ZAHID NOTES | FREE STUDY MATERIAL FOR STUDENTS

9th Class Mathematics Notes Balochistan Board (BISE Quetta) Full Book Notes

Complete chapter-wise notes, worked examples and original practice questions for all 17 chapters of the Balochistan Textbook Board Mathematics book for Class 9, Board of Intermediate and Secondary Education, Quetta.

Board: Board of Intermediate and Secondary Education (BISE), Quetta — Balochistan

Class: 9th Class | **Subject:** Mathematics | **Book:** Balochistan Textbook Board, Quetta

Medium: English

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How to Use These Notes

These notes follow the 17 chapters of the Balochistan Textbook Board Mathematics book for Class 9. Each chapter gives you the key ideas in plain words, the formulas you must remember, one fully worked example, five short questions with answers, two long questions with answer outlines, and ten multiple-choice questions (MCQs). The answers to all MCQs are collected in the answer key at the end of the book.

How to study: read the notes of one chapter, copy the formulas onto a revision card, work through the example with a pen in your hand, then answer the short and long questions without looking — and only then try the MCQs. Finish every chapter this way before the exam, and revise the formulas again in the last week.

Paper Pattern — What Is Confirmed

> **Total marks for Mathematics (Class 9, BISE Quetta): 75 (theory).** This total is given in the board's revised Scheme of Studies for SSC-I (notification dated 30 June 2026).
>
> **MCQ / short-question / long-question split for Quetta:** no official split for BISE Quetta could be verified from a published board document at the time of writing, so no split is printed here. Do not rely on another board's split — confirm the current split from the latest BBISE Quetta model paper issued by your school or the board.

> **Important — two board notices to check before your exam.** (1) BBISE announced a **one-time** proportionate reduction in syllabus for the SSC Annual Examination 2026 (notification dated 25 September 2026): for Grade 9 Mathematics, **Chapters 3 and 11** were excluded for that examination only. These notes still cover all 17 chapters, because the reduction was described as one-time — confirm with your school whether it still applies to your session. (2) BBISE has introduced a revised syllabus from the 2026 session, and its new model papers also test topics such as sets, probability, statistics, trigonometry, percentages, exponential growth and bearings. If your school has issued the new book, study those topics from your class work as well. The chapter list in this book is the published Balochistan Textbook Board book list, verified from two independent sources.

Contents

1. Matrices and Determinants	2. Real and Complex Numbers
3. Logarithms	4. Algebraic Expressions and Algebraic Formulae
5. Factorization	6. Algebraic Manipulation
7. Linear Equations and Inequalities	8. Linear Graphs & Their Application
9. Introduction to Coordinate Geometry	10. Congruent Triangles
11. Parallelograms and Triangles	12. Line Bisectors and Angle Bisectors
13. Sides and Angles of a Triangle	14. Ratio and Proportion
15. Pythagoras Theorem	16. Theorems Related with Area

17. Practical Geometry – Triangles MCQ Answer Key (all chapters)

All notes and questions in this book are original study material written for Zahid Notes. They explain the ideas of the textbook in fresh words; they do not reproduce textbook text. Work every question yourself — memorised answers do not survive a changed question.

Chapter 1: Matrices and Determinants

A matrix is a rectangular arrangement of numbers in rows and columns, written inside square brackets. Matrices let us organise data and solve two linear equations at once. The idea was developed by Arthur Cayley in the 19th century.

- **Order of a matrix:** a matrix with m rows and n columns is of order $m \times n$. The entry in row i, column j is written a_{ij}.
- **Types:** row matrix (one row), column matrix (one column), square matrix (rows = columns), zero or null matrix (all entries 0), diagonal matrix (non-zero entries only on the main diagonal), scalar matrix (diagonal matrix with equal diagonal entries), identity matrix I (scalar matrix with 1s on the diagonal).
- **Transpose:** A^t is formed by turning rows into columns. A matrix is **symmetric** if $A^t = A$ and **skew-symmetric** if $A^t = -A$.
- **Addition and subtraction:** only matrices of the same order can be added or subtracted, entry by entry. Addition is commutative and associative.
- **Multiplication by a number:** kA multiplies every entry by k. **Matrix multiplication:** AB is defined only when the number of columns of A equals the number of rows of B; the (i, j) entry of AB is the dot product of row i of A with column j of B. Matrix multiplication is not commutative in general.
- **Determinant of a 2 × 2 matrix:** if A = [[a, b], [c, d]] then $|A| = ad - bc$. A square matrix is **singular** if its determinant is 0, and **non-singular** if it is not 0.
- **Inverse:** if $|A| \neq 0$, then $A^{-1} = (1/|A|) \times [[d, -b], [-c, a]]$. Also $(AB)^{-1} = B^{-1}A^{-1}$.
- **Solving equations:** the system $a_1x + b_1y = c_1$, $a_2x + b_2y = c_2$ can be written as $AX = B$. By the **matrix inversion method**, $X = A^{-1}B$. By **Cramer's rule**, $x = |A_x| / |A|$ and $y = |A_y| / |A|$, where A_x and A_y are formed by replacing the x-column and y-column of A with B.

> **Remember:** $|A| = ad - bc$ for $A = [[a, b], [c, d]]$ | $A^{-1} = (1/|A|)\ [[d, -b], [-c, a]]$ | $(AB)^{-1} = B^{-1}A^{-1}$ | $AA^{-1} = I$

Worked example. Find the inverse of A = [[2, 3], [1, 4]] and use it to solve 2x + 3y = 8, x + 4y = 9.

Solution: $|A| = (2)(4) - (3)(1) = 8 - 3 = 5 \neq 0$, so A is non-singular. $A^{-1} = (1/5)\ [[4, -3], [-1, 2]]$. Then $[x, y]^t = A^{-1}B = (1/5)\ [[4, -3], [-1, 2]]\ [8, 9]^t = (1/5)\ [32 - 27, -8 + 18]^t = (1/5)\ [5, 10]^t = [1, 2]^t$. So x = 1, y = 2. Check: 2(1) + 3(2) = 8 ✓ and 1 + 4(2) = 9 ✓.

Short Questions

1. Define a square matrix and give an example. **Ans:** A matrix with the same number of rows and columns, e.g. [[1, 2], [3, 4]] is of order 2 × 2.

2. If A = [[5, −2], [3, 1]], find |A|. **Ans:** $|A| = (5)(1) - (-2)(3) = 5 + 6 = 11$.

3. What is the transpose of [[2, 0], [−1, 5]]? **Ans:** [[2, −1], [0, 5]] — rows become columns.

4. When is the product AB of two matrices defined? **Ans:** When the number of columns of A equals the number of rows of B.

5. State the condition for a matrix to be singular. **Ans:** Its determinant is zero (|A| = 0); then its inverse does not exist.

Long Questions

1. If A = [[1, 2], [3, 4]] and B = [[2, 0], [1, 3]], verify that $(AB)^t = B^tA^t$. **Ans:** AB = [[4, 6], [10, 12]], so $(AB)^t$ = [[4, 10], [6, 12]]. B^tA^t = [[2, 1], [0, 3]] [[1, 3], [2, 4]] = [[4, 10], [6, 12]]. Both sides are equal, hence verified.

2. Solve by Cramer's rule: 3x + 2y = 7, 2x − y = 4. **Ans:** |A| = (3)(−1) − (2)(2) = −7. $|A_x|$ = (7)(−1) − (2)(4) = −15, so x = (−15)/(−7) = 15/7. $|A_y|$ = (3)(4) − (7)(2) = −2, so y = (−2)/(−7) = 2/7. Solution set = {(15/7, 2/7)}.

Multiple Choice Questions

1. The order of a matrix with 3 rows and 4 columns is:
(a) 4 × 3 (b) 3 × 4
(c) 12 (d) 3 + 4

2. A matrix with all entries zero is called a: (a) identity matrix
(b) null matrix (c) diagonal matrix
(d) scalar matrix

3. If A = [[3, 1], [2, 5]], then |A| = (a) 13
(b) 17 (c) 15
(d) 11

4. The transpose of a row matrix is a: (a) row matrix
(b) column matrix (c) square matrix
(d) zero matrix

5. A matrix A is symmetric if: (a) $A^t = -A$
(b) $A^t = A$ (c) |A| = 0
(d) A = I

6. If A is 2 × 3 and B is 3 × 2, the order of AB is: (a) 3 × 3
(b) 2 × 2 (c) 2 × 3
(d) 3 × 2

7. The inverse of [[2, 0], [0, 2]] is: (a) [[2, 0], [0, 2]]
(b) [[1/2, 0], [0, 1/2]] (c) [[0, 2], [2, 0]]
(d) [[−2, 0], [0, −2]]

8. If |A| = 0, then A is: (a) non-singular
(b) invertible (c) singular
(d) identity

9. $(AB)^{-1}$ equals: (a) $A^{-1}B^{-1}$
(b) $B^{-1}A^{-1}$ (c) $A^{-1} + B^{-1}$
(d) $(BA)^{-1}$

10. The identity matrix of order 2 is: (a) [[0, 0], [0, 0]]
(b) [[1, 1], [1, 1]] (c) [[1, 0], [0, 1]]
(d) [[2, 0], [0, 2]]

Chapter 2: Real and Complex Numbers

Real numbers are all the numbers on the number line — natural numbers, whole numbers, integers, rational numbers and irrational numbers together. Complex numbers extend this system so that we can take the square root of a negative number.

- **Sets of numbers:** $N = \{1, 2, 3, \ldots\}$, $W = \{0, 1, 2, \ldots\}$, $Z = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, $Q = \{p/q : p, q \in Z, q \neq 0\}$. $N \subset W \subset Z \subset Q \subset R$.
- **Rational vs irrational:** a rational number has a terminating or repeating decimal expansion; an irrational number (like $\sqrt{2}$ or π) has a non-terminating, non-repeating decimal expansion and cannot be written as p/q.
- **Properties of real numbers:** closure, commutative, associative and distributive laws for addition and multiplication; additive identity 0, multiplicative identity 1; additive inverse of a is $-a$, multiplicative inverse (reciprocal) of $a \neq 0$ is $1/a$.
- **Radicals and exponents:** $\sqrt{a}$ means the non-negative square root. $(\sqrt{a})^2 = a$ for $a \geq 0$. Laws: $a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $(a^m)^n = a^{mn}$, $a^0 = 1$, $a^{-n} = 1/a^n$. Also $a^{1/n} = \sqrt[n]{a}$.
- **Rationalising the denominator:** multiply top and bottom by the conjugate of the denominator. For example, $1/(\sqrt{a} + \sqrt{b})$ is multiplied by $(\sqrt{a} - \sqrt{b})/(\sqrt{a} - \sqrt{b})$.
- **Complex numbers:** $i = \sqrt{(-1)}$, so $i^2 = -1$. A complex number is $z = a + bi$, where a is the real part and b the imaginary part. Powers of i repeat every four: $i, -1, -i, 1, i, \ldots$
- **Operations on complex numbers:** $(a + bi) \pm (c + di) = (a \pm c) + (b \pm d)i$. $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$. The **conjugate** of $a + bi$ is $a - bi$, and $(a + bi)(a - bi) = a^2 + b^2$ (a real number). Division is done by multiplying top and bottom by the conjugate of the denominator.

> **Remember:** $i^2 = -1$ | $i^4 = 1$ | conjugate of $a + bi$ is $a - bi$ | $(a + bi)(a - bi) = a^2 + b^2$ | $a^m \times a^n = a^{m+n}$

Worked example. Simplify $(2 + 3i)(1 - 2i)$ and write the result in the form $a + bi$.

Solution: $(2 + 3i)(1 - 2i) = 2(1) + 2(-2i) + 3i(1) + 3i(-2i) = 2 - 4i + 3i - 6i^2 = 2 - i - 6(-1) = 2 - i + 6 = 8 - i$. So $a = 8$, $b = -1$.

Short Questions

1. Is $0.333\ldots$ a rational or irrational number? Why? **Ans:** Rational — it is a repeating decimal equal to $1/3$.

2. Find the value of i^{27}. **Ans:** $27 = 4 \times 6 + 3$, so $i^{27} = i^3 = -i$.

3. Write the conjugate of $5 - 2i$. **Ans:** $5 + 2i$.

4. Simplify $\sqrt{50}$. **Ans:** $\sqrt{50} = \sqrt{(25 \times 2)} = 5\sqrt{2}$.

5. State the multiplicative inverse of 7. **Ans:** $1/7$, because $7 \times (1/7) = 1$.

Long Questions

1. Rationalise the denominator of $3/(2 + \sqrt{5})$ and simplify. **Ans:** Multiply by $(2 - \sqrt{5})/(2 - \sqrt{5})$: $3(2 - \sqrt{5})/((2)^2 - (\sqrt{5})^2) = 3(2 - \sqrt{5})/(4 - 5) = 3(2 - \sqrt{5})/(-1) = -6 + 3\sqrt{5}$.

2. If $z_1 = 3 + 2i$ and $z_2 = 1 - 4i$, find $z_1 + z_2$ and $z_1 \times z_2$. **Ans:** $z_1 + z_2 = 4 - 2i$. $z_1 \times z_2 = 3 + 2i - 12i - 8i^2 = 3 - 10i + 8 = 11 - 10i$.

Multiple Choice Questions

1. Which of the following is an irrational number?
(a) 2/3 (b) 0.25
(c) √3 (d) −5

2. The value of i^4 is: (a) i
(b) −1 (c) 1
(d) −i

3. The conjugate of 4 + 7i is: (a) −4 + 7i
(b) 4 − 7i (c) −4 − 7i
(d) 7 + 4i

4. √8 × √2 equals: (a) √10
(b) 4 (c) 16
(d) 2√2

5. The additive identity in real numbers is: (a) 1
(b) −1 (c) 0
(d) 10

6. (2 + i)(2 − i) equals: (a) 3
(b) 5 (c) 4 − i
(d) 4 + i

7. Which set is the smallest: N, W, Z, Q? (a) Q
(b) Z (c) W
(d) N

8. $5^0 \times 5^3$ equals: (a) 5^3
(b) 0 (c) 5^0
(d) 25

9. The real part of 6 − 5i is: (a) −5
(b) 5 (c) 6
(d) −6

10. π is an example of a/an: (a) rational number
(b) integer (c) irrational number
(d) whole number

Chapter 3: Logarithms

A logarithm is the inverse of an exponent. If $a^x = y$, then $\log_a y = x$. Logarithms turn multiplication into addition and powers into multiplication, which is why they were used for centuries to simplify long calculations — and why they still appear in science, from measuring earthquakes to sound.

- **Definition:** $\log_a y = x$ exactly when $a^x = y$, with $a > 0$, $a \neq 1$, $y > 0$. The number a is the base, y is the argument.
- **Common logarithm:** $\log_{10}$ is written simply as log. **Natural logarithm:** $\log_e$ is written as ln. In this book, log means $\log_{10}$ unless another base is stated.
- **Basic values:** $\log_a 1 = 0$, $\log_a a = 1$, $\log_a a^x = x$, $a^{\log_a y} = y$.
- **Laws of logarithms:** (i) $\log(mn) = \log m + \log n$; (ii) $\log(m/n) = \log m - \log n$; (iii) $\log m^n = n \log m$.
- **Change of base:** $\log_a b = \log b / \log a$ (using any convenient base, usually 10).
- **Characteristic and mantissa:** in log x, the whole-number part is the characteristic and the decimal part is the mantissa. For a number with n digits before the decimal point, the characteristic is $n - 1$; for a number smaller than 1 with k zeros just after the decimal point, the characteristic is $-(k + 1)$. The mantissa is always positive and is read from a log table.
- **Antilogarithm:** if $\log x = y$, then $x = \text{antilog } y$. To find a number from its log, take the antilog of the mantissa and place the decimal point using the characteristic.
- **Scientific notation:** a number written as $a \times 10^n$ with $1 \leq a < 10$. Its log has characteristic n and mantissa log a — a quick way to handle very large or very small numbers.

> **Remember:** $\log(mn) = \log m + \log n$ | $\log(m/n) = \log m - \log n$ | $\log m^n = n \log m$ | $\log_a 1 = 0$, $\log_a a = 1$

Worked example. If $\log 2 = 0.3010$ and $\log 3 = 0.4771$, find $\log 72$.

Solution: $72 = 8 \times 9 = 2^3 \times 3^2$. So $\log 72 = \log 2^3 + \log 3^2 = 3 \log 2 + 2 \log 3 = 3(0.3010) + 2(0.4771) = 0.9030 + 0.9542 = 1.8572$.

Short Questions

1. Write $5^3 = 125$ in logarithmic form. **Ans:** $\log_5 125 = 3$.

2. Find the value of $\log_2 32$. **Ans:** $2^5 = 32$, so $\log_2 32 = 5$.

3. What is the characteristic of log 0.0047? **Ans:** 0.0047 has two zeros after the decimal point before the first significant digit, so the characteristic is −3.

4. Simplify $\log 1000 - \log 10$. **Ans:** $\log(1000/10) = \log 100 = 2$.

5. If $\log 5 = 0.6990$, find log 25. **Ans:** $\log 25 = \log 5^2 = 2 \log 5 = 2(0.6990) = 1.3980$.

Long Questions

1. Prove that $\log(m/n) = \log m - \log n$ using the definition of logarithms. **Ans:** Let $\log m = x$ and $\log n = y$, so $m = a^x$ and $n = a^y$ (base a). Then $m/n = a^x/a^y = a^{x-y}$, so $\log(m/n) = x - y = \log m - \log n$.

2. Using log tables: find the value of (4.56 × 78.9) / 12.3, given log 4.56 = 0.6590, log 78.9 = 1.8971, log 12.3 = 1.0899. **Ans:** log of the result = 0.6590 + 1.8971 − 1.0899 = 1.4662. The antilog of 0.4662 is about 2.925, and the characteristic 1 places the decimal point one place right: about 29.25.

Multiple Choice Questions

1. If $2^x = 16$, then x = (a) 2 (b) 4 (c) 8 (d) 16

2. $\log_{10} 100$ equals: (a) 1 (b) 2 (c) 10 (d) 100

3. log 1 is equal to: (a) 1 (b) 10 (c) 0 (d) not defined

4. log 8 + log 2 equals: (a) log 10 (b) log 16 (c) log 6 (d) log 4

5. log 27 − log 3 equals: (a) log 24 (b) log 9 (c) log 30 (d) log 81

6. The characteristic of log 567 is: (a) 2 (b) 3 (c) 1 (d) 0

7. log 5^3 in expanded form is: (a) 3 + log 5 (b) 5 log 3 (c) 3 log 5 (d) log 15

8. If log x = 2, then x = (a) 20 (b) 2 (c) 100 (d) 200

9. The mantissa of a logarithm is always: (a) negative (b) zero (c) positive (d) an integer

10. $\log_7 7$ equals: (a) 0 (b) 7 (c) 1 (d) 49

Chapter 4: Algebraic Expressions and Algebraic Formulae

Algebra is arithmetic with letters. A term is a product of numbers and variables; an expression is a sum of terms. The formulae in this chapter are the working tools for the whole book — learn them by using them, not by staring at them.

- **Terms:** in $3x^2y$, the coefficient is 3 and the literal part is x^2y. Terms with the same literal part are **like terms** and can be added; unlike terms cannot be combined.
- **Polynomials:** an expression with non-negative integer powers of one variable, e.g. $2x^3 - 5x + 1$. Its degree is the highest power of the variable. A polynomial of degree 1 is linear, degree 2 quadratic, degree 3 cubic.
- **Addition and subtraction:** add or subtract like terms only. **Multiplication:** multiply coefficients, add the powers of the same variable; every term of the first expression multiplies every term of the second.
- **Square formulae:** $(a + b)^2 = a^2 + 2ab + b^2$; $(a - b)^2 = a^2 - 2ab + b^2$; $(a + b)(a - b) = a^2 - b^2$.
- **Cube formulae:** $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$; $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$; $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$; $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.
- **More identities:** $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$; $(x + a)(x + b) = x^2 + (a + b)x + ab$; if $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$.
- **Surds:** an irrational root such as $\sqrt{2}$ or $\sqrt[3]{5}$ is called a surd. Like surds can be added; surds are multiplied and divided by their radicands. Rationalising a denominator uses the conjugate, as in Chapter 2.

> **Remember:** $(a + b)^2 = a^2 + 2ab + b^2$ | $(a - b)^2 = a^2 - 2ab + b^2$ | $a^2 - b^2 = (a + b)(a - b)$ | $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Worked example. Find the value of $x^2 + 1/x^2$ if $x + 1/x = 5$.

Solution: Square both sides: $(x + 1/x)^2 = 5^2$, so $x^2 + 2(x)(1/x) + 1/x^2 = 25$, i.e. $x^2 + 2 + 1/x^2 = 25$. Hence $x^2 + 1/x^2 = 25 - 2 = 23$.

Short Questions

1. Expand $(2x + 3)^2$. **Ans:** $(2x)^2 + 2(2x)(3) + 3^2 = 4x^2 + 12x + 9$.

2. What is the degree of $7x^4 - 2x^2 + 9$? **Ans:** 4 (the highest power of x).

3. Factorise $a^2 - 49$ using a formula. **Ans:** $a^2 - 7^2 = (a + 7)(a - 7)$.

4. Simplify $3a + 5b - a + 2b$. **Ans:** $(3a - a) + (5b + 2b) = 2a + 7b$.

5. Write $(x + 4)(x + 6)$ as a trinomial. **Ans:** $x^2 + (4 + 6)x + 24 = x^2 + 10x + 24$.

Long Questions

1. If $a + b = 7$ and $ab = 12$, find $a^2 + b^2$ and $a^3 + b^3$. **Ans:** $a^2 + b^2 = (a + b)^2 - 2ab = 49 - 24 = 25$. $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = 343 - 3(12)(7) = 343 - 252 = 91$.

2. Prove that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$. **Ans:** $(a + b + c)^2 = (a + b + c)(a + b + c)$. Multiplying every term: $a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$. Hence proved.

Multiple Choice Questions

1. $(a + b)^2$ equals: (a) $a^2 + b^2$
(b) $a^2 + 2ab + b^2$ (c) $a^2 - 2ab + b^2$
(d) $a^2 + ab + b^2$

2. The degree of $5x^3 + 2x$ is: (a) 2
(b) 5 (c) 3
(d) 1

3. $a^2 - b^2$ factorises as: (a) $(a - b)^2$
(b) $(a + b)(a - b)$ (c) $(a + b)^2$
(d) $(a - b)(a - b)$

4. $(x + 5)(x + 3)$ equals: (a) $x^2 + 8x + 15$
(b) $x^2 + 15x + 8$ (c) $x^2 + 2x + 15$
(d) $x^2 + 8x + 8$

5. Which pair are like terms? (a) 3x and 3y
(b) $4x^2$ and $5x^2$ (c) 2x and $2x^2$
(d) 7 and 7x

6. $(2a - 3b)^2$ equals: (a) $4a^2 - 12ab + 9b^2$
(b) $4a^2 + 12ab + 9b^2$ (c) $4a^2 - 9b^2$
(d) $2a^2 - 6ab + 3b^2$

7. $a^3 + b^3$ equals: (a) $(a + b)(a^2 + ab + b^2)$
(b) $(a + b)(a^2 - ab + b^2)$ (c) $(a - b)(a^2 + ab + b^2)$
(d) $(a + b)^3$

8. The coefficient of x^2 in $6x^2 - 4x + 1$ is: (a) 6
(b) -4 (c) 1
(d) 2

9. $(a - b)^3$ equals: (a) $a^3 - b^3$
(b) $a^3 - 3a^2b + 3ab^2 - b^3$ (c) $a^3 + 3a^2b + 3ab^2 + b^3$
(d) $a^3 - 3ab + b^3$

10. If $a + b + c = 0$, then $a^3 + b^3 + c^3$ equals: (a) 0
(b) abc (c) 3abc
(d) $-3abc$

Chapter 5: Factorization

Factorization is the reverse of multiplication: we write an expression as a product of its factors. It is the key step in solving quadratic equations and simplifying fractions, so practise it until the patterns are automatic.

- **Common factor:** take out the highest common factor of all terms, e.g. $6x^2 + 9x = 3x(2x + 3)$.
- **Grouping:** group terms so that each group has a common factor, e.g. $ax + ay + bx + by = a(x + y) + b(x + y) = (x + y)(a + b)$.
- **Difference of squares:** $a^2 - b^2 = (a + b)(a - b)$. Watch for hidden squares, e.g. $x^4 - 16 = (x^2 + 4)(x^2 - 4) = (x^2 + 4)(x + 2)(x - 2)$.
- **Perfect square trinomials:** $a^2 + 2ab + b^2 = (a + b)^2$ and $a^2 - 2ab + b^2 = (a - b)^2$.
- **Quadratic trinomials:** to factorise $x^2 + px + q$, find two numbers with sum p and product q. For $ax^2 + bx + c$, split the middle term: find two numbers with product ac and sum b.
- **Sum and difference of cubes:** $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$; $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.
- **HCF and LCM of polynomials:** factorise each polynomial fully; the HCF is the product of the common factors (lowest powers), and the LCM is the product of all factors (highest powers).
- **Remainder theorem:** when a polynomial p(x) is divided by $(x - a)$, the remainder is $p(a)$. **Factor theorem:** $(x - a)$ is a factor of $p(x)$ if and only if $p(a) = 0$.

> **Remember:** $x^2 + px + q = (x + m)(x + n)$ where $m + n = p$ and $mn = q$ | $p(a) = 0 \Leftrightarrow (x - a)$ is a factor of $p(x)$

Worked example. Factorise $2x^2 + 7x + 3$ by splitting the middle term.

Solution: $ac = 2 \times 3 = 6$. Two numbers with product 6 and sum 7 are 6 and 1. So $2x^2 + 7x + 3 = 2x^2 + 6x + x + 3 = 2x(x + 3) + 1(x + 3) = (x + 3)(2x + 1)$. Check by multiplying back: $(x + 3)(2x + 1) = 2x^2 + 7x + 3$ ✓.

Short Questions

1. Factorise $12x^2 - 3$. **Ans:** $3(4x^2 - 1) = 3(2x + 1)(2x - 1)$.

2. Factorise $x^2 + 7x + 12$. **Ans:** Numbers 3 and 4 (sum 7, product 12): $(x + 3)(x + 4)$.

3. Factorise $8x^3 - 27$. **Ans:** $(2x)^3 - 3^3 = (2x - 3)(4x^2 + 6x + 9)$.

4. If $p(x) = x^2 - 5x + 6$, find the remainder when divided by $(x - 2)$. **Ans:** $p(2) = 4 - 10 + 6 = 0$, so the remainder is 0.

5. Factorise $x^2 - 10x + 25$. **Ans:** $(x - 5)^2$, a perfect square.

Long Questions

1. Factorise completely: $3x^3 - 12x$. **Ans:** $3x(x^2 - 4) = 3x(x + 2)(x - 2)$. First take the common factor $3x$, then the difference of squares.

2. Find the HCF and LCM of $x^2 - 9$ and $x^2 - 6x + 9$. **Ans:** $x^2 - 9 = (x + 3)(x - 3)$; $x^2 - 6x + 9 = (x - 3)^2$. Common factor $(x - 3)$: HCF $= (x - 3)$. LCM $= (x + 3)(x - 3)^2$.

Multiple Choice Questions

1. The factorisation of $x^2 - 16$ is: (a) $(x - 4)^2$
(b) $(x + 4)(x - 4)$ (c) $(x + 4)^2$
(d) $(x - 8)(x + 2)$

2. $x^2 + 5x + 6$ factorises as: (a) $(x + 1)(x + 6)$
(b) $(x + 2)(x + 3)$ (c) $(x - 2)(x - 3)$
(d) $(x + 5)(x + 1)$

3. The HCF of $12x$ and $18x^2$ is: (a) $6x$
(b) $36x^2$ (c) $6x^2$
(d) $3x$

4. $a^3 - 8$ factorises as: (a) $(a - 2)(a^2 + 2a + 4)$
(b) $(a - 2)(a^2 - 2a + 4)$ (c) $(a + 2)(a^2 + 2a + 4)$
(d) $(a - 2)^3$

5. If $p(3) = 0$, then a factor of $p(x)$ is: (a) $(x + 3)$
(b) $(x - 3)$ (c) $(3x - 1)$
(d) $(x + 1)$

6. $4x^2 + 12x + 9$ is: (a) $(2x + 3)^2$
(b) $(2x - 3)^2$ (c) $(4x + 3)(x + 3)$
(d) $(2x + 9)(2x + 1)$

7. The common factor of $15a^2b$ and $10ab^2$ is: (a) $10ab$
(b) $5ab$ (c) $15ab$
(d) $5a^2b^2$

8. $x^2 - 7x + 10$ factorises as: (a) $(x - 2)(x - 5)$
(b) $(x + 2)(x + 5)$ (c) $(x - 10)(x + 1)$
(d) $(x - 7)(x + 3)$

9. The remainder when $x^2 + 3x + 2$ is divided by $(x + 1)$ is:
(a) 0 (b) 2
(c) 6 (d) -2

10. $x^4 - 1$ factorises completely as: (a) $(x^2 + 1)(x^2 - 1)$
(b) $(x^2 + 1)(x + 1)(x - 1)$ (c) $(x + 1)^4$
(d) $(x - 1)(x^3 + 1)$

Chapter 6: Algebraic Manipulation

Algebraic manipulation means changing the form of an expression without changing its value — simplifying fractions, cancelling factors, and making a subject of a formula. It is the everyday skill behind every later chapter.

- **Simplifying fractions:** factorise numerator and denominator fully, then cancel common factors. Never cancel a term that is added — only factors can be cancelled.
- **Multiplying and dividing fractions:** to multiply, multiply numerators and denominators; to divide, multiply by the reciprocal of the divisor. Factorise first — it makes cancelling easy.
- **Adding and subtracting fractions:** take the LCM of the denominators, convert each fraction, then add or subtract the numerators.
- **Square roots of expressions:** $\sqrt{(a^2b)} = a\sqrt{b}$ for $a \geq 0$; simplify surds by taking out perfect squares, e.g. $\sqrt{72} = \sqrt{(36 \times 2)} = 6\sqrt{2}$.
- **Making a subject:** to make x the subject of a formula, undo the operations on x in reverse order — use inverse operations step by step, keeping the equation balanced.
- **Changing the subject with fractions and roots:** if x appears in a denominator, multiply through by it; if x is under a square root, square both sides (and check the result, since squaring can introduce extra values).
- **Common errors to avoid:** $(a + b)^2 \neq a^2 + b^2$; $\sqrt{(a + b)} \neq \sqrt{a} + \sqrt{b}$; cancelling terms instead of factors; forgetting to change signs when moving terms across the equals sign.

> **Remember:** $a/b \div c/d = a/b \times d/c$ | $\sqrt{(ab)} = \sqrt{a} \times \sqrt{b}$ | to change the subject, undo operations in reverse order

Worked example. Simplify $(x^2 - 9)/(x^2 + 5x + 6)$.

Solution: Factorise: $x^2 - 9 = (x + 3)(x - 3)$; $x^2 + 5x + 6 = (x + 2)(x + 3)$. So the fraction $= (x + 3)(x - 3) / ((x + 2)(x + 3)) = (x - 3)/(x + 2)$, provided $x \neq -3$ and $x \neq -2$.

Short Questions

1. Simplify $(6x^2)/(9x)$. **Ans:** $(2x)/3$.

2. Make t the subject of $v = u + at$. **Ans:** $at = v - u$, so $t = (v - u)/a$.

3. Simplify $\sqrt{48}$. **Ans:** $\sqrt{(16 \times 3)} = 4\sqrt{3}$.

4. Add $1/x + 2/x$. **Ans:** $3/x$.

5. Simplify $(x^2 - 4)/(x + 2)$. **Ans:** $(x - 2)(x + 2)/(x + 2) = x - 2$.

Long Questions

1. Simplify $(x^2 - 5x + 6)/(x^2 - 9) \times (x + 3)/(x - 2)$. **Ans:** Factorise: $(x - 2)(x - 3)/((x + 3)(x - 3)) \times (x + 3)/(x - 2)$. Cancel $(x - 3)$, $(x + 3)$ and $(x - 2)$: the result is 1.

2. Make x the subject of $y = (2x + 1)/(x - 3)$. **Ans:** $y(x - 3) = 2x + 1 \rightarrow xy - 3y = 2x + 1 \rightarrow xy - 2x = 3y + 1 \rightarrow x(y - 2) = 3y + 1 \rightarrow x = (3y + 1)/(y - 2)$.

Multiple Choice Questions

1. $(x^2 - 25)/(x - 5)$ simplifies to: (a) $x - 5$ (b) $x + 5$ (c) $x^2 + 5$ (d) 5

2. $\sqrt{75}$ equals: (a) $5\sqrt{3}$ (b) $3\sqrt{5}$ (c) $25\sqrt{3}$ (d) 15

3. If $s = ut + (1/2)at^2$, the subject is: (a) u (b) t (c) s (d) a

4. $1/a + 1/b$ equals: (a) $2/(a + b)$ (b) $(a + b)/ab$ (c) $1/(a + b)$ (d) $(b + a)/2$

5. $(3x/4) \div (9x/8)$ equals: (a) 2/3 (b) 3/2 (c) 27/32 (d) 6

6. To divide by a fraction we: (a) divide straight across (b) multiply by its reciprocal (c) add the fractions (d) invert the dividend

7. $(x + 1)/(x + 1)$ equals $(x \neq -1)$: (a) 0 (b) x (c) 1 (d) $x + 1$

8. $\sqrt{(16x^2)}$ $(x \geq 0)$ equals: (a) 16x (b) 4x (c) 8x (d) $4x^2$

9. If $y = 3x + 5$, then x in terms of y is: (a) $(y + 5)/3$ (b) $(y - 5)/3$ (c) $3y - 5$ (d) $(y - 3)/5$

10. The LCM of $(x + 1)$ and $(x - 1)$ is: (a) $x^2 - 1$ (b) $x + 1$ (c) 1 (d) $x^2 + 1$

Chapter 7: Linear Equations and Inequalities

A linear equation in one variable has the form ax + b = 0; a linear equation in two variables has the form ax + by = c. An inequality uses <, >, ≤ or ≥ instead of =. The rules are almost the same — with one important twist for inequalities.

- **Solving in one variable:** collect the x-terms on one side and the numbers on the other, then divide by the coefficient of x. Always check the answer by substitution.
- **Simultaneous equations:** two equations in x and y. **Elimination:** make the coefficients of one variable equal, then add or subtract to remove it. **Substitution:** express one variable from one equation and put it in the other. **Graphical method:** the solution is the point where the two lines cross.
- **Number of solutions:** the lines may cross once (unique solution), be parallel (no solution), or be the same line (infinitely many solutions).
- **Inequalities:** adding or subtracting the same number from both sides keeps the inequality true; multiplying or dividing by a positive number keeps it true; **multiplying or dividing by a negative number reverses the inequality sign**.
- **Solving in one variable:** solve like an equation, remembering the reversal rule, and write the answer as an interval or on a number line, e.g. 2x + 3 < 9 gives x < 3.
- **Word problems:** choose letters for the unknowns, write the equations from the words, solve, then check that the answer makes sense in the story.

> **Remember:** ax + by = c | multiplying/dividing an inequality by a negative number reverses the sign | parallel lines: no solution; same line: infinitely many solutions

Worked example. Solve by elimination: 2x + y = 7 and x − y = 2.

Solution: Add the two equations: (2x + y) + (x − y) = 7 + 2, so 3x = 9 and x = 3. Put x = 3 in the second equation: 3 − y = 2, so y = 1. Solution set = {(3, 1)}. Check: 2(3) + 1 = 7 ✓.

Short Questions

1. Solve 5x − 7 = 18. **Ans:** 5x = 25, so x = 5.

2. Solve 3x + 2 > 11. **Ans:** 3x > 9, so x > 3.

3. If x + y = 10 and x − y = 4, find x and y. **Ans:** Adding gives 2x = 14, x = 7; then y = 3.

4. What happens to the sign when an inequality is multiplied by −2? **Ans:** The inequality sign reverses (e.g. < becomes >).

5. Solve −4x ≤ 12. **Ans:** Divide by −4 and reverse the sign: x ≥ −3.

Long Questions

1. The sum of two numbers is 24 and their difference is 6. Find the numbers. **Ans:** Let the numbers be x and y: x + y = 24, x − y = 6. Adding: 2x = 30, x = 15; then y = 9. The numbers are 15 and 9.

2. Solve the system by substitution: y = 2x + 1 and 3x + y = 11. **Ans:** Put y = 2x + 1 in the second equation: 3x + 2x + 1 = 11 → 5x = 10 → x = 2. Then y = 2(2) + 1 = 5. Solution set = {(2, 5)}.

Multiple Choice Questions

1. The solution of $2x + 5 = 13$ is: (a) $x = 3$ (b) $x = 4$ (c) $x = 5$ (d) $x = 9$

2. If $x + y = 5$ and $x - y = 1$, then $x =$ (a) 2 (b) 3 (c) 4 (d) 1

3. The solution of $4x < 20$ is: (a) $x < 5$ (b) $x > 5$ (c) $x < 4$ (d) $x > 4$

4. Multiplying $-x > 3$ by -1 gives: (a) $x > -3$ (b) $x < -3$ (c) $x > 3$ (d) $x < 3$

5. The pair (2, 3) satisfies: (a) $x + y = 6$ (b) $x - y = 1$ (c) $2x + y = 7$ (d) $x + 2y = 5$

6. Two parallel lines have: (a) one solution (b) no solution (c) two solutions (d) infinitely many solutions

7. Solve $3(x - 2) = 12$: (a) $x = 2$ (b) $x = 4$ (c) $x = 6$ (d) $x = 10$

8. The inequality $x \geq 2$ means x is: (a) less than 2 (b) greater than or equal to 2 (c) equal to 2 only (d) less than or equal to 2

9. If $2x - 3y = 7$ and $x = 5$, then $y =$ (a) 1 (b) -1 (c) 3 (d) 0

10. The equation $x/2 + 1 = 5$ has solution: (a) $x = 2$ (b) $x = 8$ (c) $x = 12$ (d) $x = 10$

Chapter 8: Linear Graphs & Their Application

A linear equation in two variables has a straight-line graph. Reading graphs is a practical skill: it turns tables of numbers into pictures that show trends, speeds and costs at a glance.

- **Plotting points:** each solution (x, y) of ax + by = c is a point on the line. Two points are enough to draw it; a third point is a useful check.
- **Intercepts:** the x-intercept is where the line meets the x-axis (y = 0); the y-intercept is where it meets the y-axis (x = 0).
- **Gradient (slope):** for the line y = mx + c, m is the gradient and c is the y-intercept. The gradient between two points (x_1, y_1) and (x_2, y_2) is $m = (y_2 - y_1)/(x_2 - x_1)$. Parallel lines have equal gradients.
- **Forms of a line:** slope-intercept form y = mx + c; point-slope form $y - y_1 = m(x - x_1)$; two-point form using the gradient between two points.
- **Graphical solution:** draw both lines of a simultaneous pair; their point of intersection is the solution. If the lines are parallel, there is no solution.
- **Distance–time graphs:** the gradient is the speed; a flat (horizontal) part means the object is at rest; a steeper line means faster movement.
- **Conversion graphs:** a straight line can convert one quantity into another, e.g. Celsius to Fahrenheit (F = (9/5)C + 32) or currency conversion at a fixed rate.

> **Remember:** y = mx + c (m = gradient, c = y-intercept) | $m = (y_2 - y_1)/(x_2 - x_1)$ | F = (9/5)C + 32

Worked example. Draw the graph of 2x + y = 6 and find its intercepts and gradient.

Solution: Rearrange: y = −2x + 6, so the gradient is −2 and the y-intercept is 6. When y = 0, 2x = 6, so x = 3: the x-intercept is (3, 0). Plot (0, 6) and (3, 0) and join them with a straight line. A check point: when x = 1, y = 4, which lies on the line ✓.

Short Questions

1. What is the gradient of y = 4x − 7? **Ans:** 4.

2. Find the y-intercept of 3x + 2y = 12. **Ans:** Put x = 0: 2y = 12, y = 6. The intercept is (0, 6).

3. Find the gradient between (1, 2) and (5, 10). **Ans:** m = (10 − 2)/(5 − 1) = 8/4 = 2.

4. What does a horizontal line on a distance–time graph mean? **Ans:** The object is at rest (distance is not changing).

5. Write y = 3x + 2 in the form ax + by + c = 0. **Ans:** 3x − y + 2 = 0.

Long Questions

1. Draw the graphs of x + y = 5 and x − y = 1 on the same axes and find the solution of the system. **Ans:** The first line passes through (0, 5) and (5, 0); the second passes through (0, −1) and (1, 0). They meet at (3, 2), so the solution is x = 3, y = 2. Check: 3 + 2 = 5 ✓ and 3 − 2 = 1 ✓.

2. A taxi charges a fixed fee of Rs. 100 plus Rs. 20 per kilometre. Write the cost equation, draw its graph, and find the cost of a 12 km ride. **Ans:** If the distance is d km, cost C = 100 + 20d. The

graph is a straight line with y-intercept 100 and gradient 20. For d = 12, C = 100 + 240 = Rs. 340.

Multiple Choice Questions

1. The graph of $y = 2x + 1$ is a: (a) circle (b) parabola (c) straight line (d) curve

2. The gradient of $y = -3x + 5$ is: (a) 5 (b) 3 (c) −3 (d) −5

3. The y-intercept of $y = 7x - 2$ is: (a) 7 (b) −2 (c) 2 (d) 0

4. The point (2, 6) lies on: (a) $y = x + 2$ (b) $y = 2x + 2$ (c) $y = 3x$ (d) $y = x + 6$

5. Two lines with the same gradient are: (a) perpendicular (b) intersecting (c) parallel (d) the same point

6. The x-intercept of $x + 2y = 4$ is: (a) (0, 2) (b) (4, 0) (c) (0, 4) (d) (2, 0)

7. On a distance–time graph, speed equals the: (a) intercept (b) area (c) gradient (d) length

8. The line $y = 5$ is: (a) vertical (b) horizontal (c) diagonal (d) curved

9. If $C = (5/9)(F - 32)$, then $F = 212$ gives $C =$ (a) 100 (b) 0 (c) 32 (d) 180

10. The gradient between (0, 0) and (4, 8) is: (a) 4 (b) 8 (c) 2 (d) 1/2

Chapter 9: Introduction to Coordinate Geometry

Coordinate geometry joins algebra and geometry: every point is a pair of numbers (x, y), and every line or curve is an equation. It was introduced by René Descartes, which is why the system is called the Cartesian plane.

- **The Cartesian plane:** the x-axis (horizontal) and y-axis (vertical) cross at the origin O(0, 0) and divide the plane into four quadrants: I (+, +), II (−, +), III (−, −), IV (+, −).
- **Coordinates:** in P(x, y), x is the abscissa (distance from the y-axis) and y is the ordinate (distance from the x-axis). Points on the x-axis have y = 0; points on the y-axis have x = 0.
- **Distance formula:** the distance between $A(x_1, y_1)$ and $B(x_2, y_2)$ is $AB = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$. It is just Pythagoras' theorem in disguise.
- **Midpoint formula:** the midpoint of AB is $((x_1 + x_2)/2, (y_1 + y_2)/2)$.
- **Section formula:** a point dividing AB in the ratio m:n is $((mx_2 + nx_1)/(m + n), (my_2 + ny_1)/(m + n))$. For the midpoint, m = n = 1.
- **Collinear points:** three points are collinear if they lie on one straight line — check by showing that the distance between the two outer points equals the sum of the two smaller distances, or that the slopes are equal.
- **Area of a triangle:** for vertices (x_1, y_1), (x_2, y_2), (x_3, y_3), area $= (1/2)|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$. If the area is zero, the points are collinear.

> **Remember:** $AB = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$ | Midpoint $= ((x_1 + x_2)/2, (y_1 + y_2)/2)$

Worked example. Find the distance between A(2, 3) and B(6, 6), and the midpoint of AB.

Solution: $AB = \sqrt{((6 - 2)^2 + (6 - 3)^2)} = \sqrt{(16 + 9)} = \sqrt{25} = 5$ units. Midpoint $= ((2 + 6)/2, (3 + 6)/2) = (4, 4.5)$.

Short Questions

1. In which quadrant does the point (−4, 5) lie? **Ans:** Quadrant II (x negative, y positive).

2. Find the distance between (0, 0) and (3, 4). **Ans:** $\sqrt{(9 + 16)} = \sqrt{25} = 5$ units.

3. What is the midpoint of (2, 8) and (6, 2)? **Ans:** $((2 + 6)/2, (8 + 2)/2) = (4, 5)$.

4. Write the coordinates of a point on the y-axis with ordinate −3. **Ans:** (0, −3).

5. Find the distance of the point (5, 12) from the origin. **Ans:** $\sqrt{(25 + 144)} = \sqrt{169} = 13$ units.

Long Questions

1. Show that the points A(1, 2), B(4, 6) and C(7, 10) are collinear. **Ans:** $AB = \sqrt{(9 + 16)} = 5$; $BC = \sqrt{(9 + 16)} = 5$; $AC = \sqrt{(36 + 64)} = 10$. Since $AB + BC = 5 + 5 = 10 = AC$, the points lie on one straight line — they are collinear.

2. Find the area of the triangle with vertices (1, 2), (4, 6) and (7, 3). **Ans:** Area $= (1/2)|1(6 - 3) + 4(3 - 2) + 7(2 - 6)| = (1/2)|3 + 4 - 28| = (1/2)|-21| = 10.5$ square units.

Multiple Choice Questions

1. The point (0, 0) is called the: (a) abscissa
(b) ordinate (c) origin
(d) quadrant

2. The point (−3, −2) lies in quadrant: (a) I
(b) II (c) III
(d) IV

3. The distance between (1, 1) and (4, 5) is: (a) 3
(b) 4 (c) 5
(d) 7

4. The midpoint of (0, 6) and (4, 2) is: (a) (2, 4)
(b) (4, 2) (c) (2, 2)
(d) (4, 4)

5. In the point (7, −2), the abscissa is: (a) −2
(b) 7 (c) 5
(d) 9

6. Points on the x-axis have: (a) x = 0
(b) y = 0 (c) x = y
(d) x = 1

7. The distance of (6, 8) from the origin is: (a) 14
(b) 10 (c) 48
(d) 2

8. If the area of a triangle formed by three points is zero, the points are:
(a) vertices of a triangle (b) collinear
(c) concyclic (d) parallel

9. The midpoint of (−2, 4) and (2, −4) is: (a) (0, 0)
(b) (2, 2) (c) (−2, −2)
(d) (4, 8)

10. The plane is divided into quadrants by: (a) one axis
(b) the two axes (c) the origin only
(d) a circle

Chapter 10: Congruent Triangles

Two triangles are congruent if one can be placed exactly on top of the other — same shape, same size. Congruence is proved with just three matching parts, not all six. The word comes from the Latin *congruere*, to agree.

- **Meaning:** triangle ABC ≅ triangle DEF means all three sides and all three angles match in order. The order of the letters matters — it shows which parts correspond.
- **SSS:** if the three sides of one triangle equal the three sides of another, the triangles are congruent.
- **SAS:** if two sides and the included angle (the angle between those two sides) of one triangle equal those of another, the triangles are congruent.
- **ASA:** if two angles and the included side of one triangle equal those of another, the triangles are congruent.
- **AAS:** if two angles and a non-included side of one triangle equal those of another, the triangles are congruent (because the third angles must also be equal).
- **RHS:** if the hypotenuse and one side of a right-angled triangle equal the hypotenuse and one side of another right-angled triangle, the triangles are congruent.
- **Corresponding parts:** once triangles are proved congruent, any matching side or angle is equal — written as c.p.c.t. (corresponding parts of congruent triangles).
- **Isosceles triangle theorem:** if two sides of a triangle are equal, the angles opposite them are equal; and conversely, if two angles are equal, the sides opposite them are equal.

> **Remember:** SSS, SAS, ASA, AAS, RHS | SSA is NOT a congruence rule (the "ambiguous case") | congruent triangles have equal area and equal perimeter

Worked example. In triangle ABC, AB = AC. Prove that ∠B = ∠C.

Solution: Draw the bisector of ∠A meeting BC at D. In triangles ABD and ACD: AB = AC (given), AD is common, and ∠BAD = ∠CAD (AD bisects ∠A). So the triangles are congruent by SAS. Hence ∠B = ∠C by c.p.c.t. — the base angles of an isosceles triangle are equal.

Short Questions

1. State the SAS rule of congruence. **Ans:** Two sides and the included angle of one triangle equal to the corresponding parts of another make the triangles congruent.

2. In triangle PQR, PQ = PR. Which angles are equal? **Ans:** ∠Q = ∠R (angles opposite the equal sides).

3. Why is SSA not a valid congruence rule? **Ans:** Two different triangles can share two sides and a non-included angle, so SSA does not fix the triangle.

4. If triangle ABC ≅ triangle XYZ, write the side equal to BC. **Ans:** YZ (the corresponding side in the same position).

5. Which rule applies to two right-angled triangles with equal hypotenuses and one equal side? **Ans:** RHS.

Long Questions

1. In triangles ABC and DEF, AB = DE, BC = EF and AC = DF. Prove the triangles are congruent and name the rule. **Ans:** All three sides of one triangle equal the three sides of the other, so the triangles are congruent by the SSS rule. Then all corresponding angles are equal by c.p.c.t.

2. ABCD is a parallelogram with diagonal AC. Prove that triangles ABC and CDA are congruent. **Ans:** In a parallelogram, opposite sides are equal: AB = CD and BC = AD; AC is common. So the triangles are congruent by SSS. Hence ∠BAC = ∠DCA and ∠BCA = ∠DAC (alternate angles), which also shows AB ∥ CD and BC ∥ AD.

Multiple Choice Questions

1. Which is a valid congruence rule? (a) SSA
(b) AAA (c) SSS
(d) ASS

2. In triangle ABC ≅ triangle PQR, the angle equal to ∠A is:
(a) ∠Q (b) ∠R
(c) ∠P (d) ∠B

3. The included angle of sides AB and AC is: (a) ∠B
(b) ∠C (c) ∠A
(d) ∠ABC

4. RHS rule applies to triangles that are: (a) equilateral
(b) isosceles (c) right-angled
(d) scalene

5. If two angles of a triangle are 50° and 60°, the third angle is:
(a) 60° (b) 70°
(c) 80° (d) 110°

6. In an isosceles triangle, the angles opposite equal sides are:
(a) supplementary (b) equal
(c) 90° (d) different

7. AAA proves triangles are: (a) congruent
(b) similar only (c) equal in area only
(d) right-angled

8. The common side in triangles ABD and CBD sharing BD is:
(a) AB (b) BD
(c) BC (d) AD

9. Congruent triangles have: (a) equal shape, different size
(b) equal size, different shape (c) equal shape and size
(d) no equal parts

10. c.p.c.t. stands for: (a) common parts of congruent triangles
(b) corresponding parts of congruent triangles (c) congruent parts of common triangles
(d) central parts of congruent triangles

Chapter 11: Parallelograms and Triangles

A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Its properties connect directly with triangles: a diagonal divides a parallelogram into two congruent triangles, and triangles on the same base between the same parallels have equal area.

- **Definition:** a quadrilateral with both pairs of opposite sides parallel is a parallelogram. A rectangle has all angles 90°; a rhombus has all sides equal; a square is both a rectangle and a rhombus.
- **Properties of a parallelogram:** opposite sides are equal; opposite angles are equal; diagonals bisect each other; consecutive angles are supplementary (add to 180°); each diagonal divides it into two congruent triangles.
- **Tests for a parallelogram:** a quadrilateral is a parallelogram if any one of these holds — both pairs of opposite sides are equal; both pairs of opposite angles are equal; one pair of opposite sides is equal and parallel; the diagonals bisect each other.
- **Midpoint theorem:** the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length. Conversely, a line through the midpoint of one side parallel to another side bisects the third side.
- **Triangles on the same base:** triangles on the same base and between the same parallel lines have equal areas. A triangle and a parallelogram on the same base between the same parallels: the triangle's area is half the parallelogram's area.
- **Area facts:** area of a parallelogram = base × height. Area of a triangle = (1/2) × base × height. Triangles on equal bases with equal heights have equal areas.

> **Remember:** Area of parallelogram = base × height | Area of triangle = (1/2) × base × height | diagonals of a parallelogram bisect each other

Worked example. ABCD is a parallelogram with AB = 8 cm, BC = 5 cm and ∠A = 70°. Find ∠B, ∠C, ∠D and CD.

Solution: Opposite sides are equal, so CD = AB = 8 cm. Opposite angles are equal, so ∠C = ∠A = 70°. Consecutive angles are supplementary, so ∠B = 180° − 70° = 110°, and ∠D = ∠B = 110°.

Short Questions

1. Define a parallelogram. **Ans:** A quadrilateral with both pairs of opposite sides parallel.

2. In a parallelogram, one angle is 100°. Find its opposite angle. **Ans:** 100° (opposite angles are equal).

3. State the midpoint theorem. **Ans:** The line joining the midpoints of two sides of a triangle is parallel to the third side and half as long.

4. Find the area of a parallelogram with base 12 cm and height 7 cm. **Ans:** 12 × 7 = 84 cm².

5. What is the relation between the areas of a triangle and a parallelogram on the same base between the same parallels? **Ans:** The triangle's area is half the parallelogram's area.

Long Questions

1. Prove that the diagonals of a parallelogram bisect each other. **Ans:** Let ABCD be a parallelogram with diagonals AC and BD meeting at O. In triangles AOB and COD: AB = CD (opposite sides), ∠OAB = ∠OCD and ∠OBA = ∠ODC (alternate angles, since AB ∥ CD). So the triangles are congruent by ASA, giving AO = CO and BO = DO — the diagonals bisect each other.

2. Two triangles ABC and DBC stand on the same base BC and between the same parallels AD and BC. Prove their areas are equal. **Ans:** Both triangles have the same base BC, and their heights are the same because the distance between the parallel lines AD and BC is constant. Area = (1/2) × base × height, so both areas are equal.

Multiple Choice Questions

1. Opposite angles of a parallelogram are: (a) supplementary
(b) equal (c) 90°
(d) complementary

2. The diagonals of a parallelogram: (a) are equal
(b) bisect each other (c) are perpendicular
(d) are parallel

3. A rhombus has: (a) all angles 90°
(b) all sides equal (c) one pair of parallel sides
(d) no parallel sides

4. Area of a parallelogram with base 9 cm and height 4 cm is:
(a) 13 cm² (b) 26 cm²
(c) 36 cm² (d) 18 cm²

5. The line joining the midpoints of two sides of a triangle is:
(a) equal to the third side (b) half the third side
(c) twice the third side (d) perpendicular to it

6. Consecutive angles of a parallelogram add to:
(a) 90° (b) 180°
(c) 270° (d) 360°

7. A square is a: (a) rectangle only
(b) rhombus only (c) rectangle and rhombus
(d) trapezium

8. Triangles on the same base between the same parallels have:
(a) equal perimeters (b) equal areas
(c) equal heights only (d) equal bases only

9. Each diagonal of a parallelogram divides it into:
(a) two equal triangles (b) two squares
(c) four triangles (d) two rectangles

10. The sum of the angles of a quadrilateral is: (a) 180°
(b) 270° (c) 360°
(d) 540°

Chapter 12: Line Bisectors and Angle Bisectors

A bisector cuts something into two equal parts. The perpendicular bisector of a line and the bisector of an angle are two of the most useful constructions in geometry — and each has a clean "locus" property that exam questions love.

- **Perpendicular bisector:** the line that cuts a line segment at its midpoint at right angles (90°). Every point on it is the same distance from the two ends of the segment.
- **Locus property (perpendicular bisector):** the perpendicular bisector of AB is the set of all points equidistant from A and B. If a point P is equidistant from A and B, then P lies on the perpendicular bisector of AB.
- **Angle bisector:** the ray that divides an angle into two equal angles. Every point on it is the same distance from the two arms of the angle.
- **Locus property (angle bisector):** the bisector of an angle is the set of all points inside the angle that are equidistant from its two sides.
- **Circumcentre:** the three perpendicular bisectors of the sides of a triangle meet at one point, the circumcentre, which is equidistant from all three vertices and is the centre of the circle through them (the circumcircle).
- **Incentre:** the three angle bisectors of a triangle meet at one point, the incentre, which is equidistant from all three sides and is the centre of the circle touching them (the incircle).
- **Construction steps (perpendicular bisector):** with compasses centred at each end of the segment, draw two arcs of equal radius (more than half the segment) on both sides; join the two points where the arcs cross.
- **Construction steps (angle bisector):** draw an arc from the vertex cutting both arms; from those two points draw two equal arcs that cross; join the vertex to the crossing point.

> **Remember:** perpendicular bisector: equidistant from the ends | angle bisector: equidistant from the sides | circumcentre: bisectors of sides meet | incentre: bisectors of angles meet

Worked example. A point P is 5 cm from A and 5 cm from B. Where must P lie?

Solution: P is equidistant from A and B, so by the locus property P lies on the perpendicular bisector of the segment AB. Every point on that line is 5 cm from each end only at the two points where the line is at that distance — but the key fact is that P must be on the perpendicular bisector of AB.

Short Questions

1. Define the perpendicular bisector of a segment. **Ans:** The line through the midpoint of the segment, perpendicular to it.

2. What is the locus of points equidistant from two fixed points A and B? **Ans:** The perpendicular bisector of AB.

3. Where do the angle bisectors of a triangle meet? **Ans:** At the incentre, which is equidistant from the three sides.

4. What is the circumcentre of a triangle? **Ans:** The point where the perpendicular bisectors of the sides meet; it is equidistant from the vertices.

5. A point inside an angle is 3 cm from one arm and 3 cm from the other. What can you say about it? **Ans:** It lies on the bisector of the angle.

Long Questions

1. Prove that any point on the perpendicular bisector of a segment is equidistant from its ends. **Ans:** Let M be the midpoint of AB and P a point on its perpendicular bisector. In triangles PAM and PBM: AM = BM, ∠PMA = ∠PMB = 90°, PM is common. So the triangles are congruent by SAS, giving PA = PB. Hence P is equidistant from A and B.

2. Describe how to construct the bisector of angle ABC with ruler and compasses, and say why the construction works. **Ans:** With centre B draw an arc cutting BA at D and BC at E. With centres D and E draw two arcs of equal radius that meet at F. Join BF — this is the bisector. It works because triangles BDF and BEF are congruent by SSS (BD = BE, DF = EF, BF common), so ∠DBF = ∠EBF.

Multiple Choice Questions

1. A perpendicular bisector meets a segment at: (a) 45° at its end
(b) 90° at its midpoint (c) 90° at its end
(d) 60° at its midpoint

2. Points on the perpendicular bisector of AB are equidistant from: (a) A only (b) B only
(c) A and B (d) the midpoint only

3. The incentre is equidistant from the: (a) vertices
(b) sides (c) midpoints
(d) altitudes

4. The circumcentre is the centre of the: (a) incircle
(b) circumcircle (c) triangle
(d) median

5. An angle bisector divides an angle into: (a) two unequal parts
(b) two equal parts (c) three parts
(d) right angles

6. The perpendicular bisectors of a triangle meet at the: (a) incentre (b) circumcentre
(c) centroid (d) orthocentre

7. To draw a perpendicular bisector, the compass radius must be: (a) less than half the segment (b) more than half the segment
(c) equal to the segment (d) any radius

8. A point on an angle bisector is equidistant from the: (a) vertex and a side (b) two arms of the angle
(c) opposite side (d) two vertices

9. The circumcentre of a right-angled triangle lies at the: (a) vertex of the right angle (b) midpoint of the hypotenuse
(c) centroid (d) incentre

10. The three angle bisectors of a triangle are: (a) parallel
(b) concurrent (c) perpendicular
(d) equal in length

Chapter 13: Sides and Angles of a Triangle

The sides and angles of a triangle are linked: a longer side always sits opposite a larger angle. This chapter collects the inequalities that compare sides and angles — the facts you use when a question asks "which is greater?" rather than "how much?"

- **Angle sum:** the three interior angles of a triangle add to 180°. If two angles are known, the third is 180° minus their sum.
- **Exterior angle:** an exterior angle of a triangle equals the sum of the two opposite interior angles. It is also greater than either of those two interior angles.
- **Greater side, greater angle:** in any triangle, the larger side is opposite the larger angle, and the larger angle is opposite the larger side. Equal sides have equal opposite angles.
- **Triangle inequality:** the sum of any two sides of a triangle is greater than the third side. So three lengths can form a triangle only if the sum of the two shorter ones is greater than the longest.
- **Shortest distance:** the perpendicular from a point to a line is the shortest distance from the point to the line. Any other (oblique) segment from the point to the line is longer.
- **Side–angle comparison in one triangle:** if AB > AC in triangle ABC, then ∠C > ∠B; if ∠C > ∠B, then AB > AC.
- **Two triangles, two sides:** if two sides of one triangle equal two sides of another but the included angle of the first is larger, then the third side of the first is larger (the hinge theorem).

> **Remember:** angles of a triangle add to 180° | exterior angle = sum of the two opposite interior angles | sum of any two sides > third side

Worked example. Can lengths 4 cm, 7 cm and 12 cm form a triangle? Why or why not?

Solution: No. The two shorter sides add to 4 + 7 = 11 cm, which is less than the longest side, 12 cm. By the triangle inequality the sum of any two sides must be greater than the third side, so these lengths cannot form a triangle.

Short Questions

1. Two angles of a triangle are 35° and 65°. Find the third angle. **Ans:** 180° − 35° − 65° = 80°.

2. In triangle ABC, AB = 9 cm, BC = 5 cm, CA = 7 cm. Which is the largest angle? **Ans:** ∠C, because it is opposite the longest side AB.

3. State the triangle inequality. **Ans:** The sum of any two sides of a triangle is greater than the third side.

4. An exterior angle of a triangle is 120°. One opposite interior angle is 50°. Find the other. **Ans:** 120° − 50° = 70°.

5. Which is the shortest distance from a point to a line? **Ans:** The perpendicular distance from the point to the line.

Long Questions

1. In triangle ABC, $\angle A = 40°$, $\angle B = 70°$ and $\angle C = 70°$. Arrange the sides in order of length. **Ans:** The largest angles are $\angle B = \angle C = 70°$, opposite sides AC and AB, so AC = AB are the longest sides; $\angle A = 40°$ is smallest, so its opposite side BC is shortest. Order: AB = AC > BC.

2. Prove that the sum of any two sides of a triangle is greater than the third side. **Ans:** In triangle ABC, extend BA to D so that AD = AC, and join DC. Then $\angle ACD = \angle ADC$ (isosceles triangle). But $\angle BCD > \angle ACD$, so $\angle BCD > \angle ADC$, i.e. $\angle BCD > \angle BDC$. In triangle BCD, the greater angle lies opposite the greater side, so BD > BC. But BD = BA + AD = BA + AC, so BA + AC > BC. The same argument works for the other pairs.

Multiple Choice Questions

1. The angles of a triangle add to: (a) 90° (b) 180° (c) 270° (d) 360°

2. An exterior angle of a triangle equals: (a) one interior angle (b) the sum of all three interior angles (c) the sum of the two opposite interior angles (d) 90°

3. In a triangle, the longest side is opposite the: (a) smallest angle (b) largest angle (c) right angle only (d) exterior angle

4. Which set of lengths can form a triangle? (a) 2, 3, 6 (b) 3, 4, 5 (c) 1, 2, 4 (d) 5, 5, 11

5. The shortest distance from a point to a line is the: (a) oblique segment (b) perpendicular segment (c) longest segment (d) parallel segment

6. If two sides of a triangle are equal, the angles opposite them are: (a) supplementary (b) equal (c) 90° (d) complementary

7. In triangle XYZ, XY = 10, YZ = 6, ZX = 8. The smallest angle is: (a) $\angle X$ (b) $\angle Y$ (c) $\angle Z$ (d) all equal

8. An exterior angle of 100° has one opposite interior angle 40°. The other is: (a) 40° (b) 60° (c) 80° (d) 140°

9. The hinge theorem compares: (a) two angles (b) third sides when two sides are equal and included angles differ (c) areas only (d) perimeters only

10. If $\angle A = 30°$ and $\angle B = 60°$ in triangle ABC, the longest side is: (a) BC (b) AC (c) AB (d) cannot tell

Chapter 14: Ratio and Proportion

A ratio compares two quantities of the same kind; a proportion says two ratios are equal. Ratios run through daily life — recipes, maps, mixtures, money — and through geometry, where similar figures are built on equal ratios.

- **Ratio:** the ratio of a to b ($b \neq 0$) is a:b or a/b. It has no units. A ratio is in lowest terms when a and b have no common factor except 1. Ratios can be compounded: a:b and b:c give a:c.
- **Proportion:** a:b :: c:d, or a/b = c/d, means the two ratios are equal. Then a, b, c, d are in proportion; a and d are the extremes, b and c the means. **Cross-multiplication:** ad = bc.
- **Mean proportional:** if a:b :: b:c, then b is the mean proportional between a and c, and $b^2 = ac$, so $b = \sqrt{(ac)}$.
- **Fourth proportional:** if a:b :: c:x, then x = bc/a. **Third proportional:** if a:b :: b:x, then $x = b^2/a$.
- **Continued proportion:** a, b, c are in continued proportion if a/b = b/c.
- **Properties (componendo and dividendo):** if a/b = c/d, then (a + b)/b = (c + d)/d (componendo) and (a − b)/b = (c − d)/d (dividendo); combining gives (a + b)/(a − b) = (c + d)/(c − d) (componendo-dividendo).
- **Direct and inverse proportion:** y is directly proportional to x ($y \propto x$) if y = kx; y is inversely proportional to x ($y \propto 1/x$) if y = k/x, where k is a constant.
- **Similar triangles:** two triangles are similar if their corresponding angles are equal and their corresponding sides are in the same ratio. The ratio of their areas equals the square of the ratio of corresponding sides.

> **Remember:** a/b = c/d ⇒ ad = bc | mean proportional $b = \sqrt{(ac)}$ | (a + b)/(a − b) = (c + d)/(c − d)

Worked example. Divide Rs. 1,200 between Ali and Bilal in the ratio 3:5. How much does each get?

Solution: Total parts = 3 + 5 = 8. One part = 1200/8 = Rs. 150. Ali gets 3 × 150 = Rs. 450; Bilal gets 5 × 150 = Rs. 750. Check: 450 + 750 = 1200 ✓.

Short Questions

1. Simplify the ratio 18:24. **Ans:** Divide by 6: 3:4.

2. Find x if 5:7 :: x:21. **Ans:** 5/7 = x/21 → x = 15.

3. Find the mean proportional between 4 and 9. **Ans:** $\sqrt{(4 \times 9)} = \sqrt{36} = 6$.

4. If a:b = 2:3 and b:c = 4:5, find a:c. **Ans:** a:c = 8:15 (a/b × b/c = 2/3 × 4/5 = 8/15).

5. What is the fourth proportional to 3, 4 and 6? **Ans:** x = (4 × 6)/3 = 8.

Long Questions

1. If a/b = c/d, prove that (a + b)/(a − b) = (c + d)/(c − d). **Ans:** Let a/b = c/d = k, so a = bk and c = dk. Then (a + b)/(a − b) = (bk + b)/(bk − b) = (k + 1)/(k − 1). Similarly (c + d)/(c − d) = (dk + d)/(dk − d) = (k + 1)/(k − 1). Both sides are equal, hence proved.

2. Two similar triangles have corresponding sides 6 cm and 9 cm. If the area of the smaller is 40 cm^2, find the area of the larger. **Ans:** Ratio of sides = 6:9 = 2:3. Ratio of areas = $2^2:3^2$ = 4:9. So the larger area = 40 × 9/4 = 90 cm^2.

Multiple Choice Questions

1. The ratio 20:25 in lowest terms is: (a) 5:4
(b) 4:5 (c) 20:25
(d) 1:5

2. If a/b = c/d, then ad equals: (a) ab
(b) bc (c) cd
(d) ac

3. The mean proportional between 9 and 16 is: (a) 12
(b) 25 (c) 7
(d) 144

4. If x:8 :: 3:4, then x = (a) 6
(b) 24 (c) 12
(d) 4

5. The extremes in a:b :: c:d are: (a) b and c
(b) a and d (c) a and b
(d) c and d

6. If $y \propto x$ and y = 10 when x = 2, then y when x = 5 is:
(a) 20 (b) 25
(c) 15 (d) 50

7. The fourth proportional to 2, 3 and 4 is: (a) 5
(b) 6 (c) 8
(d) 12

8. Two similar triangles have sides in ratio 1:2. Their areas are in ratio:
(a) 1:2 (b) 1:4
(c) 2:1 (d) 4:1

9. A ratio has: (a) units of both quantities
(b) no units (c) only the first unit
(d) only the second unit

10. If a/b = 3/4, then (a + b)/b equals: (a) 7/4
(b) 3/7 (c) 4/3
(d) 7/3

Chapter 15: Pythagoras Theorem

Pythagoras' theorem is the most famous result in school geometry: in a right-angled triangle, the square of the hypotenuse equals the sum of the squares of the other two sides. It is used everywhere — building, navigation, and even finding distances on a map.

- **The theorem:** in a right-angled triangle with legs a and b and hypotenuse c, $a^2 + b^2 = c^2$. The hypotenuse is the side opposite the right angle and is always the longest side.
- **Converse:** if $a^2 + b^2 = c^2$ for the sides of a triangle, then the triangle is right-angled, with the right angle opposite the side c.
- **Finding a leg:** if the hypotenuse and one leg are known, the other leg is $\sqrt{(c^2 - a^2)}$.
- **Pythagorean triples:** whole-number triples satisfying $a^2 + b^2 = c^2$, e.g. (3, 4, 5), (5, 12, 13), (8, 15, 17), (7, 24, 25). Multiples of a triple are also triples, e.g. (6, 8, 10).
- **Testing for a right angle:** check whether the squares satisfy $a^2 + b^2 = c^2$. If $a^2 + b^2 > c^2$, the angle opposite c is acute; if $a^2 + b^2 < c^2$, it is obtuse.
- **Applications:** the diagonal of a rectangle of sides l and b is $\sqrt{(l^2 + b^2)}$; the distance formula of Chapter 9 is Pythagoras in coordinates; the height of a ladder problem is a standard application.

> **Remember:** $a^2 + b^2 = c^2$ | triples: (3, 4, 5), (5, 12, 13), (8, 15, 17), (7, 24, 25)

Worked example. A ladder 10 m long leans against a wall with its foot 6 m from the wall. How high up the wall does it reach?

Solution: The ladder, wall and ground form a right-angled triangle with hypotenuse 10 m and one leg 6 m. Height = $\sqrt{(10^2 - 6^2)} = \sqrt{(100 - 36)} = \sqrt{64} = 8$ m. The ladder reaches 8 m up the wall.

Short Questions

1. Find the hypotenuse of a right triangle with legs 9 cm and 12 cm. **Ans:** $\sqrt{(81 + 144)} = \sqrt{225} = 15$ cm.

2. Is (6, 8, 10) a Pythagorean triple? **Ans:** Yes: $6^2 + 8^2 = 36 + 64 = 100 = 10^2$.

3. The hypotenuse is 13 cm and one leg is 5 cm. Find the other leg. **Ans:** $\sqrt{(169 - 25)} = \sqrt{144} = 12$ cm.

4. State the converse of Pythagoras' theorem. **Ans:** If $a^2 + b^2 = c^2$ in a triangle, the triangle is right-angled opposite side c.

5. Find the diagonal of a rectangle 8 cm by 15 cm. **Ans:** $\sqrt{(64 + 225)} = \sqrt{289} = 17$ cm.

Long Questions

1. Prove Pythagoras' theorem: in a right-angled triangle ABC with right angle at C, $AB^2 = AC^2 + BC^2$. **Ans:** Draw CD perpendicular to the hypotenuse AB. Triangles ACD and ABC are similar (common angle at A, both right-angled), so $AC/AB = AD/AC$, giving $AC^2 = AB \times AD$. Similarly, triangles BCD and BAC are similar, giving $BC^2 = AB \times BD$. Adding: $AC^2 + BC^2 = AB(AD + BD) = AB \times AB = AB^2$. Hence proved.

2. The sides of a triangle are 7 cm, 24 cm and 25 cm. Is it right-angled? Which angle is the right angle? **Ans:** $7^2 + 24^2 = 49 + 576 = 625 = 25^2$, so by the converse of Pythagoras' theorem the triangle is right-angled. The right angle is opposite the longest side (25 cm).

Multiple Choice Questions

1. In a right triangle, the hypotenuse is: (a) the shortest side
(b) the longest side (c) equal to a leg
(d) the base

2. If legs are 5 and 12, the hypotenuse is: (a) 13
(b) 17 (c) 11
(d) 15

3. Which is a Pythagorean triple? (a) 2, 3, 4
(b) 5, 12, 13 (c) 4, 5, 6
(d) 6, 7, 8

4. If $a^2 + b^2 < c^2$, the angle opposite c is: (a) acute
(b) right (c) obtuse
(d) 180°

5. The diagonal of a square of side 1 is: (a) 1
(b) 2 (c) $\sqrt{2}$
(d) $\sqrt{3}$

6. A triangle with sides 9, 12, 15 is: (a) acute-angled
(b) right-angled (c) obtuse-angled
(d) equilateral

7. If the hypotenuse is 25 and one leg is 7, the other leg is:
(a) 18 (b) 24
(c) 20 (d) 32

8. Pythagoras' theorem applies to triangles that are:
(a) equilateral (b) right-angled
(c) isosceles (d) scalene

9. The length of the hypotenuse with legs 8 and 15 is:
(a) 23 (b) 17
(c) 19 (d) 21

10. In triangle ABC, $\angle B = 90°$, AB = 6, BC = 8. AC equals:
(a) 14 (b) 10
(c) 12 (d) 9

Chapter 16: Theorems Related with Area

Area theorems tell us when two different shapes have the same area. They are proved by cutting and rearranging, not by measuring — and they are the reason the area formulae of triangles and parallelograms fit together so neatly.

- **Parallelograms on the same base:** parallelograms on the same base (or equal bases) and between the same parallel lines have equal areas. Their heights are the same, so base × height is the same.
- **Parallelograms on equal bases:** if two parallelograms stand on equal bases and between the same parallels, their areas are equal.
- **Triangle and parallelogram:** if a triangle and a parallelogram stand on the same base and between the same parallels, the area of the triangle is half the area of the parallelogram.
- **Triangles on the same base:** triangles on the same base (or equal bases) and between the same parallels have equal areas. This was used in Chapter 11 and is proved again here from the parallelogram result.
- **Triangles on equal bases:** triangles on equal bases with the same altitude have equal areas. Equal area does not mean congruent — two triangles can have equal areas with quite different shapes.
- **Median divides area:** a median of a triangle divides it into two triangles of equal area, because they have equal bases (the median bisects the side) and the same height.
- **Area and congruence:** congruent figures have equal areas, but the converse is false: equal area does not imply congruence.

> **Remember:** same base + same parallels ⇒ equal area | triangle = (1/2) × parallelogram on the same base | a median halves the area of a triangle

Worked example. ABCD is a parallelogram and ABE is a triangle on the same base AB between the same parallels. If the area of ABCD is 48 cm², find the area of triangle ABE.

Solution: A triangle and a parallelogram on the same base between the same parallels: area of triangle = (1/2) × area of parallelogram = (1/2) × 48 = 24 cm².

Short Questions

1. Two parallelograms stand on the same base between the same parallels. What can you say about their areas? **Ans:** Their areas are equal.

2. A triangle and a parallelogram are on the same base between the same parallels. The triangle's area is 30 cm². Find the parallelogram's area. **Ans:** 60 cm² (the parallelogram is double the triangle).

3. What does a median do to the area of a triangle? **Ans:** It divides the triangle into two triangles of equal area.

4. Do equal areas mean two triangles are congruent? **Ans:** No — congruent figures have equal areas, but equal area does not imply congruence.

5. Find the area of a triangle with base 10 cm and height 6 cm. **Ans:** (1/2) × 10 × 6 = 30 cm².

Long Questions

1. Prove that parallelograms on the same base and between the same parallels are equal in area. **Ans:** Let ABCD and ABEF be parallelograms on base AB between parallels AB and FC. Each has area = base × height, and the height is the same (the distance between the parallels), so their areas are equal. In detail: the two parallelograms share the same base AB and the same altitude, so base × height is identical for both.

2. In triangle ABC, D is the midpoint of BC and AD is a median. Prove that the areas of triangles ABD and ACD are equal. **Ans:** Both triangles have the same height (the perpendicular from A to BC) and equal bases BD = DC (D is the midpoint). Area = (1/2) × base × height, so both areas are equal.

Multiple Choice Questions

1. Parallelograms on the same base between the same parallels have:
(a) equal perimeters (b) equal areas
(c) equal diagonals (d) equal sides

2. A triangle on the same base as a parallelogram between the same parallels has area:
(a) equal to the parallelogram (b) double the parallelogram
(c) half the parallelogram (d) one quarter of it

3. A median divides a triangle into two triangles of:
(a) equal perimeter (b) equal area
(c) equal angles (d) equal sides

4. Triangles on equal bases and the same altitude have:
(a) equal areas (b) equal perimeters
(c) equal angles (d) equal hypotenuses

5. If two triangles have equal areas, they are: (a) always congruent
(b) always similar (c) not necessarily congruent
(d) always right-angled

6. Area of a triangle = (a) base × height
(b) (1/2) × base × height (c) 2 × base × height
(d) base + height

7. Parallelograms on equal bases between the same parallels are equal in:
(a) perimeter (b) area
(c) diagonal (d) angle

8. Congruent figures have: (a) equal areas
(b) different areas (c) zero area
(d) double area

9. The area of a parallelogram is 50 cm^2. The triangle on the same base between the same parallels has area: (a) 100 cm^2
(b) 50 cm^2 (c) 25 cm^2
(d) 75 cm^2

10. Two triangles on the same base between the same parallels have equal:
(a) sides (b) angles
(c) areas (d) medians

Chapter 17: Practical Geometry – Triangles

Practical geometry is geometry done with ruler and compasses. The skill is accuracy: a sharp pencil, a steady compass, and arcs that cross cleanly. Learn each construction by doing it, not by reading it.

- **Tools:** a ruler (for straight lines and measuring), a pair of compasses (for circles and arcs), a protractor (for measuring angles) and a set square. Never use a protractor where a compass construction is asked for.
- **Construct a triangle (SSS):** given three sides, draw the base, then draw arcs from each end with radii equal to the other two sides; their intersection is the third vertex.
- **Construct a triangle (SAS):** given two sides and the included angle, draw one side, construct the angle at one end, mark the second side on the new ray, and join the ends.
- **Construct a triangle (ASA):** given two angles and the included side, draw the side, construct the two angles at its ends, and the rays meet at the third vertex.
- **Construct a triangle (RHS):** given the hypotenuse and one side of a right-angled triangle, draw the known side, erect a perpendicular at one end, and draw an arc of the hypotenuse length from the other end to meet the perpendicular.
- **Construct an angle:** angles of 60° and 120° come from an equilateral-triangle arc; bisect to get 30°, 45°, 15°, 75° and 90°. An angle of 22.5° is the bisection of 45°.
- **Construct a triangle from perimeter and base angles:** given the perimeter and the two base angles, draw a line of the perimeter length, construct the two base angles at its ends, bisect those angles, and the bisectors meet at the apex; the perpendicular bisectors of the two segments from the apex to the ends give the base vertices.
- **Checking your construction:** measure the finished triangle against the given data (sides with a ruler, angles with a protractor). A construction that does not match its data is wrong, however neat it looks.

> **Remember:** SSS: three sides | SAS: two sides + included angle | ASA: two angles + included side | RHS: hypotenuse + one side

Worked example. Construct triangle ABC with AB = 5 cm, BC = 6 cm and AC = 7 cm.

Solution: Draw AB = 5 cm. With centre A and radius 7 cm draw an arc; with centre B and radius 6 cm draw another arc. The arcs meet at C. Join AC and BC. Check: measure AC = 7 cm and BC = 6 cm. Triangle ABC is the required triangle.

Short Questions

1. Which instruments are used for a compass construction? **Ans:** Ruler and compasses (no protractor for the construction itself).

2. How is a 30° angle constructed? **Ans:** Construct a 60° angle (equilateral-triangle arc) and bisect it.

3. What data is needed for an SSS construction? **Ans:** The lengths of all three sides.

4. In an SAS construction, which angle must be given? **Ans:** The included angle — the angle between the two given sides.

5. How do you check a finished construction? **Ans:** Measure its sides and angles against the given data.

Long Questions

1. Construct a right-angled triangle with hypotenuse 10 cm and one side 6 cm. Describe the steps. **Ans:** Draw the side of 6 cm, say AB. At A, erect a perpendicular (construct a 90° angle). With centre B and radius 10 cm (the hypotenuse), draw an arc cutting the perpendicular at C. Join BC. Triangle ABC is right-angled at A with hypotenuse 10 cm and AB = 6 cm. Check: AC should measure 8 cm (6-8-10 triple).

2. Construct triangle PQR with QR = 6 cm, ∠Q = 60° and ∠R = 45°. Describe the steps. **Ans:** Draw QR = 6 cm. At Q construct a 60° angle (equilateral arc) and at R construct a 45° angle (bisect 90°). The two rays meet at P. Triangle PQR is the required triangle (∠P = 75°). Check the angles with a protractor.

Multiple Choice Questions

1. For an SSS construction we need: (a) two sides
(b) three sides (c) three angles
(d) two angles

2. A 60° angle is constructed from: (a) a square
(b) an equilateral triangle arc (c) a circle
(d) a protractor only

3. Bisecting a 90° angle gives: (a) 30°
(b) 60° (c) 45°
(d) 15°

4. In an SAS construction, the given angle must be the: (a) largest angle (b) smallest angle
(c) included angle (d) exterior angle

5. The RHS construction is for triangles that are: (a) equilateral (b) right-angled
(c) isosceles (d) acute-angled

6. Bisecting 30° gives: (a) 10°
(b) 15° (c) 20°
(d) 45°

7. To construct a triangle from its perimeter, you first draw a line equal to the: (a) longest side (b) perimeter
(c) base (d) altitude

8. A compass is used to draw: (a) straight lines
(b) arcs and circles (c) right angles only
(d) parallel lines only

9. An ASA construction needs: (a) two sides and an angle
(b) three sides (c) two angles and the included side
(d) the hypotenuse

10. The first step in constructing a triangle with sides 5, 6 and 7 cm is to draw:

(a) an arc of 7 cm (b) the base of 5 cm

(c) a 60° angle (d) a circle

MCQ Answer Key — All Chapters

Check your answers here. If you got a question wrong, go back to the notes of that chapter and work the idea again — do not just memorise the letter.

Chapter	1	2	3	4	5	6	7	8	9	10
1. Matrices and Determinants	b	b	a	b	b	b	b	c	b	c
2. Real and Complex Numbers	c	c	b	b	c	b	d	a	c	c
3. Logarithms	b	b	c	b	b	a	c	c	c	c
4. Algebraic Expressions	b	c	b	a	b	a	b	a	b	c
5. Factorization	b	b	a	a	b	a	b	a	a	b
6. Algebraic Manipulation	b	a	c	b	a	b	c	b	b	a
7. Linear Equations and Inequalities	b	b	a	b	c	b	c	b	a	b
8. Linear Graphs	c	c	b	c	c	b	c	b	a	c
9. Coordinate Geometry	c	c	c	a	b	b	b	b	a	b
10. Congruent Triangles	c	c	c	c	b	b	b	b	c	b
11. Parallelograms and Triangles	b	b	b	c	b	b	c	b	a	c
12. Line and Angle Bisectors	b	c	b	b	b	b	b	b	b	b
13. Sides and Angles of a Triangle	b	c	b	b	b	b	a	b	b	c
14. Ratio and Proportion	b	b	a	a	b	b	b	b	b	a
15. Pythagoras Theorem	b	a	b	c	c	b	b	b	b	b
16. Theorems Related with Area	b	c	b	a	c	b	b	a	c	c
17. Practical Geometry – Triangles	b	b	c	c	b	b	b	b	c	b

Final Revision Checklist

- Matrices: determinant, inverse, Cramer's rule, matrix inversion method.
- Numbers: laws of exponents, rationalising, i-powers, conjugate of a complex number.
- Logarithms: three laws, characteristic and mantissa, change of base.
- Algebra: square and cube formulae, factorisation methods, remainder and factor theorems.
- Equations: elimination and substitution, the inequality reversal rule.
- Graphs and coordinates: y = mx + c, gradient, distance and midpoint formulae.
- Geometry: SSS / SAS / ASA / AAS / RHS, parallelogram properties, Pythagoras, area theorems, constructions.

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