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9th Class Mathematics Notes AJK Board (BISE Mirpur) Full Book Notes

Complete unit-wise notes, worked examples and original practice questions for all 11 units of the National Book Foundation (NBF) Mathematics book for Class 9, Board of Intermediate and Secondary Education (BISE) Mirpur, Azad Jammu and Kashmir.

Board: Board of Intermediate and Secondary Education (BISE) Mirpur, Azad Jammu and Kashmir

Class: 9th Class | **Subject:** Mathematics | **Book:** National Book Foundation (NBF)

Medium: English

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How to Use These Notes

These notes follow the 11 units of the National Book Foundation Mathematics book for Class 9, AJK Board (BISE Mirpur), in the same order as the book. Each unit gives the key concepts in plain words, the formulas you must memorise, worked examples with every step shown, a key-terms table, and original practice questions: 10 multiple choice questions, 5 short questions and 2 long questions per unit. The answers to all multiple choice questions are in the answer key at the end of the book.

Study tip: read the concepts first, cover the worked examples and try them yourself, then attempt the practice questions without looking at the notes. Revise the formula box of each unit the night before the paper.

Paper Pattern (AJK BISE Mirpur)

> **Total marks: 75.** There are no practical marks in 9th class Mathematics.
>
> The section-wise split of the paper (multiple choice, short questions and long questions) is set by AJK BISE Mirpur in its current model paper. Check the current model paper on ajkbise.net for your session before the examination. No pairing scheme is printed in these notes.

Source: AJK BISE SSC-I result data (Mathematics-I out of 75 marks); model paper: ajkbise.net.

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Unit 1: Real Numbers

Real numbers are all the numbers on the number line: natural numbers, whole numbers, integers, rational numbers and irrational numbers. This unit builds the foundation for the whole book: how numbers are classified, how surds and exponents work, and how the number system is used in calculations.

1.1 Classification of Real Numbers

Natural numbers N = {1, 2, 3, ...}; whole numbers W = {0, 1, 2, 3, ...}; integers Z = {..., -2, -1, 0, 1, 2, ...}.

A rational number can be written as p/q where p and q are integers and q is not zero. Its decimal form either terminates or repeats, for example 1/4 = 0.25 and 1/3 = 0.333...

An irrational number cannot be written as p/q. Its decimal form neither terminates nor repeats, for example sqrt(2), sqrt(3) and pi.

Every rational and every irrational number is a real number. The set of real numbers is denoted by R, and R = Q union Q', where Q' is the set of irrational numbers.

- Real numbers are ordered on the number line: a number further to the right is greater.
- Between any two real numbers there are infinitely many real numbers.
- The absolute value |x| is the distance of x from zero on the number line, so |x| is never negative.

1.2 Properties of Real Numbers

Closure, commutative, associative, identity and inverse properties hold for addition and multiplication of real numbers.

Distributive property: a(b + c) = ab + ac. This is used constantly in algebra.

Additive identity is 0 and multiplicative identity is 1. The additive inverse of a is -a; the multiplicative inverse (reciprocal) of a non-zero a is 1/a.

If ab = 0 then a = 0 or b = 0. This zero-product fact is used to solve many equations.

1.3 Radicals and Surds

The nth root of a is written as the nth root symbol; sqrt(a) is the square root and the cube root of a is written with index 3.

A surd is an irrational root, such as sqrt(2) or the cube root of 5. sqrt(4) = 2 is not a surd because it is rational.

Like surds can be added or subtracted: 3 sqrt(2) + 5 sqrt(2) = 8 sqrt(2). Unlike surds cannot be combined this way.

Surds are multiplied by multiplying the numbers inside the root: sqrt(a) x sqrt(b) = sqrt(ab).

Rationalising the denominator means removing a root from the denominator by multiplying top and bottom by a suitable surd, for example 1/sqrt(2) = sqrt(2)/2.

1.4 Exponents and Scientific Notation

For a non-zero base a: a^m x a^n = a^(m+n); a^m / a^n = a^(m-n); (a^m)^n = a^(mn); a^0 = 1; a^(-n) = 1/a^n.

Scientific notation writes a number as a x 10^n where 1 <= a < 10 and n is an integer, for example 0.00045 = 4.5 x 10^(-4).

Laws of exponents are used to simplify expressions before evaluating them, which avoids large and messy numbers.

Key Formulas

a^m x a^n = a^(m+n); a^m / a^n = a^(m-n); (a^m)^n = a^(mn)

a^0 = 1 (a not 0); a^(-n) = 1 / a^n; (ab)^n = a^n b^n

sqrt(a) x sqrt(b) = sqrt(ab); sqrt(a) / sqrt(b) = sqrt(a/b) (b > 0)

(sqrt(a))^2 = a (a >= 0); |x| = x if x >= 0, -x if x < 0

Scientific notation: a x 10^n, 1 <= a < 10

Worked Example 1: Simplify (2^3 x 2^5) / 2^4

Step 1: Add powers in the numerator: 2^3 x 2^5 = 2^(3+5) = 2^8.

Step 2: Divide: 2^8 / 2^4 = 2^(8-4) = 2^4.

Step 3: 2^4 = 16. Answer: 16.

Worked Example 2: Rationalise 3 / sqrt(5)

Step 1: Multiply numerator and denominator by sqrt(5): (3 sqrt(5)) / (sqrt(5) x sqrt(5)).

Step 2: sqrt(5) x sqrt(5) = 5.

Step 3: Answer: (3 sqrt(5)) / 5.

Key Terms

Key Term	Meaning
Rational number	A number that can be written as p/q with q not zero; decimal terminates or repeats.
Irrational number	A number that cannot be written as p/q; decimal neither terminates nor repeats.
Surd	An irrational root such as sqrt(2) or the cube root of 7.
Rationalising	Removing a surd from the denominator of a fraction.
Scientific notation	Writing a number as a x 10^n with 1 <= a < 10.
Absolute value	The distance of a number from zero on the number line.

Practice Questions

Multiple Choice Questions

1. Which of the following is an irrational number?

(a) 1/3 (b) 0.25

(c) sqrt(2) (d) 7/8

2. The value of 5^0 is:

(a) 0 (b) 1
(c) 5 (d) 25

3. 2^(-3) equals:

(a) -8 (b) 8
(c) 1/8 (d) -1/8

4. sqrt(12) simplifies to:

(a) 2 sqrt(3) (b) 3 sqrt(2)
(c) 6 sqrt(2) (d) 4 sqrt(3)

5. Which set is {0, 1, 2, 3, ...}?

(a) Natural numbers (b) Whole numbers
(c) Integers (d) Rational numbers

6. The rationalised form of 1/sqrt(3) is:

(a) sqrt(3) (b) sqrt(3)/3
(c) 3/sqrt(3) (d) 1/3

7. 0.333... in fraction form is:

(a) 1/3 (b) 3/10
(c) 33/100 (d) 1/9

8. 3 sqrt(5) + 2 sqrt(5) equals:

(a) 5 sqrt(5) (b) 5 sqrt(10)
(c) 6 sqrt(5) (d) sqrt(5)

9. In scientific notation, 72000 is written as:

(a) 72 x 10^3 (b) 7.2 x 10^4
(c) 0.72 x 10^5 (d) 7.2 x 10^3

10. The absolute value of -9 is:

(a) -9 (b) 0
(c) 9 (d) 1/9

Short Questions

1. Define a rational number and give two examples.
2. Show that sqrt(3) is an irrational number by stating its decimal property.
3. Simplify 4^5 / 4^2 using the laws of exponents.
4. Rationalise the denominator of 5 / sqrt(7).
5. Write 0.000062 in scientific notation.

Long Questions

1. Simplify (3 sqrt(2) + 2 sqrt(3))(sqrt(2) - sqrt(3)) and rationalise the result if needed. Show all steps.
2. Using the laws of exponents, simplify (2^4 x 3^2 x 2^(-1)) / (3 x 2^2) and express the answer as a fraction in lowest terms.

Unit 2: Logarithms

A logarithm is the inverse of an exponent: if a^x = y then log base a of y = x. Logarithms turn multiplication into addition and division into subtraction, which is why they were used for centuries in calculation, and they remain essential in science, finance and computer work.

2.1 Definition and Basic Ideas

If a^x = y (a > 0, a not 1), then log base a of y = x. The number a is the base, y is the argument and x is the logarithm.

Common logarithms use base 10 and are written log y. Natural logarithms use base e (about 2.718) and are written ln y.

log base a of 1 = 0, because a^0 = 1. log base a of a = 1, because a^1 = a.

The logarithm of a negative number or of zero is not defined for real numbers.

2.2 Laws of Logarithms

Product law: log(mn) = log m + log n.

Quotient law: log(m/n) = log m - log n.

Power law: log(m^n) = n log m.

Change of base: log base a of b = log b / log a, using any convenient base on the right.

These laws hold for any valid base, including base 10 and base e.

2.3 Characteristic and Mantissa

In a common logarithm, the part before the decimal point is the characteristic and the part after it is the mantissa.

The characteristic is found from the position of the decimal point: for a number greater than 1 it is one less than the number of digits before the point; for a number less than 1 it is negative.

The mantissa is always positive and is found from logarithm tables or a calculator; it depends only on the digits of the number, not on the position of the decimal point.

Antilogarithm is the reverse process: if log x = 2.3010 then x = antilog 2.3010 = 200.

2.4 Applications

Logarithms solve exponential equations: if 2^x = 32, taking logs gives x log 2 = log 32, so x = 5.

They are used in measuring earthquake strength (Richter scale), sound level (decibels) and acidity (pH), where quantities span many orders of magnitude.

Compound interest and population growth problems are solved by taking logarithms of both sides.

> ### Key Formulas
>
> a^x = y <=> log base a of y = x
>
> log(mn) = log m + log n; log(m/n) = log m - log n; log(m^n) = n log m
>
> log base a of 1 = 0; log base a of a = 1
>
> log base a of b = log b / log a (change of base)

log x = characteristic + mantissa (mantissa >= 0)

Worked Example 1: Solve 3^x = 81

Step 1: Take log of both sides: x log 3 = log 81.

Step 2: x = log 81 / log 3.

Step 3: log 81 = 1.9085 and log 3 = 0.4771, so x = 1.9085 / 0.4771 = 4. Answer: x = 4.

Worked Example 2: Simplify log 1000 - log 10

Step 1: Use the quotient law: log 1000 - log 10 = log(1000/10) = log 100.

Step 2: log 100 = 2, since 10^2 = 100. Answer: 2.

Key Terms

Key Term	Meaning
Logarithm	The exponent to which a base must be raised to give a number.
Common logarithm	A logarithm with base 10.
Natural logarithm	A logarithm with base e, written ln.
Characteristic	The whole-number part of a common logarithm.
Mantissa	The positive decimal part of a common logarithm.
Antilogarithm	The number whose logarithm is a given value.

Practice Questions

Multiple Choice Questions

1. If 2^x = 16, then x equals:

(a) 2 (b) 3
(c) 4 (d) 8

2. log 1000 to base 10 is:

(a) 2 (b) 3
(c) 10 (d) 100

3. log(mn) equals:

(a) log m x log n (b) log m + log n
(c) log m - log n (d) n log m

4. log base 5 of 5 is:

(a) 0 (b) 1
(c) 5 (d) 25

5. log base a of 1 is always:

(a) 1 (b) 0
(c) a (d) -1

6. log 8 - log 2 equals log of:

(a) 4 (b) 6
(c) 10 (d) 16

7. The mantissa of a common logarithm is always:

(a) negative (b) zero
(c) positive (d) an integer

8. log(x^3) in terms of log x is:

(a) 3 + log x (b) 3 log x
(c) log x / 3 (d) (log x)^3

9. If log x = 2, then x equals:

(a) 2 (b) 20
(c) 100 (d) 200

10. The base of a natural logarithm is:

(a) 10 (b) 2
(c) e (d) pi

Short Questions

1. Write 5^3 = 125 in logarithmic form.
2. Evaluate log base 2 of 64 without a calculator.
3. Use the laws of logarithms to expand log(7x^2).
4. Find the characteristic of log 0.0045.
5. Solve 4^x = 64 by taking logarithms.

Long Questions

1. Simplify log 45 + log 20 - log 9 + log 2, giving the answer as a single logarithm and then evaluating it.
2. A population doubles every 6 years. Use logarithms to find how long it takes to grow to 8 times its present size. Show your working.

Unit 3: Sets and Relations

A set is a well-defined collection of distinct objects. Sets give mathematics a precise language for grouping things, and relations describe how elements of one set are connected with elements of another. Functions, studied later, are a special kind of relation.

3.1 Sets and Their Notation

A set is written in roster form by listing its elements, for example A = {1, 2, 3}, or in set-builder form, for example A = {x : x is a prime number less than 10}.

The empty set, written {} or the empty-set symbol, has no elements. A singleton set has exactly one element.

A set A is a subset of B, written A subset B, if every element of A is also in B. Every set is a subset of itself, and the empty set is a subset of every set.

A set with n elements has 2^n subsets. A proper subset is a subset that is not equal to the whole set.

3.2 Operations on Sets

Union: A union B contains all elements in A or in B (or both). Intersection: A intersection B contains only elements common to both.

Difference: A - B contains elements in A but not in B. Complement: A' contains all elements of the universal set not in A.

De Morgan's laws: (A union B)' = A' intersection B' and (A intersection B)' = A' union B'.

Venn diagrams show sets as overlapping circles; the overlapping region is the intersection.

Number of elements: n(A union B) = n(A) + n(B) - n(A intersection B).

3.3 Cartesian Product and Relations

The Cartesian product A x B is the set of all ordered pairs (a, b) with a in A and b in B. Order matters: (1, 2) is not the same as (2, 1).

If A has m elements and B has n elements, then A x B has m x n elements.

A relation from A to B is a subset of A x B. It is a set of ordered pairs linking elements of A with elements of B.

The domain of a relation is the set of first elements of its pairs; the range is the set of second elements.

3.4 Types of Relations

A relation on a set can be reflexive (every element relates to itself), symmetric (if a relates to b then b relates to a) or transitive (if a relates to b and b relates to c, then a relates to c).

A relation that is reflexive, symmetric and transitive is called an equivalence relation.

A function is a relation in which every element of the domain has exactly one image in the range.

> ### Key Formulas
>
> Number of subsets of an n-element set = 2^n

n(A union B) = n(A) + n(B) - n(A intersection B)

n(A x B) = n(A) x n(B)

(A union B)' = A' intersection B'; (A intersection B)' = A' union B'

A - B = {x : x in A and x not in B}

Worked Example 1: If A = {1, 2, 3, 4} and B = {3, 4, 5}, find A union B and A intersection B

Step 1: Union collects every element once: A union B = {1, 2, 3, 4, 5}.

Step 2: Intersection collects common elements only: A intersection B = {3, 4}.

Step 3: Check: n(A union B) = 5 = 4 + 3 - 2. Correct.

Worked Example 2: Find the domain and range of R = {(1, 2), (2, 4), (3, 6)}

Step 1: Domain is the set of first elements: {1, 2, 3}.

Step 2: Range is the set of second elements: {2, 4, 6}.

Step 3: Each domain element has one image, so R is also a function.

Key Terms

Key Term	Meaning
Set	A well-defined collection of distinct objects.
Subset	A set whose every element belongs to another set.
Universal set	The set containing all elements under discussion.
Cartesian product	The set of all ordered pairs from two sets.
Relation	A subset of a Cartesian product; a set of ordered pairs.
Domain and range	The sets of first and second elements of a relation.

Practice Questions

Multiple Choice Questions

1. Which of these is a set?

(a) {1, 2, 2, 3}
(b) The set of tall students
(c) {2, 4, 6, 8}
(d) The set of good books

2. The number of subsets of {a, b, c} is:

(a) 3
(b) 6
(c) 8
(d) 9

3. If A = {1, 2} and B = {2, 3}, A intersection B is:

(a) {1, 2, 3}
(b) {2}
(c) {1, 3}
(d) {}

4. The empty set is denoted by:

(a) {0}
(b) 0
(c) {}
(d) { } with 0

5. If n(A) = 4 and n(B) = 3, then n(A x B) is:

(a) 7 (b) 12
(c) 1 (d) 64

6. The domain of {(1, 5), (2, 6)} is:

(a) {5, 6} (b) {1, 2}
(c) {1, 2, 5, 6} (d) {6}

7. A - B contains elements that are:

(a) in B only (b) in A but not in B
(c) in both A and B (d) in neither

8. De Morgan's law: (A union B)' equals:

(a) A' union B' (b) A' intersection B'
(c) A union B (d) A intersection B

9. A relation in which each domain element has one image is a:

(a) set (b) subset
(c) function (d) union

10. Which pair is an ordered pair?

(a) {1, 2} (b) (2, 1)
(c) 1 + 2 (d) [1, 2]

Short Questions

1. List all subsets of {x, y}.
2. If A = {2, 4, 6} and B = {1, 2, 3, 4}, find A - B and B - A.
3. Draw and describe a Venn diagram for A intersection B when A and B overlap.
4. Find the Cartesian product of A = {1, 2} and B = {a, b}.
5. State the domain and range of R = {(0, 1), (1, 2), (2, 3)}.

Long Questions

1. In a class of 40 students, 25 play cricket, 18 play football and 8 play both. Using the formula for n(A union B), find how many students play neither game. Show the Venn diagram reasoning.
2. Let A = {1, 2, 3} and B = {4, 5}. Write A x B, then define a relation R from A to B by the rule 'a is less than b' and list its domain and range.

Unit 4: Factorization and Algebraic Manipulation

Factorization is the reverse of expansion: writing an expression as a product of its factors. It is the key skill for simplifying algebraic fractions and for solving quadratic equations, so this unit is practised until it becomes automatic.

4.1 Common Factors and Grouping

Take out the highest common factor first: 6x^2 + 9x = 3x(2x + 3). Always check for a common factor before trying other methods.

Grouping: in ax + ay + bx + by, group as a(x + y) + b(x + y) = (x + y)(a + b).

A factor can itself be a bracket; look for repeated bracket expressions.

4.2 Special Identities

Difference of two squares: a^2 - b^2 = (a - b)(a + b).

Perfect square: a^2 + 2ab + b^2 = (a + b)^2 and a^2 - 2ab + b^2 = (a - b)^2.

Sum and difference of cubes: a^3 + b^3 = (a + b)(a^2 - ab + b^2); a^3 - b^3 = (a - b)(a^2 + ab + b^2).

Recognising the pattern is faster than guessing: check whether the expression is a square, a difference of squares, or a cube.

4.3 Factorizing Quadratics

For x^2 + bx + c, find two numbers that multiply to c and add to b, then write (x + p)(x + q).

For ax^2 + bx + c with a not 1, split the middle term: find two numbers that multiply to a x c and add to b, then factor by grouping.

Always check the answer by expanding the factors back.

The zero-product rule then solves equations: if (x - 2)(x + 5) = 0 then x = 2 or x = -5.

4.4 Algebraic Fractions and HCF/LCM

Cancel common factors in a fraction only when they are factors of the whole numerator and denominator.

To add or subtract algebraic fractions, use the LCM of the denominators as the common denominator.

The HCF of algebraic expressions is the product of the common factors with the lowest powers; the LCM uses the highest powers.

Simplify (x^2 - 9)/(x + 3) by factorizing the numerator first: (x - 3)(x + 3)/(x + 3) = x - 3.

> ### Key Formulas
>
> a^2 - b^2 = (a - b)(a + b)
>
> a^2 + 2ab + b^2 = (a + b)^2; a^2 - 2ab + b^2 = (a - b)^2
>
> a^3 + b^3 = (a + b)(a^2 - ab + b^2)
>
> a^3 - b^3 = (a - b)(a^2 + ab + b^2)

(x + a)(x + b) = x^2 + (a + b)x + ab

Worked Example 1: Factorize x^2 + 7x + 12

Step 1: Need two numbers that multiply to 12 and add to 7: 3 and 4.

Step 2: x^2 + 7x + 12 = (x + 3)(x + 4).

Step 3: Check: (x + 3)(x + 4) = x^2 + 7x + 12. Correct.

Worked Example 2: Factorize 2x^2 + 7x + 3

Step 1: a x c = 6. Two numbers multiplying to 6 and adding to 7 are 6 and 1.

Step 2: Split the middle term: 2x^2 + 6x + x + 3.

Step 3: Group: 2x(x + 3) + 1(x + 3) = (x + 3)(2x + 1).

Key Terms

Key Term	Meaning
Factor	A number or expression that divides another exactly.
HCF	The highest common factor of two or more expressions.
LCM	The lowest common multiple of two or more expressions.
Perfect square trinomial	A trinomial that is the square of a binomial.
Algebraic fraction	A fraction whose numerator or denominator contains algebra.

Practice Questions

Multiple Choice Questions

1. The factorization of x^2 - 16 is:

(a) (x - 4)(x - 4)
(b) (x + 4)(x - 4)
(c) (x - 8)(x + 2)
(d) (x + 16)(x - 1)

2. The HCF of 12x^2 and 18x is:

(a) 6x
(b) 6x^2
(c) 36x
(d) 3x

3. x^2 + 5x + 6 factorizes as:

(a) (x + 1)(x + 6)
(b) (x + 2)(x + 3)
(c) (x - 2)(x - 3)
(d) (x + 5)(x + 1)

4. a^3 - b^3 equals:

(a) (a - b)(a^2 + ab + b^2)
(b) (a - b)(a^2 - ab + b^2)
(c) (a + b)(a^2 - ab + b^2)
(d) (a - b)^3

5. The factors of 3x + 6 are:

(a) 3(x + 2)
(b) 3(x + 6)
(c) x(3 + 6)
(d) 6(x + 1)

6. x^2 + 2x + 1 is a:

(a) difference of squares
(b) perfect square
(c) cube
(d) prime

7. The LCM of x and x^2 is:

(a) x
(b) x^2
(c) x^3
(d) 1

8. (x - 3)(x + 3) expands to:

(a) x^2 - 9
(b) x^2 + 9
(c) x^2 - 6x + 9
(d) x^2 - 6

9. Factorize 5a^2 - 20:

(a) 5(a^2 - 4)
(b) 5(a - 2)(a + 2)
(c) 5a(a - 4)
(d) Both first and second forms

10. If (x - 2)(x + 5) = 0, then x is:

(a) 2 or -5
(b) -2 or 5
(c) 2 or 5
(d) -2 or -5

Short Questions

1. Factorize 8x^2 - 2x by taking out the common factor.
2. Factorize x^2 - 11x + 24.
3. Use the difference of two squares to factorize 49 - y^2.
4. Simplify (x^2 - 25)/(x - 5).
5. Find the HCF of 6a^2b and 9ab^2.

Long Questions

1. Factorize 6x^2 + 11x - 10 completely by splitting the middle term, and then solve 6x^2 + 11x - 10 = 0.
2. Simplify (x^2 - 4)/(x^2 + 4x + 4) x (x + 2)/(x - 2), showing each factorization step.

Unit 5: Linear Equations and Inequalities

A linear equation has the variable raised to the power 1 only, and its graph is a straight line. Solving linear equations and inequalities is a daily tool in mathematics: it turns word problems about money, distance and mixtures into answers.

5.1 Linear Equations in One Variable

A linear equation in one variable has the form ax + b = 0 with a not zero; its solution is x = -b/a.

Solve by doing the same operation on both sides: add or subtract to collect constants, then divide by the coefficient of x.

Always check the answer by substituting it back into the original equation.

An equation with fractions is cleared by multiplying every term by the LCM of the denominators.

5.2 Simultaneous Linear Equations

Two linear equations in two variables have one solution (intersecting lines), no solution (parallel lines) or infinitely many solutions (the same line).

Elimination method: multiply the equations so that one variable has equal coefficients, then add or subtract to remove it.

Substitution method: solve one equation for one variable and substitute into the other.

Word problems are solved by defining the two unknowns, forming two equations, and solving them together.

5.3 Linear Inequalities

An inequality uses <, >, <= or >=. Its solution is a range of values, shown on a number line or in interval notation.

Adding or subtracting the same number on both sides does not change the inequality sign.

Multiplying or dividing both sides by a NEGATIVE number reverses the inequality sign. This is the most common source of errors.

A compound inequality such as -2 < 3x - 1 <= 8 is solved by doing the same operations on all three parts.

5.4 Absolute Value Equations and Inequalities

|x| = a (a >= 0) means x = a or x = -a.

|x| < a means -a < x < a. |x| > a means x < -a or x > a.

First isolate the absolute value expression, then split into two cases.

> ### Key Formulas
>
> ax + b = 0 => x = -b/a (a not 0)
>
> If a < b and c < 0, then ac > bc (sign reverses)
>
> |x| = a => x = a or x = -a
>
> |x| < a => -a < x < a; |x| > a => x < -a or x > a

Solution of ax + by = c and dx + ey = f is found by elimination or substitution

Worked Example 1: Solve 3x - 7 = 11

Step 1: Add 7 to both sides: 3x = 18.

Step 2: Divide by 3: x = 6.

Step 3: Check: 3(6) - 7 = 18 - 7 = 11. Correct.

Worked Example 2: Solve the system x + y = 10 and x - y = 4

Step 1: Add the equations: (x + y) + (x - y) = 10 + 4, so 2x = 14 and x = 7.

Step 2: Substitute x = 7 into x + y = 10: y = 3.

Step 3: Check in the second equation: 7 - 3 = 4. Correct. Solution: x = 7, y = 3.

Key Terms

Key Term	Meaning
Linear equation	An equation in which each variable has power 1.
Simultaneous equations	Two or more equations solved together for the same unknowns.
Inequality	A statement comparing two expressions with <, >, <= or >=.
Solution set	All values that satisfy an equation or inequality.
Elimination	Removing one variable by adding or subtracting equations.

Practice Questions

Multiple Choice Questions

1. The solution of 2x + 6 = 0 is:

(a) x = 3 (b) x = -3
(c) x = 6 (d) x = -6

2. If x > 5, then -2x is:

(a) greater than -10 (b) less than -10
(c) equal to -10 (d) greater than 10

3. The solution of x - 4 = 9 is:

(a) x = 5 (b) x = 13
(c) x = -5 (d) x = 36

4. Solving x + y = 5 and x - y = 1 gives:

(a) x = 3, y = 2 (b) x = 2, y = 3
(c) x = 4, y = 1 (d) x = 1, y = 4

5. |x| = 7 gives:

(a) x = 7 only (b) x = -7 only
(c) x = 7 or x = -7 (d) no solution

6. The inequality 2x < 10 is solved as:

(a) x < 5 (b) x > 5
(c) x < 20 (d) x > 20

7. Multiplying both sides of 3 < 7 by -1 gives:

(a) -3 < -7 (b) -3 > -7
(c) 3 > 7 (d) -3 = -7

8. The solution set of x >= 2 in interval form is:

(a) (2, infinity) (b) [2, infinity)
(c) (-infinity, 2] (d) [2, 2]

9. 5x = 45 gives x equal to:

(a) 5 (b) 9
(c) 40 (d) 225

10. Two parallel lines give a system with:

(a) one solution (b) no solution
(c) two solutions (d) infinite solutions

Short Questions

1. Solve 4x - 9 = 15.
2. Solve the inequality 3x + 2 < 14 and show the solution on a number line.
3. Solve |2x - 1| = 5.
4. Solve x + 2y = 7 and 2x - y = 4 by elimination.
5. The sum of two numbers is 24 and their difference is 6. Find the numbers.

Long Questions

1. A shopkeeper sells pens at Rs. 25 each and notebooks at Rs. 60 each. A student buys 12 items for Rs. 505. Form two equations and find how many pens and how many notebooks were bought.
2. Solve the compound inequality -3 <= 2x + 1 < 9, write the solution in interval notation, and represent it on a number line.

Unit 6: Trigonometry and Bearing

Trigonometry studies the ratios between the sides of a right-angled triangle. These ratios let us find unknown lengths and angles, measure heights of buildings and trees, and describe directions using bearings.

6.1 Trigonometric Ratios

In a right-angled triangle, for an acute angle theta: sin theta = opposite / hypotenuse; cos theta = adjacent / hypotenuse; tan theta = opposite / adjacent.

tan theta = sin theta / cos theta, and sin^2 theta + cos^2 theta = 1.

The ratios depend only on the angle, not on the size of the triangle, because all similar right triangles give the same ratios.

Standard angles: sin 30 = 1/2, cos 30 = sqrt(3)/2, tan 30 = 1/sqrt(3); sin 45 = cos 45 = 1/sqrt(2), tan 45 = 1; sin 60 = sqrt(3)/2, cos 60 = 1/2, tan 60 = sqrt(3).

6.2 Finding Sides and Angles

To find an unknown side, choose the ratio that links the known side, the unknown side and the given angle, then solve.

To find an unknown angle, use the inverse ratio: theta = sin^(-1)(opposite/hypotenuse), and similarly for cos^(-1) and tan^(-1).

Always check that the hypotenuse is the longest side and that an angle found is acute in a right triangle.

6.3 Angles of Elevation and Depression

The angle of elevation is measured upwards from the horizontal to the line of sight; the angle of depression is measured downwards from the horizontal.

The angle of depression from point A to point B equals the angle of elevation from B to A (alternate angles between parallel horizontal lines).

Heights and distances are found by drawing a right triangle and using tan of the angle of elevation or depression.

6.4 Bearings

A bearing is the direction of one point from another, measured clockwise from north and written with three digits, for example 060 degrees or 235 degrees.

North is 000 degrees, east is 090 degrees, south is 180 degrees and west is 270 degrees.

The back bearing differs by 180 degrees: if the bearing of B from A is x degrees, the bearing of A from B is x + 180 degrees (if x < 180) or x - 180 degrees (if x > 180).

Bearings are used in navigation, surveying and map work, often combined with trigonometry to find distances.

> ### Key Formulas
>
> sin theta = opposite/hypotenuse; cos theta = adjacent/hypotenuse; tan theta = opposite/adjacent

tan theta = sin theta / cos theta

sin^2 theta + cos^2 theta = 1

sin 30 = 1/2; cos 30 = sqrt(3)/2; tan 45 = 1; sin 60 = sqrt(3)/2

Back bearing = bearing + 180 degrees (if bearing < 180) or bearing - 180 degrees

Worked Example 1: In a right triangle, hypotenuse = 10 cm and the angle is 30 degrees. Find the opposite side.

Step 1: sin 30 = opposite / hypotenuse = opposite / 10.

Step 2: sin 30 = 1/2, so opposite / 10 = 1/2.

Step 3: Opposite = 5 cm.

Worked Example 2: A ladder leans against a wall. Its foot is 3 m from the wall and it makes 60 degrees with the ground. How long is the ladder?

Step 1: cos 60 = adjacent / hypotenuse = 3 / ladder.

Step 2: cos 60 = 1/2, so 1/2 = 3 / ladder.

Step 3: Ladder = 6 m.

Key Terms

Key Term	Meaning
Hypotenuse	The side opposite the right angle; the longest side.
Angle of elevation	The angle above the horizontal line of sight.
Angle of depression	The angle below the horizontal line of sight.
Bearing	Direction measured clockwise from north, in three digits.
Trigonometric ratio	The ratio of two sides of a right triangle for a given angle.

Practice Questions

Multiple Choice Questions

1. sin theta equals:

(a) adjacent/hypotenuse
(b) opposite/hypotenuse
(c) opposite/adjacent
(d) hypotenuse/opposite

2. tan 45 degrees equals:

(a) 0
(b) 1/2
(c) 1
(d) sqrt(3)

3. sin^2 theta + cos^2 theta equals:

(a) 0
(b) 1
(c) 2
(d) tan theta

4. The bearing of east from north is:

(a) 000 degrees
(b) 090 degrees
(c) 180 degrees
(d) 270 degrees

5. cos 60 degrees equals:

(a) 1/2 (b) sqrt(3)/2
(c) 1 (d) 0

6. If the bearing of B from A is 070 degrees, the bearing of A from B is:

(a) 070 degrees (b) 110 degrees
(c) 250 degrees (d) 290 degrees

7. tan theta in terms of sin and cos is:

(a) sin/cos (b) cos/sin
(c) sin x cos (d) 1/sin

8. The angle of depression is measured from:

(a) the vertical (b) the horizontal, downwards
(c) north (d) the hypotenuse

9. sin 90 degrees equals:

(a) 0 (b) 1/2
(c) 1 (d) undefined

10. In a right triangle the longest side is the:

(a) opposite (b) adjacent
(c) hypotenuse (d) base

Short Questions

1. Write the three trigonometric ratios for an angle theta in a right triangle.
2. Find the value of sin 60 degrees x cos 30 degrees.
3. A tower casts a shadow 12 m long when the angle of elevation of the sun is 45 degrees. Find the height of the tower.
4. Convert the bearing 315 degrees into a compass direction description.
5. Find angle theta if tan theta = 1.

Long Questions

1. From the top of a 40 m cliff, the angle of depression to a boat is 30 degrees. Find the horizontal distance of the boat from the foot of the cliff. Draw the diagram and show all steps.
2. A ship sails 12 km on a bearing of 060 degrees from port P, then 16 km on a bearing of 150 degrees. Use trigonometry to find its distance from P and the bearing of P from the ship's final position.

Unit 7: Coordinate Geometry

Coordinate geometry joins algebra and geometry: points are described by ordered pairs (x, y), and lines and shapes are studied with formulas. The distance formula, midpoint formula and slope are the working tools of this unit.

7.1 The Cartesian Plane

A point is written as (x, y): x is the horizontal distance from the origin (abscissa) and y is the vertical distance (ordinate).

The plane is divided into four quadrants by the x-axis and y-axis. In quadrant I both coordinates are positive; in quadrant II x is negative and y is positive; in quadrant III both are negative; in quadrant IV x is positive and y is negative.

Points on the x-axis have y = 0; points on the y-axis have x = 0.

7.2 Distance Between Two Points

The distance between P(x1, y1) and Q(x2, y2) is sqrt((x2 - x1)^2 + (y2 - y1)^2), from Pythagoras' theorem.

Distance is always non-negative, and the distance from a point to itself is zero.

The formula is used to check whether a triangle is isosceles, equilateral or right-angled, and to find the radius of a circle from its centre and a point on it.

7.3 Midpoint and Section Formula

The midpoint of P(x1, y1) and Q(x2, y2) is ((x1 + x2)/2, (y1 + y2)/2).

A point dividing the segment PQ in the ratio m:n has coordinates ((n x1 + m x2)/(m + n), (n y1 + m y2)/(m + n)).

The midpoint formula is the special case m = n = 1 of the section formula.

7.4 Slope (Gradient) of a Line

The slope of the line through (x1, y1) and (x2, y2) is m = (y2 - y1)/(x2 - x1), the change in y over the change in x.

Parallel lines have equal slopes. Perpendicular lines have slopes whose product is -1.

A horizontal line has slope 0; a vertical line has undefined slope.

The equation of a line with slope m through (x1, y1) is y - y1 = m(x - x1); the slope-intercept form is y = mx + c, where c is the y-intercept.

> ### Key Formulas
>
> Distance PQ = sqrt((x2 - x1)^2 + (y2 - y1)^2)
>
> Midpoint = ((x1 + x2)/2, (y1 + y2)/2)
>
> Section (m:n) = ((n x1 + m x2)/(m + n), (n y1 + m y2)/(m + n))
>
> Slope m = (y2 - y1)/(x2 - x1)
>
> Line: y - y1 = m(x - x1); y = mx + c; parallel: m1 = m2; perpendicular: m1 x m2 = -1

Worked Example 1: Find the distance between A(1, 2) and B(4, 6)

Step 1: Distance = sqrt((4 - 1)^2 + (6 - 2)^2) = sqrt(3^2 + 4^2).

Step 2: = sqrt(9 + 16) = sqrt(25) = 5 units.

Worked Example 2: Find the slope of the line through (2, 3) and (6, 11)

Step 1: m = (11 - 3)/(6 - 2) = 8/4.

Step 2: m = 2. The line rises 2 units for each unit across.

Key Terms

Key Term	Meaning
Abscissa	The x-coordinate of a point.
Ordinate	The y-coordinate of a point.
Slope	The steepness of a line; change in y over change in x.
Midpoint	The point exactly halfway between two points.
Y-intercept	The y-value where a line crosses the y-axis.

Practice Questions

Multiple Choice Questions

1. The distance between (0, 0) and (3, 4) is:

(a) 5 (b) 7
(c) 12 (d) 25

2. The midpoint of (2, 6) and (4, 10) is:

(a) (3, 8) (b) (6, 16)
(c) (1, 2) (d) (3, 4)

3. The slope of a horizontal line is:

(a) 1 (b) 0
(c) undefined (d) -1

4. The point (-3, 5) lies in quadrant:

(a) I (b) II
(c) III (d) IV

5. The slope of the line through (1, 2) and (3, 8) is:

(a) 2 (b) 3
(c) 4 (d) 6

6. Two lines are perpendicular if the product of their slopes is:

(a) 1 (b) 0
(c) -1 (d) 2

7. The y-intercept of y = 3x + 5 is:

(a) 3 (b) 5
(c) -3 (d) -5

8. The origin has coordinates:

(a) (1, 1) (b) (0, 0)
(c) (1, 0) (d) (0, 1)

9. A point on the x-axis has:

(a) $x = 0$ (b) $y = 0$
(c) $x = y$ (d) $x = 1$

10. The distance formula is based on:

(a) the area formula (b) Pythagoras' theorem
(c) the midpoint rule (d) the slope formula

Short Questions

1. Find the distance between P(-2, 3) and Q(4, -5).
2. Find the midpoint of the segment joining (5, -3) and (-1, 7).
3. Find the slope of the line joining (0, 4) and (3, 10).
4. Write the equation of the line with slope 2 passing through (1, 5).
5. Check whether the points (0, 0), (3, 0) and (0, 4) form a right-angled triangle.

Long Questions

1. Show that the points A(1, 1), B(4, 5) and C(7, 1) form an isosceles triangle, and find its area.
2. A line passes through (2, 3) and is perpendicular to the line $y = 2x + 1$. Find the equation of the line and its x-intercept.

Unit 8: Geometry of Straight Lines

This unit studies angles formed by straight lines: angles on a straight line, angles at a point, vertically opposite angles, and the angles made when a transversal cuts parallel lines. These facts are the building blocks of all later geometry.

8.1 Angles on a Straight Line and at a Point

Angles on a straight line add up to 180 degrees; this is called a supplementary pair when two angles add to 180 degrees.

Angles around a point add up to 360 degrees.

Complementary angles add up to 90 degrees.

Vertically opposite angles are equal: when two straight lines cross, the angles opposite each other are equal.

8.2 Parallel Lines and a Transversal

A transversal is a line that crosses two or more other lines.

When a transversal cuts parallel lines: corresponding angles are equal; alternate angles are equal; co-interior angles are supplementary (add to 180 degrees).

Conversely, if corresponding angles are equal, or alternate angles are equal, or co-interior angles are supplementary, then the two lines are parallel.

These angle facts are used to prove lines parallel and to find unknown angles in diagrams.

8.3 Triangles and Angle Sum

The angles of a triangle add up to 180 degrees.

An exterior angle of a triangle equals the sum of the two opposite interior angles.

In an isosceles triangle the angles opposite the equal sides are equal.

The angles of an equilateral triangle are each 60 degrees.

8.4 Congruent Triangles

Two triangles are congruent if they have exactly the same shape and size; all corresponding sides and angles are equal.

Congruence conditions: SSS (three sides), SAS (two sides and the included angle), ASA (two angles and the included side), AAS (two angles and a non-included side), RHS (right angle, hypotenuse and one side).

Congruence is used to prove that segments or angles in a figure are equal.

Key Formulas

Angles on a straight line = 180 degrees; angles at a point = 360 degrees

Complementary angles sum to 90 degrees; supplementary angles sum to 180 degrees

Parallel lines cut by a transversal: corresponding equal; alternate equal; co-interior supplementary

Triangle angle sum = 180 degrees

Exterior angle = sum of the two opposite interior angles

Worked Example 1: Two angles on a straight line are (2x + 10) degrees and (3x - 20) degrees. Find x.

Step 1: Angles on a straight line sum to 180: (2x + 10) + (3x - 20) = 180.

Step 2: 5x - 10 = 180, so 5x = 190 and x = 38.

Step 3: The angles are 86 degrees and 94 degrees.

Worked Example 2: In triangle ABC, angle A = 50 degrees and angle B = 70 degrees. Find angle C.

Step 1: Angles sum to 180: angle C = 180 - 50 - 70.

Step 2: Angle C = 60 degrees.

Key Terms

Key Term	Meaning
Transversal	A line crossing two or more lines.
Corresponding angles	Angles in the same relative position at each intersection.
Alternate angles	Angles on opposite sides of the transversal, inside the two lines.
Co-interior angles	Angles on the same side of the transversal, inside the two lines.
Congruent	Exactly the same shape and size.

Practice Questions

Multiple Choice Questions

1. Angles on a straight line add up to:

(a) 90 degrees (b) 180 degrees
(c) 270 degrees (d) 360 degrees

2. Two angles that add to 90 degrees are called:

(a) supplementary (b) complementary
(c) vertically opposite (d) adjacent

3. Vertically opposite angles are:

(a) supplementary (b) complementary
(c) equal (d) 90 degrees

4. When a transversal cuts parallel lines, alternate angles are:

(a) supplementary (b) equal
(c) 90 degrees (d) 180 degrees

5. The angles of a triangle add up to:

(a) 90 degrees (b) 180 degrees
(c) 270 degrees (d) 360 degrees

6. An exterior angle of a triangle equals:

(a) the adjacent interior angle
(b) the sum of the two opposite interior angles
(c) 90 degrees
(d) 180 degrees

7. Congruence condition with three sides is:

(a) SAS
(b) ASA
(c) SSS
(d) RHS

8. Angles around a point add up to:

(a) 180 degrees
(b) 270 degrees
(c) 360 degrees
(d) 90 degrees

9. Co-interior angles on parallel lines are:

(a) equal
(b) supplementary
(c) complementary
(d) vertically opposite

10. Each angle of an equilateral triangle is:

(a) 45 degrees
(b) 60 degrees
(c) 90 degrees
(d) 30 degrees

Short Questions

1. Find the complement of 35 degrees and the supplement of 110 degrees.
2. Two straight lines cross, making one angle 72 degrees. Find the other three angles.
3. In a triangle, two angles are 65 degrees and 55 degrees. Find the third angle.
4. A transversal cuts two parallel lines and one corresponding angle is 115 degrees. Find the co-interior angle on the same side.
5. State the four main conditions for two triangles to be congruent.

Long Questions

1. Two parallel lines are cut by a transversal. One alternate angle is (4x - 10) degrees and the other is (2x + 30) degrees. Find x and all eight angles formed.
2. Prove that the exterior angle of a triangle is equal to the sum of the two opposite interior angles, and use the result to find the exterior angle when the opposite interior angles are 48 degrees and 62 degrees.

Unit 9: Geometry and Polygons

A polygon is a closed plane figure made of straight line segments. This unit covers the angle sums of polygons, the special properties of quadrilaterals, and the circle: its parts, angles and tangents.

9.1 Polygons and Their Angles

A polygon with n sides has interior angles summing to (n - 2) x 180 degrees.

The exterior angles of any convex polygon add up to 360 degrees.

A regular polygon has all sides and all angles equal. Each interior angle of a regular n-sided polygon is (n - 2) x 180 / n degrees.

A polygon is named by its number of sides: triangle (3), quadrilateral (4), pentagon (5), hexagon (6), octagon (8).

9.2 Quadrilaterals

A quadrilateral has four sides and its interior angles sum to 360 degrees.

Parallelogram: opposite sides are parallel and equal; opposite angles are equal; diagonals bisect each other.

Rectangle: a parallelogram with all angles 90 degrees; diagonals are equal.

Rhombus: a parallelogram with all sides equal; diagonals bisect each other at right angles.

Square: a rectangle and a rhombus together; all sides equal and all angles 90 degrees.

Trapezium: a quadrilateral with one pair of opposite sides parallel.

Kite: two pairs of adjacent sides equal; one diagonal is bisected by the other.

9.3 The Circle: Parts and Angles

The centre, radius, diameter, chord, arc, sector and segment are the parts of a circle. Diameter = 2 x radius.

The angle at the centre is twice the angle at the circumference standing on the same arc.

Angles in the same segment are equal. The angle in a semicircle is a right angle.

Opposite angles of a cyclic quadrilateral add up to 180 degrees.

The perpendicular from the centre to a chord bisects the chord.

9.4 Tangents to a Circle

A tangent touches a circle at exactly one point and is perpendicular to the radius at the point of contact.

Tangents drawn from the same external point to a circle are equal in length.

The alternate segment theorem: the angle between a tangent and a chord at the point of contact equals the angle in the alternate segment.

> ### Key Formulas
>
> Interior angle sum of an n-sided polygon = (n - 2) x 180 degrees
>
> Exterior angle sum of any convex polygon = 360 degrees

Each interior angle of a regular n-gon = (n - 2) x 180 / n degrees

Circumference = 2 pi r; Area of circle = pi r^2

Angle at centre = 2 x angle at circumference (same arc)

Diameter = 2 x radius

Worked Example 1: Find each interior angle of a regular hexagon

Step 1: Sum of interior angles = (6 - 2) x 180 = 720 degrees.

Step 2: Each angle = 720 / 6 = 120 degrees.

Worked Example 2: The angle at the centre of a circle is 100 degrees. Find the angle at the circumference on the same arc.

Step 1: Angle at the circumference = half the angle at the centre.

Step 2: 100 / 2 = 50 degrees.

Key Terms

Key Term	Meaning
Polygon	A closed figure made of straight line segments.
Regular polygon	A polygon with all sides and all angles equal.
Cyclic quadrilateral	A quadrilateral whose four vertices lie on a circle.
Chord	A line segment joining two points on a circle.
Tangent	A line that touches a circle at exactly one point.
Sector	The region of a circle bounded by two radii and an arc.

Practice Questions

Multiple Choice Questions

1. The interior angle sum of a pentagon is:

(a) 360 degrees (b) 540 degrees
(c) 720 degrees (d) 900 degrees

2. Each interior angle of a regular hexagon is:

(a) 90 degrees (b) 108 degrees
(c) 120 degrees (d) 135 degrees

3. The exterior angles of any convex polygon add to:

(a) 180 degrees (b) 270 degrees
(c) 360 degrees (d) 540 degrees

4. A quadrilateral with all sides equal and all angles 90 degrees is a:

(a) rhombus (b) rectangle
(c) square (d) kite

5. The angle in a semicircle is:

(a) 45 degrees (b) 60 degrees
(c) 90 degrees (d) 180 degrees

6. Opposite angles of a cyclic quadrilateral are:

(a) equal (b) supplementary
(c) complementary (d) 90 degrees

7. The circumference of a circle of radius 7 cm is about:

(a) 22 cm (b) 44 cm
(c) 154 cm (d) 14 cm

8. A tangent to a circle is perpendicular to the:

(a) chord (b) radius at the point of contact
(c) diameter (d) arc

9. The diagonals of a rhombus bisect each other at:

(a) 45 degrees (b) 60 degrees
(c) 90 degrees (d) 180 degrees

10. A hexagon has how many sides?

(a) 5 (b) 6
(c) 7 (d) 8

Short Questions

1. Find the sum of the interior angles of an octagon.
2. Each interior angle of a regular polygon is 135 degrees. How many sides does it have?
3. State two properties of the diagonals of a rectangle.
4. In a circle, the angle at the centre is 84 degrees. Find the angle at the circumference on the same arc.
5. Two tangents are drawn from an external point to a circle; one is 9 cm long. How long is the other? Give the reason.

Long Questions

1. Prove that the sum of the interior angles of a quadrilateral is 360 degrees, and use the result to find the fourth angle when the other three are 95 degrees, 80 degrees and 110 degrees.
2. ABCD is a cyclic quadrilateral with angle A = 70 degrees and angle B = 105 degrees. Find angles C and D, and explain each step using the cyclic quadrilateral property.

Unit 10: Practical Geometry

Practical geometry is geometry done with instruments: ruler, compasses, protractor and set squares. The skill is accuracy: a construction is only correct if every step is followed in order and every measurement is checked.

10.1 Basic Constructions

Bisecting a line segment: draw two arcs of equal radius centred at the endpoints; the line joining the arc intersections bisects the segment at right angles.

Bisecting an angle: draw an arc across both arms, then two equal arcs from the points where it meets the arms; the line from the vertex through their intersection bisects the angle.

Drawing a perpendicular from a point to a line, and drawing a perpendicular at a point on a line, both use equal arcs to find two points the same distance away.

Constructing a parallel line through a given point uses equal corresponding angles.

10.2 Constructing Standard Angles

An angle of 60 degrees is constructed from an equilateral triangle: two arcs of the same radius give the 60-degree point.

Bisecting 60 degrees gives 30 degrees; bisecting 90 degrees gives 45 degrees.

An angle of 90 degrees is constructed as the perpendicular at a point, or by combining 60 degrees and 30 degrees.

Angles such as 15 degrees and 75 degrees are made by further bisection and addition of standard angles.

10.3 Constructing Triangles

A triangle can be constructed from three sides (SSS), two sides and the included angle (SAS), or two angles and a side (ASA/AAS).

Steps: draw the base to scale, mark the given lengths with compasses, and find the third vertex where the arcs cross.

A triangle is impossible if the sum of any two sides is not greater than the third side.

Constructing a triangle with a given base, base angles and perimeter, or with a given altitude, follows the same arc method with extra steps.

10.4 Constructing Quadrilaterals and Circles

A quadrilateral is constructed from enough given sides and angles, usually by dividing it into two triangles along a diagonal.

A circle is constructed from its centre and radius; a circle through three non-collinear points has its centre at the intersection of the perpendicular bisectors of the joins.

Circumscribing a circle about a triangle: the centre is the intersection of the perpendicular bisectors of the sides.

Inscribing a circle in a triangle: the centre is the intersection of the angle bisectors.

Key Formulas

Bisector of 60 degrees = 30 degrees; bisector of 90 degrees = 45 degrees

Triangle possible only if sum of any two sides > third side

Circumcentre = intersection of perpendicular bisectors of the sides

Incentre = intersection of the angle bisectors

Perpendicular bisector: every point on it is equidistant from the two endpoints

Worked Example 1: Construct an angle of 30 degrees

Step 1: Construct a 60-degree angle using two arcs of equal radius.

Step 2: Bisect the 60-degree angle: draw two equal arcs inside it and join their intersection to the vertex.

Step 3: Each half is 30 degrees. Check with a protractor.

Worked Example 2: Can a triangle have sides 3 cm, 4 cm and 8 cm?

Step 1: Check the shortest two sides: 3 + 4 = 7.

Step 2: 7 is not greater than 8, so the arcs would not meet.

Step 3: No, such a triangle cannot be constructed.

Key Terms

Key Term	Meaning
Bisector	A line or ray dividing a segment or angle into two equal parts.
Perpendicular bisector	A line that cuts a segment at its midpoint at 90 degrees.
Circumcentre	The centre of the circle passing through all vertices of a triangle.
Incentre	The centre of the circle touching all sides of a triangle.
Locus	The set of all points satisfying a given condition.

Practice Questions

Multiple Choice Questions

1. Bisecting a 90-degree angle gives:

(a) 30 degrees (b) 45 degrees
(c) 60 degrees (d) 15 degrees

2. The perpendicular bisector of a segment passes through its:

(a) endpoint (b) midpoint at 90 degrees
(c) vertex (d) centre of arc

3. A triangle with sides 5 cm, 6 cm and 7 cm:

(a) cannot be drawn (b) can be drawn
(c) is right-angled (d) is equilateral

4. The circumcentre of a triangle is the intersection of the:

(a) angle bisectors (b) perpendicular bisectors
(c) medians (d) altitudes

5. An angle of 60 degrees is constructed using:

(a) a protractor only (b) an equilateral triangle construction
(c) a 45-degree bisection (d) a set square only

6. Bisecting a 60-degree angle gives:

(a) 20 degrees (b) 30 degrees
(c) 45 degrees (d) 15 degrees

7. The incentre is the centre of the circle that:

(a) passes through the vertices (b) touches all three sides
(c) has the longest side as diameter (d) is outside the triangle

8. A triangle cannot be formed if the sum of two sides is:

(a) greater than the third (b) equal to the third
(c) less than the third (d) Both second and third options

9. To draw a parallel line through a point, we copy the:

(a) corresponding angle (b) opposite angle
(c) right angle (d) arc length

10. The perpendicular from the centre of a circle to a chord:

(a) is the radius (b) bisects the chord
(c) is a tangent (d) is the diameter

Short Questions

1. Describe the steps to bisect a line segment of length 8 cm.
2. How would you construct an angle of 45 degrees without a protractor?
3. List the three sets of measurements that are enough to construct a triangle.
4. Can a triangle have sides 4 cm, 9 cm and 5 cm? Give the reason.
5. Where is the circumcentre of a right-angled triangle located?

Long Questions

1. Construct a triangle ABC with AB = 6 cm, BC = 5 cm and AC = 7 cm. Describe every construction step in order, and state how you would check the result.
2. Construct a circle through three non-collinear points P, Q and R. Describe the construction steps and explain why the perpendicular bisectors give the centre.

Unit 11: Basic Statistics

Statistics turns raw data into information: averages that summarise a group, measures of spread that show how varied it is, and graphs that make patterns visible. This unit covers the mean, median, mode, range, and the bar chart, histogram and pie chart.

11.1 Data and Frequency

Raw data is unorganised information; arranging it in a frequency table makes it readable.

In a frequency table, each value or class interval has a frequency: the number of times it occurs.

A class interval such as 10-20 has lower limit 10, upper limit 20 and class size 10. The class mark (midpoint) is (10 + 20)/2 = 15.

Discrete data takes separate values (number of children); continuous data can take any value in a range (height, weight).

11.2 Measures of Central Tendency

Mean: add all values and divide by the number of values. For grouped data, mean = sum(f x class mark) / sum(f).

Median: the middle value when data is arranged in order. For an even number of values it is the average of the two middle values.

Mode: the value that occurs most often. In grouped data the modal class is the class with the highest frequency.

The mean is affected by very large or small values; the median is not, so the median is better for data with extreme values.

11.3 Measures of Spread

Range = highest value - lowest value. It is the simplest measure of spread.

A larger range or a larger spread means the data is more varied; a small range means the values are close together.

Quartiles divide ordered data into four parts; the interquartile range (Q3 - Q1) describes the spread of the middle half of the data.

11.4 Graphical Representation

A bar chart shows categories with bars whose heights are the frequencies; bars are separated by gaps.

A histogram shows continuous grouped data with bars joined together; the area of each bar represents frequency.

A pie chart shows parts of a whole as sectors of a circle; each sector angle = (frequency / total) x 360 degrees.

A frequency polygon joins the midpoints of the tops of histogram bars.

Choose the graph that fits the data: bar chart for categories, histogram for continuous groups, pie chart for shares of a whole.

Key Formulas

Mean = sum of values / number of values

Grouped mean = sum(f x class mark) / sum(f)

Range = highest value - lowest value

Median position = (n + 1)/2 th value (ordered data)

Pie sector angle = (frequency / total frequency) x 360 degrees

Class mark = (lower limit + upper limit) / 2

Worked Example 1: Find the mean of 4, 8, 6, 10, 2

Step 1: Sum = 4 + 8 + 6 + 10 + 2 = 30.

Step 2: Number of values = 5.

Step 3: Mean = 30 / 5 = 6.

Worked Example 2: Find the median of 7, 3, 9, 5, 1

Step 1: Arrange in order: 1, 3, 5, 7, 9.

Step 2: The middle (3rd) value is 5.

Step 3: Median = 5.

Key Terms

Key Term	Meaning
Frequency	The number of times a value or class occurs.
Mean	The average: sum of values divided by their number.
Median	The middle value of ordered data.
Mode	The most frequently occurring value.
Range	The difference between the highest and lowest values.
Histogram	A graph of continuous grouped data with joined bars.

Practice Questions

Multiple Choice Questions

1. The mean of 2, 4, 6, 8 is:

(a) 4 (b) 5
(c) 6 (d) 20

2. The median of 3, 1, 4, 1, 5 is:

(a) 1 (b) 3
(c) 4 (d) 5

3. The mode of 2, 3, 3, 5, 3, 4 is:

(a) 2 (b) 3
(c) 4 (d) 5

4. The range of 12, 5, 20, 8 is:

(a) 8 (b) 12
(c) 15 (d) 20

5. The class mark of 20-30 is:

(a) 20 (b) 25
(c) 30 (d) 10

6. In a pie chart, a frequency that is one quarter of the total has a sector angle of:

(a) 45 degrees (b) 90 degrees
(c) 120 degrees (d) 180 degrees

7. A histogram is used for:

(a) categorical data (b) continuous grouped data
(c) shares of a whole (d) two variables

8. The median is preferred over the mean when the data has:

(a) equal values (b) extreme values
(c) no mode (d) few values

9. The sum of frequencies in a table equals the:

(a) mean (b) total number of observations
(c) range (d) mode

10. A bar chart is best for showing:

(a) continuous data (b) categories and their frequencies
(c) parts of a whole circle (d) the median

Short Questions

1. Find the mean and median of 10, 15, 20, 25, 30.
2. The marks of 6 students are 12, 18, 15, 20, 14, 17. Find the range.
3. Find the class mark of the interval 40-50.
4. In a pie chart, a category has frequency 15 out of a total of 60. Find its sector angle.
5. State one difference between a bar chart and a histogram.

Long Questions

1. The weights (in kg) of 10 students are: 42, 48, 45, 50, 44, 52, 47, 49, 46, 51. Find the mean, median and range, and say which average best represents the group.
2. A survey of 72 students' favourite subjects gave: Mathematics 24, Science 18, English 12, Urdu 18. Draw up the angle for each subject in a pie chart and describe how the chart would be drawn.

MCQ Answer Key

Answers to all 110 multiple choice questions, unit by unit. Check your answers after attempting the questions without help.

Unit 1	1.c 2.b 3.c 4.a 5.b 6.b 7.a 8.a 9.b 10.c
Unit 2	1.c 2.b 3.b 4.b 5.b 6.a 7.c 8.b 9.c 10.c
Unit 3	1.c 2.c 3.b 4.c 5.b 6.b 7.b 8.b 9.c 10.b
Unit 4	1.b 2.a 3.b 4.a 5.a 6.b 7.b 8.a 9.b 10.a
Unit 5	1.b 2.b 3.b 4.a 5.c 6.a 7.b 8.b 9.b 10.b
Unit 6	1.b 2.c 3.b 4.b 5.a 6.c 7.a 8.b 9.c 10.c
Unit 7	1.a 2.a 3.b 4.b 5.b 6.c 7.b 8.b 9.b 10.b
Unit 8	1.b 2.b 3.c 4.b 5.b 6.b 7.c 8.c 9.b 10.b
Unit 9	1.b 2.c 3.c 4.c 5.c 6.b 7.b 8.b 9.c 10.b
Unit 10	1.b 2.b 3.b 4.b 5.b 6.b 7.b 8.d 9.a 10.b
Unit 11	1.b 2.b 3.b 4.c 5.b 6.b 7.b 8.b 9.b 10.b

All practice questions in this book are original, written for these notes. Verify the current paper pattern and model paper on ajkbise.net for your session. www.zahidenotes.com