the graph is a straight line. The slope a is the rise over run: for each step of 1 in x, y changes by a. The y-intercept b is the point where the line meets the y-axis. For example, in f(x) = 3x + 2, the line rises 3 units for each 1 unit of x and crosses the y-axis at (0, 2). The x-intercept is found by solving 3x + 2 = 0, giving x = -2/3.

MCQs

1. The graph of f(x) = 2x + 3 is a:

(a) Parabola

(b) Straight line

(c) Circle

(d) Hyperbola

2. The y-intercept of f(x) = x^2 - 5 is:

(a) 5

(b) -5

(c) 0

(d) 1

3. A parabola with a less than 0 opens:

(a) Upward

(b) Downward

(c) Left

(d) Right

4. The vertex of a parabola is its:

(a) Intercept

(b) Turning point

(c) Slope

(d) Asymptote

5. The x-intercepts of f(x) = x^2 - 9 are:

(a) 3 only

(b) -3 only

(c) 3 and -3

(d) 9 and -9

6. For f(x) = 4x - 2, the slope is:

(a) -2

(b) 2

(c) 4

(d) 1/4

Answer key: 1. (b) 2. (b) 3. (b) 4. (b) 5. (c) 6. (c)

Chapter 11: Loci and Construction

Summary

A locus is the set of all points that satisfy a given condition. The locus of points at a fixed distance r from a fixed point O is a circle with centre O and radius r. The locus of points equidistant from two fixed points A and B is the perpendicular bisector of AB, the line that cuts AB at its midpoint at a right angle. The locus of points at a fixed distance from a straight line consists of two lines parallel to it, one on each side. The locus of points equidistant from two intersecting lines is the pair of angle bisectors of the angles formed. These ideas are the basis of geometric construction.

Constructions are made with a ruler, compass and protractor, without measuring lengths from a scale. To bisect an angle, draw an arc cutting both arms, then from each cut draw equal arcs that meet, and join the vertex to the meeting point: this line bisects the angle. To bisect a line segment, draw arcs of equal radius (more than half the segment) from both ends, and join the two points where the arcs meet: this line is the perpendicular bisector. To construct a perpendicular from a point to a line, or a triangle from given sides and angles, the same arc method is used step by step. Every construction should end with a clear, labelled diagram and a short statement of what was constructed.

Common constructions in this chapter include: bisecting an angle, bisecting a line segment, drawing a perpendicular from a point on a line and from a point outside a line, constructing a triangle given three sides (SSS), two sides and the included angle (SAS), and two angles and one side (ASA), and constructing a circle through three non-collinear points (using the perpendicular bisectors of two chords, which meet at the centre).

Key formulas and rules

- Locus: the set of all points satisfying a given condition.
- Fixed distance from a point: a circle with that point as centre.
- Equidistant from two points: the perpendicular bisector of the segment joining them.
- Fixed distance from a line: two lines parallel to it, one on each side.
- Equidistant from two intersecting lines: the angle bisectors.
- Angle bisector: divides an angle into two equal angles.
- Perpendicular bisector: cuts a segment at its midpoint at 90 degrees.
- SSS, SAS, ASA: the standard triangle construction cases.
- Centre of a circle through three points: the meeting point of the perpendicular bisectors of two chords.

Worked examples

1. Describe the locus of points 5 cm from a fixed point O. It is a circle with centre O and radius 5 cm, because every point on the circle is exactly 5 cm from O, and every point 5 cm from O lies on the circle.

2. How do you construct the perpendicular bisector of a segment AB? Open the compass to more than half of AB. With centre A, draw arcs above and below AB. With the same radius and centre B, draw arcs cutting the first arcs at P and Q. Join P and Q with a ruler: line PQ is the perpendicular bisector of AB.

Short questions

1. What is a locus?

A locus is the set of all points that satisfy a given condition, for example all points at a fixed distance from a point.

2. What is the locus of points equidistant from two fixed points?

The perpendicular bisector of the line segment joining the two points.

3. Why must the compass radius be more than half the segment when bisecting it?

So that the arcs drawn from the two ends actually meet on both sides; with a smaller radius they would not cross, and no bisector could be drawn.

4. What is the first step in bisecting an angle with a compass?

With the compass centred at the vertex, draw an arc that cuts both arms of the angle; the two cut points are then used for the next arcs.

Long questions

1. Explain how to construct a triangle ABC with AB = 6 cm, BC = 5 cm and AC = 4 cm (SSS construction).

Draw a line and mark AB = 6 cm on it. With centre A and radius 4 cm, draw an arc. With centre B and radius 5 cm, draw an arc cutting the first arc at C. Join A to C and B to C. The result is triangle ABC with the required side lengths, because each arc guarantees the distance from its centre.

2. How do you find the centre of a circle passing through three non-collinear points P, Q and R?

Join P to Q and Q to R to make two chords. Construct the perpendicular bisector of PQ: the centre must lie on it, since it is equidistant from P and Q. Construct the perpendicular bisector of QR: the centre must also lie on it. The two bisectors meet at a single point, which is equidistant from P, Q and R and is therefore the centre of the required circle.

MCQs

1. The locus of points at a fixed distance from a point is a:

(a) Line

(b) Circle

(c) Square

(d) Triangle

2. The perpendicular bisector of a segment cuts it at:

(a) 45 degrees

(b) 60 degrees

(c) 90 degrees

(d) 180 degrees

3. The locus of points equidistant from two intersecting lines is:

(a) A circle

(b) The angle bisectors

(c) A parallel line

(d) A single point

4. In an SSS construction, the three given parts are:

(a) Three angles

(b) Three sides

(c) Two sides and an angle

(d) Two angles and a side

5. An angle bisector divides an angle into:

(a) Three equal parts

(b) Two equal parts

(c) Four equal parts

(d) Unequal parts

6. To bisect a segment, the compass radius must be:

(a) Less than half the segment

(b) More than half the segment

(c) Equal to the segment

(d) Any size

Answer key: 1. (b) 2. (c) 3. (b) 4. (b) 5. (b) 6. (b)

Chapter 12: Information Handling

Summary

Information handling means collecting, organising and interpreting data. Data are collected by observation, surveys or experiments. Raw data are hard to read, so they are organised into a frequency distribution: values are grouped into class intervals (for example 0-9, 10-19), and the frequency of each class is the number of values in it. The class mark (midpoint) of a class is the average of its two limits. A table with classes, tallies and frequencies is called a frequency table. Cumulative frequency is the running total of frequencies up to each class, and it shows how many values lie below a given limit.

Data are shown with graphs. A bar chart uses bars for categories. A histogram uses touching bars for class intervals, with bar heights showing frequencies. A frequency polygon joins the midpoints of the tops of histogram bars with straight lines. A cumulative frequency curve (ogive) plots cumulative frequencies against upper class boundaries and rises to the total number of values. These graphs make patterns visible: where most values lie, whether the data are spread out, and whether any value is unusual.

Three measures describe the centre of data. The mean is the sum of all values divided by the number of values; for grouped data, it is the sum of (frequency x class mark) divided by the total frequency. The median is the middle value when the data are arranged in order (for grouped data it is read from the cumulative frequency curve at half the total frequency). The mode is the value that occurs most often (for grouped data, the class with the highest frequency is the modal class). The mean uses every value but is pulled by extreme values, while the median is safer when a few values are very large or very small.

Key formulas and rules

- Class mark = (lower limit + upper limit) / 2.
- Cumulative frequency = running total of frequencies up to that class.
- Mean (ungrouped) = sum of values / number of values.
- Mean (grouped) = sum of (frequency x class mark) / total frequency.
- Median: the middle value in ordered data; for grouped data, read at half the total frequency on the ogive.
- Mode: the most frequent value. Modal class: the class with the highest frequency.
- Histogram: touching bars for classes, heights = frequencies.
- Frequency polygon: midpoints of histogram bar tops joined by lines.
- Ogive: cumulative frequency plotted against upper class boundaries.

Worked examples

1. Find the mean of 4, 6, 8, 10, 12. Sum = 4 + 6 + 8 + 10 + 12 = 40. Number of values = 5. Mean = 40/5 = 8.

2. The classes 0-9, 10-19, 20-29 have frequencies 3, 5, 2. Find the mean using class marks. Class marks are 4.5, 14.5, 24.5. Sum of (frequency x class mark) = 3 x 4.5 + 5 x 14.5 + 2 x 24.5 = 13.5 + 72.5 + 49 = 135. Total frequency = 10. Mean = 135/10 = 13.5.

Short questions

1. What is a frequency distribution?

A table that groups data into classes and shows how many values fall in each class, so patterns in the data can be seen easily.

2. Find the class mark of the class 20-29.

Class mark = (20 + 29)/2 = 49/2 = 24.5.

3. What is the difference between a histogram and a bar chart?

A histogram uses touching bars for continuous class intervals, while a bar chart uses separated bars for distinct categories.

4. When is the median better than the mean?

When the data contain a few very large or very small values, because the median is not pulled by extreme values while the mean is.

Long questions

1. Explain how to draw a cumulative frequency curve (ogive) and how to read the median from it.

First build a table of upper class boundaries and cumulative frequencies. Plot each upper boundary against its cumulative frequency, join the points with a smooth curve starting from the lower boundary of the first class (cumulative frequency zero), and end at the total frequency. To find the median, find half the total frequency on the vertical axis, move horizontally to the curve, then drop vertically to the horizontal axis: the value read there is the median.

2. The marks of 10 students are 12, 15, 18, 12, 20, 15, 15, 10, 18, 15. Find the mean, median and mode.

Mean: sum = 150, count = 10, so mean = 15. Median: arrange in order: 10, 12, 12, 15, 15, 15, 15, 18, 18, 20. The middle two values are both 15, so median = 15. Mode: 15 occurs four times, more than any other value, so mode = 15. In this data all three measures agree.

MCQs

1. The class mark of 10-19 is:

(a) 14

(b) 14.5

(c) 15

(d) 10

2. The mean of 2, 4, 6, 8 is:

(a) 4

(b) 5

(c) 6

(d) 10

3. The median of 3, 7, 9, 12, 15 is:

(a) 7

(b) 9

(c) 12

(d) 3

4. The mode of 5, 5, 6, 7, 5, 8 is:

(a) 6

(b) 7

(c) 5

(d) 8

5. In a histogram, the bars are:

(a) Separated

(b) Touching

(c) Vertical only

(d) Horizontal only

6. Cumulative frequency is a:

(a) Difference of frequencies

(b) Running total of frequencies

(c) Average of frequencies

(d) Highest frequency

Answer key: 1. (b) 2. (b) 3. (b) 4. (c) 5. (b) 6. (b)

Chapter 13: Probability

Summary

Probability measures how likely an event is, on a scale from 0 (impossible) to 1 (certain). An experiment is an action with an uncertain result, like tossing a coin or rolling a die. The sample space is the set of all possible outcomes: for a coin, {Head, Tail}; for a die, {1, 2, 3, 4, 5, 6}. An event is a subset of the sample space, for example "getting an even number" on a die, which is {2, 4, 6}. If all outcomes are equally likely, the probability of an event E is P(E) = (number of outcomes in E) / (total number of outcomes in the sample space).

Some events are special. The complement of E, written E' or "not E", contains all outcomes not in E, and P(E) + P(E') = 1. Mutually exclusive events cannot happen together, like getting a 2 and getting a 5 on one roll; for mutually exclusive events A and B, P(A or B) = P(A) + P(B). An impossible event has probability 0 and a certain event has probability 1. Probability is used in games, weather forecasts, insurance and quality control: it never tells us what will happen in one trial, but it tells us what to expect over many trials.

Tree diagrams help with combined experiments, like tossing two coins: the sample space is {HH, HT, TH, TT}, each with probability 1/4. Counting outcomes carefully is the main skill in probability problems: list the sample space first, then count the outcomes in the event, then divide.

Key formulas and rules

- P(E) = (number of favourable outcomes) / (total number of possible outcomes).
- 0 is less than or equal to P(E), and P(E) is less than or equal to 1.
- P(E) + P(E') = 1, so P(E') = 1 - P(E).
- Mutually exclusive events: P(A or B) = P(A) + P(B).
- Impossible event: P = 0. Certain event: P = 1.
- Sample space of one coin: {Head, Tail}. Of one die: {1, 2, 3, 4, 5, 6}.
- Sample space of two coins: {HH, HT, TH, TT}.

Worked examples

1. A die is rolled once. Find the probability of getting a prime number. The sample space is {1, 2, 3, 4, 5, 6}. The prime outcomes are {2, 3, 5}, which are 3 outcomes. So P(prime) = 3/6 = 1/2.

2. A bag has 3 red balls and 2 blue balls. One ball is drawn at random. Find the probability of drawing a blue ball. Total balls = 5. Favourable outcomes (blue) = 2. So P(blue) = 2/5.

Short questions

1. What is a sample space? Give an example.

The sample space is the set of all possible outcomes of an experiment. For tossing a coin, it is {Head, Tail}.

2. A coin is tossed once. What is the probability of getting a Head?

There is 1 favourable outcome out of 2 possible outcomes, so P(Head) = 1/2.

3. If P(E) = 0.3, what is P(not E)?

P(not E) = 1 - P(E) = 1 - 0.3 = 0.7.

4. What does it mean for two events to be mutually exclusive?

They cannot happen at the same time, like getting a 1 and getting a 6 on a single roll of a die.

Long questions

1. Two coins are tossed together. List the sample space and find the probability of getting exactly one Head.

The sample space is {HH, HT, TH, TT}, with 4 equally likely outcomes. Exactly one Head occurs in {HT, TH}, which is 2 outcomes. So P(exactly one Head) = 2/4 = 1/2.

2. A card is drawn at random from a standard deck of 52 cards. Find the probability of drawing (a) a King, (b) a red card.

(a) There are 4 Kings in 52 cards, so P(King) = 4/52 = 1/13. (b) There are 26 red cards in 52, so P(red) = 26/52 = 1/2. Both use the rule P(E) = favourable outcomes / total outcomes.

MCQs

1. The probability of an impossible event is:

(a) 1

(b) 0

(c) 1/2

(d) Not defined

2. When a die is rolled, the probability of getting a 6 is:

(a) 1/2

(b) 1/3

(c) 1/6

(d) 5/6

3. The sample space of tossing two coins has how many outcomes?

(a) 2

(b) 3

(c) 4

(d) 6

4. If P(E) = 0.4, then P(not E) is:

(a) 0.4

(b) 0.6

(c) 0.8

(d) 1.4

5. Two events that cannot occur together are called:

(a) Independent

(b) Mutually exclusive

(c) Complementary

(d) Certain

6. The probability of drawing an Ace from 52 cards is:

(a) 1/13

(b) 1/26

(c) 1/52

(d) 4/13

Answer key: 1. (b) 2. (c) 3. (c) 4. (b) 5. (b) 6. (a)

Revision checklist

Before the exam, be able to do each of these without looking at the notes.

- Chapter 1: State the sets N, W, Z, Q and R, simplify radicals, rationalise a denominator, and apply the laws of exponents.
- Chapter 2: Write numbers in scientific notation, find a logarithm from its definition, and apply the product, quotient and power laws.
- Chapter 3: Find unions, intersections and complements, build a Cartesian product, and test whether a relation is a function.
- Chapter 4: Factorise by common factor, grouping, identities and quadratic trinomials, and find HCF and LCM of expressions.
- Chapter 5: Solve linear equations and inequalities, and solve simultaneous equations by elimination and substitution.
- Chapter 6: Write all six trigonometric ratios, recall the ratios of 30, 45 and 60 degrees, and solve a height and distance problem.
- Chapter 7: Apply the distance, midpoint and section formulas, find a slope, and write the equation of a line.
- Chapter 8: Build truth tables for and, or and implication, and write the converse, inverse and contrapositive of a statement.
- Chapter 9: Test triangles for similarity, apply the Pythagoras theorem, and use the area ratio of similar figures.
- Chapter 10: Sketch linear and quadratic graphs, and read intercepts, vertex and axis of symmetry from a graph.
- Chapter 11: Describe loci in words, and construct angle bisectors, perpendicular bisectors and triangles by SSS, SAS and ASA.
- Chapter 12: Build a frequency table, draw a histogram and an ogive, and find mean, median and mode.
- Chapter 13: List a sample space, find probabilities by counting outcomes, and use P(E) + P(E') = 1.