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9th Class Mathematics Notes Khyber Pakhtunkhwa (KPK) Full Book Notes

Complete chapter-wise notes, worked examples and original practice questions for all 17 chapters of the Khyber Pakhtunkhwa Textbook Board Mathematics book for Class 9.

Boards served: Peshawar, Mardan, Swat, Abbottabad, Bannu, Kohat, Malakand, D.I. Khan

Class: 9th Class | **Subject:** Mathematics | **Book:** Khyber Pakhtunkhwa Textbook Board, Peshawar

Medium: English

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How to Use These Notes

These notes follow the 17 chapters of the Khyber Pakhtunkhwa Textbook Board Mathematics book for Class 9. Each chapter has study notes, key formulae, a worked example, short and long questions with answers, and 10 multiple choice questions with an answer key.

Paper Pattern (KPK Boards)

The 9th-class Mathematics paper of the KPK boards (BISE Peshawar, Mardan, Swat, Abbottabad, Bannu, Kohat, Malakand and D.I. Khan) is of **75 marks in 3 hours**. The structure below is reproduced from the BISE Peshawar Class 9 Mathematics paper of 2025 as published by a third-party past-paper site; it is **not officially confirmed** from a board document, so students should confirm the current pattern from their board's model paper.

- **Section A — Objective:** 15 MCQs, 1 mark each = 15 marks (20 minutes).
- **Section B — Short questions:** attempt any 9 of 12 questions, 4 marks each = 36 marks.
- **Section C — Long questions:** attempt any 4 of 6 questions, 6 marks each = 24 marks.
- Total: 15 + 36 + 24 = 75 marks.

Note: one third-party pairing-scheme document gives a different split (15 + 30 + 30 = 75). The split above follows the actual 2025 Peshawar paper reproduced by the past-paper site.

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Chapter 1: Matrices and Determinants

Study Notes

- A matrix is a rectangular arrangement of numbers in rows and columns. A matrix with m rows and n columns is of order m × n.
- A square matrix has the same number of rows and columns. A diagonal matrix has all non-diagonal entries zero; an identity matrix has 1s on the main diagonal and 0s elsewhere.
- A zero (null) matrix has all entries zero. A symmetric matrix equals its transpose; a skew-symmetric matrix equals the negative of its transpose.
- Two matrices can be added or subtracted only when they have the same order; addition is done entry by entry.
- For multiplication AB, the number of columns of A must equal the number of rows of B. The product has the order (rows of A) × (columns of B).
- The transpose A^t is formed by turning rows into columns. $(AB)^t = B^tA^t$.
- A matrix is singular if its determinant is zero; otherwise it is non-singular and has an inverse.

Key Formulae

For A = [[a, b], [c, d]], det A = |A| = ad − bc.

Adjoint of [[a, b], [c, d]] = [[d, −b], [−c, a]].

A^{-1} = (1 / det A) × adj A, provided det A ≠ 0.

Solving AX = B gives X = A^{-1}B.

Worked Example

Find the inverse of A = [[5, 4], [1, 2]]. det A = (5)(2) − (4)(1) = 10 − 4 = 6. adj A = [[2, −4], [−1, 5]]. Hence A^{-1} = (1/6)[[2, −4], [−1, 5]]. Check: A × A^{-1} = I.

Short Questions with Answers — Chapter 1

Q1. Define a square matrix and give an example.
Answer: A matrix with equal numbers of rows and columns, e.g. [[2, 3], [1, 4]] is a 2 × 2 square matrix.

Q2. When can two matrices be multiplied?
Answer: When the number of columns of the first equals the number of rows of the second.

Q3. Find the determinant of [[3, 2], [5, 4]].
Answer: det = (3)(4) − (2)(5) = 12 − 10 = 2.

Q4. What is a singular matrix?
Answer: A square matrix whose determinant is zero; it has no inverse.

Q5. Write the transpose of [[1, 2, 3], [4, 5, 6]].
Answer: [[1, 4], [2, 5], [3, 6]].

Long Questions with Answers — Chapter 1

Q1. Find A^{-1} for A = [[2, 1], [3, 4]] and use it to solve 2x + y = 5, 3x + 4y = 10.
Answer: det A = 8 − 3 = 5; adj A = [[4, −1], [−3, 2]]; A^{-1} = (1/5)[[4, −1], [−3, 2]]. X = A^{-1}B = (1/5)[[4, −1], [−3, 2]] [[5], [10]] = (1/5)[[10], [5]] = [[2], [1]]. So x = 2, y = 1.

Q2. If A = [[1, 2], [0, 3]] and B = [[2, 1], [4, 0]], find AB and BA. Are they equal?
Answer: AB = [[10, 1], [12, 0]]; BA = [[2, 7], [4, 8]]. AB ≠ BA, so matrix multiplication is not commutative in general.

Multiple Choice Questions — Chapter 1

1. The order of the matrix [[1, 2], [3, 4], [5, 6]] is:

(A) 2 × 3 (B) 3 × 2
(C) 3 × 3 (D) 2 × 2

2. The identity matrix of order 2 is:

(A) [[0, 1], [1, 0]] (B) [[1, 0], [0, 1]]
(C) [[1, 1], [1, 1]] (D) [[2, 0], [0, 2]]

3. If A = [[a, b], [c, d]], then det A equals:

(A) ab − cd (B) ad − bc
(C) ac − bd (D) ad + bc

4. A matrix with all entries zero is called:

(A) identity matrix (B) diagonal matrix
(C) zero matrix (D) symmetric matrix

5. For matrices A (2 × 3) and B (3 × 4), the order of AB is:

(A) 2 × 4 (B) 3 × 3
(C) 4 × 2 (D) 2 × 3

6. The transpose of a row matrix is a:

(A) row matrix (B) column matrix
(C) square matrix (D) zero matrix

7. If det A = 0, then A is:

(A) invertible (B) singular
(C) identity (D) symmetric

8. The adjoint of [[7, 10], [−5, 4]] is:

(A) [[4, −10], [5, 7]] (B) [[7, −10], [5, 4]]
(C) [[4, 10], [−5, 7]] (D) [[7, 5], [−10, 4]]

9. If A = [[2, 0], [0, 2]], then A^{-1} is:

(A) [[1/2, 0], [0, 1/2]] (B) [[2, 0], [0, 2]]
(C) [[0, 1/2], [1/2, 0]] (D) [[−2, 0], [0, −2]]

10. $(AB)^t$ equals:

(A) A^tB^t (B) B^tA^t
(C) AB (D) BA

Answer Key: 1–B, 2–B, 3–B, 4–C, 5–A, 6–B, 7–B, 8–A, 9–A, 10–B

Chapter 2: Real and Complex Numbers

Study Notes

- Real numbers include rational and irrational numbers. Rational numbers can be written as p/q (q ≠ 0); irrational numbers cannot, and their decimals never terminate or repeat.
- A recurring decimal can be converted to a fraction: e.g. 0.333… = 1/3.
- A radical (surd) is an expression like √2 or $\sqrt[3]{5}$. A surd in simplest form has no perfect-square factor under the root and no surd in a denominator.
- Laws of exponents: $a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $(a^m)^n = a^{mn}$, $a^0 = 1$, $a^{-n} = 1/a^n$.
- Scientific notation writes a number as $a \times 10^n$ where $1 \le a < 10$ and n is an integer.
- A complex number is z = a + bi, where a is the real part, b the imaginary part, and $i^2 = -1$. The conjugate of a + bi is a − bi.
- Complex numbers are added by adding real parts and imaginary parts separately; they are multiplied like binomials, using $i^2 = -1$.

Key Formulae

$\sqrt[n]{a^m} = a^{(m/n)}$.

$|a + bi| = \sqrt{a^2 + b^2}$ (modulus of a complex number).

$(a + bi)(a - bi) = a^2 + b^2$.

0.999… = 1.

Worked Example

Simplify $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$. Using $(a + b)(a - b) = a^2 - b^2$: $(\sqrt{3})^2 - (\sqrt{2})^2 = 3 - 2 = 1$.

Short Questions with Answers — Chapter 2

Q1. Define an irrational number with an example.
Answer: A number that cannot be written as p/q; e.g. $\sqrt{2}$, π. Its decimal expansion never ends and never repeats.

Q2. Write 0.00056 in scientific notation.
Answer: 5.6×10^{-4}.

Q3. What is the imaginary part of $-7 - 4i$?
Answer: -4.

Q4. Find the conjugate of $3 + 5i$.
Answer: $3 - 5i$.

Q5. Simplify $\sqrt{50}$.
Answer: $\sqrt{50} = \sqrt{(25 \times 2)} = 5\sqrt{2}$.

Long Questions with Answers — Chapter 2

Q1. Simplify (2 + 3i)(1 − 2i) and write the answer in the form a + bi.
Answer: (2 + 3i)(1 − 2i) = 2 − 4i + 3i − 6i² = 2 − i + 6 = 8 − i.

Q2. Express 1.090909… as a fraction.
Answer: Let x = 1.090909…; 100x = 109.0909…; 100x − x = 108; 99x = 108; x = 108/99 = 12/11.

Multiple Choice Questions — Chapter 2

1. Which of the following is an irrational number?

(A) 3/4 (B) 0.25
(C) √2 (D) −5

2. The decimal 0.333… equals:

(A) 1/3 (B) 3/10
(C) 1/9 (D) 3/1

3. The value of 5^0 is:

(A) 0 (B) 1
(C) 5 (D) 25

4. The conjugate of −7 − 4i is:

(A) 7 + 4i (B) −7 + 4i
(C) 7 − 4i (D) −7 − 4i

5. √(16) × √(9) equals:

(A) 7 (B) 12
(C) 25 (D) 144

6. In scientific notation, 0.00056 is:

(A) 5.6×10^{-4} (B) 56×10^{-5}
(C) 0.56×10^{-3} (D) 5.6×10^{4}

7. The modulus of 3 + 4i is:

(A) 5 (B) 7
(C) 25 (D) 1

8. i^2 equals:

(A) 1 (B) −1
(C) 0 (D) i

9. The cube root of 343 is:

(A) 7 (B) 9
(C) 11 (D) 13

10. The additive inverse of −13 is:

(A) 1/13 (B) −13
(C) 13 (D) 0

Answer Key: 1–C, 2–A, 3–B, 4–B, 5–B, 6–A, 7–A, 8–B, 9–A, 10–C

Chapter 3: Logarithms

Study Notes

- A logarithm is the inverse of exponentiation: if $a^x = y$ ($a > 0$, $a \neq 1$), then $x = \log_a y$.
- The common logarithm uses base 10 and is written log y; the natural logarithm uses base e.
- Product law: $\log(mn) = \log m + \log n$. Quotient law: $\log(m/n) = \log m - \log n$. Power law: $\log(m^n) = n \log m$.
- $\log 1 = 0$ and $\log a = 1$ for any valid base a.
- The characteristic of a common logarithm is its integer part and the mantissa is its positive decimal part; log tables use the mantissa.
- Logarithms turn multiplication into addition and division into subtraction, which is how they were used for calculation before electronic calculators.

Key Formulae

$\log_a(mn) = \log_a m + \log_a n$.

$\log_a(m/n) = \log_a m - \log_a n$.

$\log_a(m^n) = n \log_a m$.

$\log_a 1 = 0$; $\log_a a = 1$.

Change of base: $\log_a b = \log b / \log a$.

Worked Example

Find $\log_2 32$. Since $2^5 = 32$, $\log_2 32 = 5$. Check with the power law: $\log_2 32 = \log_2 2^5 = 5 \log_2 2 = 5 \times 1 = 5$.

Short Questions with Answers — Chapter 3

Q1. Define a logarithm.
Answer: If $a^x = y$, then $x = \log_a y$; the logarithm is the exponent to which the base must be raised to get the number.

Q2. What is log 1000 to base 10?
Answer: 3, because $10^3 = 1000$.

Q3. State the product law of logarithms.
Answer: $\log(mn) = \log m + \log n$.

Q4. What is the value of $\log_5 1$?
Answer: 0.

Q5. What are the characteristic and mantissa of log 4.23?
Answer: Characteristic 0 (since $1 \le 4.23 < 10$), mantissa is the decimal part of log 4.23.

Long Questions with Answers — Chapter 3

Q1. Simplify log 8 + log 125 using logarithm laws and find its value.
Answer: log 8 + log 125 = log(8 × 125) = log 1000 = 3.

Q2. Solve for x: $\log_3 x = 4$.
Answer: $x = 3^4 = 81$.

Multiple Choice Questions — Chapter 3

1. If $2^x = 16$, then x equals:

(A) 3 (B) 4
(C) 5 (D) 8

2. log 100 equals:

(A) 1 (B) 2
(C) 10 (D) 100

3. log(mn) equals:

(A) log m − log n (B) log m + log n
(C) log m × log n (D) log m ÷ log n

4. The value of $\log_7 7$ is:

(A) 0 (B) 1
(C) 7 (D) 49

5. log 1000 − log 10 equals:

(A) 2 (B) 3
(C) 10 (D) 100

6. log(2^3) equals:

(A) 2 log 3 (B) 3 log 2
(C) log 6 (D) log 5

7. The base of a common logarithm is:

(A) e (B) 2
(C) 10 (D) 5

8. If log x = 2, then x equals:

(A) 20 (B) 100
(C) 2 (D) 10

9. log 1 to any base is:

(A) 1 (B) 0
(C) the base (D) undefined

10. $\log_a a^n$ equals:

(A) a (B) n
(C) 1 (D) 0

Answer Key: 1–B, 2–B, 3–B, 4–B, 5–A, 6–B, 7–C, 8–B, 9–B, 10–B

Chapter 4: Algebraic Expressions and Algebraic Formulas

Study Notes

- An algebraic expression is built from variables, constants and operations. A polynomial is an expression with non-negative integer powers of the variable.
- The degree of a polynomial is the highest power of the variable in it. A polynomial in one variable of degree 2 is quadratic, of degree 3 is cubic.
- A monomial has one term, a binomial two terms, and a trinomial three terms.
- Algebraic identities are equalities that hold for all values of the variables; they are used to expand and factorise quickly.
- The square of a binomial: $(a + b)^2 = a^2 + 2ab + b^2$; $(a - b)^2 = a^2 - 2ab + b^2$; and $a^2 - b^2 = (a + b)(a - b)$.
- The cube formulas: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$; $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$; $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

Key Formulae

$(a + b)^2 = a^2 + 2ab + b^2$.

$(a - b)^2 = a^2 - 2ab + b^2$.

$a^2 - b^2 = (a + b)(a - b)$.

$(a + b)^3 = a^3 + 3ab(a + b) + b^3$.

$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$.

Worked Example

Expand $(2x + 3)^2$. $(2x + 3)^2 = (2x)^2 + 2(2x)(3) + 3^2 = 4x^2 + 12x + 9$.

Short Questions with Answers — Chapter 4

Q1. What is the degree of $5x^3 - 2x + 7$?
Answer: 3.

Q2. Expand $(x - 5)^2$.
Answer: $x^2 - 10x + 25$.

Q3. Factorise $a^2 - 49$.
Answer: $(a + 7)(a - 7)$.

Q4. What is a binomial? Give an example.
Answer: An algebraic expression with two terms, e.g. $3x + 2$.

Q5. Write the formula for $a^3 + b^3$.
Answer: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

Long Questions with Answers — Chapter 4

Q1. If $x + 1/x = 5$, find $x^2 + 1/x^2$.

Answer: Squaring: $(x + 1/x)^2 = x^2 + 2 + 1/x^2 = 25$, so $x^2 + 1/x^2 = 25 - 2 = 23$.

Q2. Prove that $(m + n)^2 + (m - n)^2 = 2(m^2 + n^2)$.

Answer: LHS $= (m^2 + 2mn + n^2) + (m^2 - 2mn + n^2) = 2m^2 + 2n^2 = 2(m^2 + n^2) =$ RHS.

Multiple Choice Questions — Chapter 4

1. The degree of the polynomial $4x^5 - x^2 + 1$ is:

(A) 2 (B) 3
(C) 5 (D) 1

2. $(x + 4)^2$ equals:

(A) $x^2 + 8x + 16$ (B) $x^2 + 4x + 16$
(C) $x^2 + 16$ (D) $x^2 + 8x + 8$

3. $a^2 - b^2$ factorises as:

(A) $(a - b)^2$ (B) $(a + b)(a - b)$
(C) $(a + b)^2$ (D) $(a - b)(a - b)$

4. The expression $3x^2 + 2x - 1$ is a:

(A) monomial (B) binomial
(C) trinomial (D) constant

5. $(a + b)^3$ equals:

(A) $a^3 + b^3$ (B) $a^3 + 3a^2b + 3ab^2 + b^3$
(C) $a^3 + 3ab + b^3$ (D) $a^3 - b^3$

6. If $a + b = 7$ and $ab = 10$, then $a^2 + b^2$ equals:

(A) 29 (B) 49
(C) 39 (D) 69

7. $x^2 - 9$ is equal to:

(A) $(x - 3)^2$ (B) $(x + 3)(x - 3)$
(C) $(x + 9)(x - 9)$ (D) $(x - 9)^2$

8. The coefficient of x^2 in $5x^2 - 3x + 2$ is:

(A) 5 (B) -3
(C) 2 (D) 7

9. $(2x - 1)^2$ equals:

(A) $4x^2 - 4x + 1$ (B) $4x^2 + 4x + 1$
(C) $4x^2 - 1$ (D) $2x^2 - 4x + 1$

10. $a^3 - b^3$ factorises as:

(A) $(a - b)(a^2 + ab + b^2)$ (B) $(a - b)(a^2 - ab + b^2)$
(C) $(a + b)(a^2 - ab + b^2)$ (D) $(a - b)^3$

Answer Key: 1–C, 2–A, 3–B, 4–C, 5–B, 6–A, 7–B, 8–A, 9–A, 10–A

Chapter 5: Factorization

Study Notes

- Factorization is writing an expression as a product of its factors. It is the reverse of expansion.
- Common factor: take out the highest common factor of all terms first, e.g. $6x^2 + 9x = 3x(2x + 3)$.
- Grouping: group terms so that each group has a common factor, then factor again.
- Quadratic trinomials $x^2 + bx + c$ are factored by finding two numbers whose product is c and whose sum is b.
- Difference of squares: $a^2 - b^2 = (a + b)(a - b)$. Sum and difference of cubes: $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$.
- The remainder theorem: when a polynomial p(x) is divided by $(x - a)$, the remainder is p(a). The factor theorem: $(x - a)$ is a factor of p(x) if and only if $p(a) = 0$.

Key Formulae

$x^2 + (a + b)x + ab = (x + a)(x + b)$.

$a^2 - b^2 = (a + b)(a - b)$.

Remainder on dividing p(x) by $(x - a)$ is p(a).

Worked Example

Factorize $4y^2 + 8y + 3$. Look for two numbers with product $4 \times 3 = 12$ and sum 8: 6 and 2. $4y^2 + 6y + 2y + 3 = 2y(2y + 3) + 1(2y + 3) = (2y + 1)(2y + 3)$.

Short Questions with Answers — Chapter 5

Q1. What is factorization?
Answer: Writing an expression as a product of its factors.

Q2. Factorize $x^2 - 16$.
Answer: $(x + 4)(x - 4)$.

Q3. State the factor theorem.
Answer: $(x - a)$ is a factor of $p(x)$ if and only if $p(a) = 0$.

Q4. Find the remainder when $3x^2 + 2x - 5$ is divided by $x + 2$.
Answer: $p(-2) = 3(4) + 2(-2) - 5 = 12 - 4 - 5 = 3$.

Q5. Factorize $x^2 + 7x + 12$.
Answer: $(x + 3)(x + 4)$.

Long Questions with Answers — Chapter 5

Q1. Factorize $x^3 - 8$ completely.

Answer: $x^3 - 8 = x^3 - 2^3 = (x - 2)(x^2 + 2x + 4)$.

Q2. Show that $(x - 3)$ is a factor of $x^3 - 4x^2 + x + 6$.

Answer: $p(3) = 27 - 36 + 3 + 6 = 0$, so by the factor theorem $(x - 3)$ is a factor.

Multiple Choice Questions — Chapter 5

1. The factors of $x^2 - 9$ are:

(A) $(x - 3)(x - 3)$ (B) $(x + 3)(x - 3)$
(C) $(x + 9)(x - 9)$ (D) $(x - 9)(x + 1)$

2. The HCF of $12x^2$ and $18x$ is:

(A) $6x$ (B) $6x^2$
(C) $12x$ (D) 18

3. Factorize $x^2 + 5x + 6$:

(A) $(x + 2)(x + 3)$ (B) $(x - 2)(x - 3)$
(C) $(x + 1)(x + 6)$ (D) $(x + 5)(x + 1)$

4. The LCM of $x^2 - 9$ and $(x - 3)^2$ is:

(A) $(x - 3)^2$ (B) $(x + 3)(x - 3)^2$
(C) $(x + 3)^2(x - 3)$ (D) $x^2 - 9$

5. If $p(x) = x^2 - 3x + 2$, then $p(1)$ equals:

(A) 0 (B) 1
(C) 2 (D) 6

6. The factors of $4y^2 + 8y + 3$ are:

(A) $(2y + 1)(2y + 3)$ (B) $(2y - 1)(2y - 3)$
(C) $(4y + 1)(y + 3)$ (D) $(y + 1)(4y + 3)$

7. $x^3 + 27$ factorises as:

(A) $(x + 3)(x^2 - 3x + 9)$ (B) $(x - 3)(x^2 + 3x + 9)$
(C) $(x + 3)^3$ (D) $(x + 3)(x^2 + 3x + 9)$

8. The remainder when $x^3 - 2x + 1$ is divided by $(x - 1)$ is:

(A) 0 (B) 1
(C) -1 (D) 2

9. The HCF of $(x - 3)^2$ and $x^2 - 9$ is:

(A) $x - 3$ (B) $x + 3$
(C) $(x - 3)^2$ (D) $x^2 - 9$

10. Factorize $9x^2 - 25$:

(A) $(3x - 5)^2$ (B) $(3x + 5)(3x - 5)$
(C) $(3x + 5)^2$ (D) $(9x - 5)(x + 5)$

Answer Key: 1–B, 2–A, 3–A, 4–B, 5–A, 6–A, 7–A, 8–A, 9–A, 10–B

Chapter 6: Algebraic Manipulation

Study Notes

- Algebraic manipulation means simplifying rational expressions using factorization, HCF and LCM.
- To add or subtract rational expressions, first find the LCM of the denominators, then combine the numerators.
- A rational expression is in lowest terms when numerator and denominator have no common factor other than 1.
- The square root of a perfect-square expression can be found by factorization: $\sqrt{(a^2)} = a$ (for $a \geq 0$).
- Surds can be added or subtracted only when they are like surds; they are multiplied by multiplying radicands.
- To simplify a fraction with a surd in the denominator, rationalize it by multiplying numerator and denominator by the conjugate of the denominator.

Key Formulae

LCM of $(x - 1)$ and $(x^2 - 1)$ is $(x - 1)(x + 1)$.

$\sqrt{(x^2)} = |x|$.

$1/(\sqrt{a} + \sqrt{b}) = (\sqrt{a} - \sqrt{b})/(a - b)$.

Worked Example

Find the square root of $16y^2 + 24yz + 9z^2$ by factorization. $16y^2 + 24yz + 9z^2 = (4y + 3z)^2$, since $(4y)^2 = 16y^2$, $(3z)^2 = 9z^2$ and $2(4y)(3z) = 24yz$. So the square root is $4y + 3z$.

Short Questions with Answers — Chapter 6

Q1. Simplify $x^2/(x)$ for $x \neq 0$.
Answer: x.

Q2. What is the HCF of $x^2 - 1$ and $x - 1$?
Answer: $x - 1$.

Q3. Simplify $(x^2 - 4)/(x + 2)$.
Answer: $(x - 2)(x + 2)/(x + 2) = x - 2$, for $x \neq -2$.

Q4. Find the square root of $25a^2$.
Answer: $5a$ (for $a \geq 0$).

Q5. Rationalize $1/\sqrt{3}$.
Answer: $\sqrt{3}/3$.

Long Questions with Answers — Chapter 6

Q1. Simplify $\frac{2x}{x^2 - 1} + \frac{1}{x + 1}$.

Answer: LCM is $(x - 1)(x + 1)$. Expression $= \frac{2x + (x - 1)}{(x - 1)(x + 1)} = \frac{3x - 1}{x^2 - 1}$.

Q2. Find the HCF and LCM of $x^2 - 1$ and $x^3 - 1$.

Answer: $x^2 - 1 = (x - 1)(x + 1)$; $x^3 - 1 = (x - 1)(x^2 + x + 1)$. HCF $= x - 1$; LCM $= (x - 1)(x + 1)(x^2 + x + 1)$.

Multiple Choice Questions — Chapter 6

1. The simplified form of $(x^2 - 9)/(x - 3)$ is:

(A) $x + 3$ (B) $x - 3$
(C) $x^2 + 3$ (D) $x + 9$

2. The LCM of $(x - 1)$ and $(x + 1)$ is:

(A) $x - 1$ (B) $x + 1$
(C) $x^2 - 1$ (D) 1

3. $\sqrt{(49x^2)}$ equals $(x \geq 0)$:

(A) $7x$ (B) $49x$
(C) $7x^2$ (D) $14x$

4. $1/\sqrt{2}$ rationalized is:

(A) $\sqrt{2}/2$ (B) $2\sqrt{2}$
(C) $1/2$ (D) $\sqrt{2}$

5. The HCF of $x^2 - 4$ and $x^2 - 2x$ is:

(A) $x - 2$ (B) $x + 2$
(C) x (D) x^2

6. $(x^2 - 1)/(x - 1)$ simplifies to:

(A) $x - 1$ (B) $x + 1$
(C) $x^2 + 1$ (D) 1

7. Like surds are:

(A) $\sqrt{2}$ and $\sqrt{3}$ (B) $2\sqrt{3}$ and $5\sqrt{3}$
(C) $\sqrt{2}$ and $2\sqrt{2}$ (D) $\sqrt{5}$ and $\sqrt{7}$

8. The square root of $(x + 1)^2$ is:

(A) $x + 1$ (B) $x - 1$
(C) $(x + 1)^2$ (D) $x^2 + 1$

9. Simplify $3/\sqrt{5}$:

(A) $3\sqrt{5}/5$ (B) $\sqrt{5}/3$
(C) $3/5$ (D) $5\sqrt{3}/3$

10. The lowest form of $(2x + 4)/(x + 2)$ is:

(A) 2 (B) $x + 2$
(C) $2x$ (D) x

Answer Key: 1–A, 2–C, 3–A, 4–A, 5–A, 6–B, 7–B, 8–A, 9–A, 10–A

Chapter 7: Linear Equations and Inequalities

Study Notes

- A linear equation in one variable has the form $ax + b = 0$ ($a \neq 0$); its solution is $x = -b/a$.
- An equation is solved by doing the same operation on both sides: add, subtract, multiply or divide by the same (non-zero) number.
- Simultaneous linear equations in two variables can be solved by elimination (add or subtract to remove one variable) or by substitution.
- A system may have one solution (lines meet), no solution (parallel lines), or infinitely many solutions (same line).
- A linear inequality uses $<$, $>$, $\leq$ or $\geq$. Its solution is shown on a number line with an open circle for strict inequality and a closed circle for $\leq$ or $\geq$.
- When an inequality is multiplied or divided by a negative number, the inequality sign reverses.

Key Formulae

$ax + b = 0 \Rightarrow x = -b/a$.

If $x + y = 10$ and $x - y = 2$, then $x = 6$, $y = 4$.

Multiplying an inequality by a negative number reverses the sign.

Worked Example

Solve and plot on a number line: $3(t - 3) > 3$. $3t - 9 > 3 \Rightarrow 3t > 12 \Rightarrow t > 4$. On the number line, an open circle at 4 with shading to the right.

Short Questions with Answers — Chapter 7

Q1. Solve $5x - 3 = 17$.
Answer: $5x = 20$, so $x = 4$.

Q2. Solve and verify $\sqrt{(8y - 6)} = \sqrt{(4y + 10)}$.
Answer: Squaring both sides: $8y - 6 = 4y + 10 \Rightarrow 4y = 16 \Rightarrow y = 4$. Check: $\sqrt{(32 - 6)} = \sqrt{26} = \sqrt{(16 + 10)} = \sqrt{26}$. Correct.

Q3. What happens to an inequality when multiplied by -2?
Answer: The inequality sign reverses.

Q4. Solve $x + 7 < 12$.
Answer: $x < 5$.

Q5. Name two methods of solving simultaneous equations.
Answer: Elimination and substitution.

Long Questions with Answers — Chapter 7

Q1. Solve the simultaneous equations $2x + 3y = 12$ and $x - y = 1$.

Answer: From the second, $x = y + 1$. Substituting: $2(y + 1) + 3y = 12 \Rightarrow 5y = 10 \Rightarrow y = 2$, $x = 3$.

Q2. Solve the inequality $2x + 5 \leq 13$ and show the solution on a number line.

Answer: $2x \leq 8 \Rightarrow x \leq 4$. Number line: closed circle at 4, shading to the left.

Multiple Choice Questions — Chapter 7

1. The solution of $3x + 6 = 0$ is:

(A) $x = 2$ (B) $x = -2$
(C) $x = 6$ (D) $x = 0$

2. If $x + y = 10$ and $x - y = 10$, then x is:

(A) 0 (B) 5
(C) 10 (D) 20

3. The solution of $2x > 10$ is:

(A) $x > 5$ (B) $x < 5$
(C) $x > 20$ (D) $x < 20$

4. Solve $4x - 8 = 0$:

(A) $x = 2$ (B) $x = -2$
(C) $x = 8$ (D) $x = 4$

5. An open circle on a number line means:

(A) the value is included (B) the value is not included
(C) the value is zero (D) the value is negative

6. The equations $x + y = 5$ and $2x + 2y = 10$ have:

(A) one solution (B) no solution
(C) infinitely many solutions (D) two solutions

7. If $5x = 35$, then x equals:

(A) 5 (B) 6
(C) 7 (D) 8

8. Solve $x/3 + 2 = 5$:

(A) $x = 9$ (B) $x = 3$
(C) $x = 6$ (D) $x = 15$

9. The inequality $x \geq 3$ is shown by a:

(A) open circle at 3, shading right (B) closed circle at 3, shading right
(C) open circle at 3, shading left (D) closed circle at 3, shading left

10. The solution of $7 - x = 2$ is:

(A) $x = 5$ (B) $x = 9$
(C) $x = -5$ (D) $x = 2$

Answer Key: 1–B, 2–C, 3–A, 4–A, 5–B, 6–C, 7–C, 8–A, 9–B, 10–A

Chapter 8: Linear Graphs and Their Applications

Study Notes

- A linear equation in two variables, ax + by = c, graphs as a straight line.
- The gradient (slope) of a line measures its steepness: gradient = change in y / change in x = $(y_2 - y_1)/(x_2 - x_1)$.
- The equation y = mx + c is the slope-intercept form: m is the gradient and c is the y-intercept.
- Parallel lines have equal gradients; perpendicular lines have gradients whose product is −1.
- To draw a graph, find at least two points that satisfy the equation, plot them and join with a straight line.
- Graphs are used to solve simultaneous equations (the intersection point), to read values, and to model real situations such as distance–time and cost problems.

Key Formulae

Gradient $m = (y_2 - y_1)/(x_2 - x_1)$.

y = mx + c (m = gradient, c = y-intercept).

Parallel lines: $m_1 = m_2$. Perpendicular lines: $m_1 m_2 = -1$.

Worked Example

Find the gradient of the line through (1, 2) and (5, 10). m = (10 − 2)/(5 − 1) = 8/4 = 2. The line has equation y − 2 = 2(x − 1), i.e. y = 2x.

Short Questions with Answers — Chapter 8

Q1. What is the gradient of $y = 3x + 5$?
Answer: 3.

Q2. What is the y-intercept of $y = -2x + 7$?
Answer: 7.

Q3. Are the lines $y = 2x + 1$ and $y = 2x - 3$ parallel?
Answer: Yes, both have gradient 2.

Q4. Find the gradient of the line joining (0, 0) and (4, 8).
Answer: 8/4 = 2.

Q5. What is the gradient of a horizontal line?
Answer: 0.

Long Questions with Answers — Chapter 8

Q1. Draw the graph of $2x + y = 6$ and find where it meets the axes.
Answer: When $x = 0$, $y = 6$; when $y = 0$, $x = 3$. Plot $(0, 6)$ and $(3, 0)$ and join them. The line meets the y-axis at 6 and the x-axis at 3.

Q2. Find the equation of the line through $(2, 3)$ with gradient 4.
Answer: $y - 3 = 4(x - 2) \Rightarrow y = 4x - 5$.

Multiple Choice Questions — Chapter 8

1. The gradient of the line $y = 5x - 2$ is:

(A) -2 (B) 5
(C) 2 (D) 7

2. The y-intercept of $y = -x + 4$ is:

(A) -1 (B) 4
(C) 1 (D) -4

3. Lines with gradients 3 and $-1/3$ are:

(A) parallel (B) perpendicular
(C) the same (D) horizontal

4. The gradient of a vertical line is:

(A) 0 (B) 1
(C) undefined (D) -1

5. The line $y = 2x$ passes through:

(A) (0, 1) (B) (1, 2)
(C) (2, 1) (D) (0, 2)

6. If two lines are parallel, their gradients are:

(A) equal (B) opposite
(C) reciprocals (D) zero

7. The point (2, 6) lies on the line:

(A) $y = 2x$ (B) $y = 3x$
(C) $y = x + 2$ (D) $y = 6x$

8. The x-intercept of $2x + y = 6$ is:

(A) 3 (B) 6
(C) 2 (D) -3

9. The gradient of the line joining (1, 1) and (3, 5) is:

(A) 2 (B) 3
(C) 4 (D) 1

10. A line with gradient -1 and y-intercept 5 has equation:

(A) $y = -x + 5$ (B) $y = x - 5$
(C) $y = -5x + 1$ (D) $y = 5x - 1$

Answer Key: 1–B, 2–B, 3–B, 4–C, 5–B, 6–A, 7–B, 8–A, 9–A, 10–A

Chapter 9: Introduction to Coordinate Geometry

Study Notes

- Coordinate geometry studies geometry using coordinates on the Cartesian plane, with a horizontal x-axis and a vertical y-axis meeting at the origin (0, 0).
- A point is written as (x, y), where x is the abscissa and y is the ordinate.
- The plane is divided into four quadrants: first (+, +), second (−, +), third (−, −) and fourth (+, −).
- The distance between points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$.
- The midpoint of PQ is $((x_1 + x_2)/2, (y_1 + y_2)/2)$.
- Three points are collinear if the area of the triangle they form is zero; the area of a triangle with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) is $(1/2)|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$.

Key Formulae

Distance PQ = $\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$.

Midpoint = $((x_1 + x_2)/2, (y_1 + y_2)/2)$.

Area of triangle = $(1/2)|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$.

Worked Example

Show that the points A(0, 4), B($\sqrt{48}$, 0) and C(0, −4) form an equilateral triangle. AB = $\sqrt{(48 + 16)}$ = $\sqrt{64}$ = 8; BC = $\sqrt{(48 + 16)}$ = 8; AC = 8. All three sides are 8 units, so the triangle is equilateral.

Short Questions with Answers — Chapter 9

Q1. In which quadrant does the point (6, 9) lie?
Answer: First quadrant, since both coordinates are positive.

Q2. What is the abscissa of the point (−3, 5)?
Answer: −3.

Q3. Find the distance between (0, 0) and (3, 4).
Answer: 5 units.

Q4. Find the midpoint of (2, 6) and (4, 10).
Answer: (3, 8).

Q5. What are the coordinates of the origin?
Answer: (0, 0).

Long Questions with Answers — Chapter 9

Q1. Find the distance between A(1, 2) and B(4, 6).
Answer: AB = $\sqrt{((4 - 1)^2 + (6 - 2)^2)} = \sqrt{(9 + 16)} = \sqrt{25} = 5$ units.

Q2. Show that the points (0, 0), (3, 0) and (0, 4) form a right-angled triangle and find its area.
Answer: Sides: 3, 4 and 5 (by distance formula). $3^2 + 4^2 = 5^2$, so it is right-angled at the origin. Area = $(1/2) \times 3 \times 4 = 6$ square units.

Multiple Choice Questions — Chapter 9

1. The point (6, 9) lies in the:

(A) 1st quadrant (B) 2nd quadrant
(C) 3rd quadrant (D) 4th quadrant

2. The distance between (0, 0) and (6, 8) is:

(A) 10 (B) 14
(C) 8 (D) 6

3. The midpoint of (2, 4) and (6, 10) is:

(A) (4, 7) (B) (3, 5)
(C) (8, 14) (D) (4, 6)

4. A point with x = 0 lies on the:

(A) x-axis (B) y-axis
(C) origin (D) first quadrant

5. The ordinate of (5, −2) is:

(A) 5 (B) −2
(C) 3 (D) 7

6. Points (−1, 2), (1, 2) and (0, 2) are:

(A) collinear (B) non-collinear
(C) vertices of a triangle (D) on the y-axis

7. The distance formula is based on:

(A) Pythagoras theorem (B) area formula
(C) midpoint formula (D) gradient formula

8. The point (−4, −5) lies in the:

(A) 1st quadrant (B) 2nd quadrant
(C) 3rd quadrant (D) 4th quadrant

9. The midpoint of (0, 0) and (10, 6) is:

(A) (5, 3) (B) (10, 6)
(C) (5, 6) (D) (10, 3)

10. The abscissa of (7, 3) is:

(A) 3 (B) 7
(C) 10 (D) 4

Answer Key: 1–A, 2–A, 3–A, 4–B, 5–B, 6–A, 7–A, 8–C, 9–A, 10–B

Chapter 10: Congruent Triangles

Study Notes

- Two triangles are congruent if they have exactly the same shape and size; all corresponding sides and angles are equal.
- Congruence conditions: SSS (three sides), SAS (two sides and the included angle), ASA (two angles and the included side), AAS (two angles and a non-included side), RHS (right angle, hypotenuse and one side).
- Congruence is written with the symbol $\cong$, and the order of the letters must match corresponding vertices.
- Isosceles triangle theorem: the angles opposite the equal sides of an isosceles triangle are equal; conversely, if two angles of a triangle are equal, the sides opposite them are equal.
- In a proof, always state the condition used and which sides or angles correspond.
- Congruence is used to prove facts about parallelograms, bisectors and constructions.

Key Formulae

SSS: three sides equal.

SAS: two sides and the included angle equal.

ASA: two angles and the included side equal.

RHS: right angle, hypotenuse and a side equal.

Worked Example

Prove: if two angles of a triangle are congruent, the sides opposite them are congruent. In triangle ABC, suppose $\angle B = \angle C$. Draw the bisector of $\angle A$ meeting BC at D. In triangles ABD and ACD, $\angle BAD = \angle CAD$, AD is common, and $\angle B = \angle C$, so by AAS the triangles are congruent. Hence AB = AC, the sides opposite the equal angles.

Short Questions with Answers — Chapter 10

Q1. What does congruent mean for triangles?
Answer: Same shape and same size, with all corresponding sides and angles equal.

Q2. State the SSS condition.
Answer: Two triangles are congruent if the three sides of one equal the three sides of the other.

Q3. State the RHS condition.
Answer: Two right-angled triangles are congruent if the hypotenuse and one side of one equal those of the other.

Q4. In an isosceles triangle, which angles are equal?
Answer: The angles opposite the equal sides.

Q5. What symbol is used for congruence?
Answer: ≅

Long Questions with Answers — Chapter 10

Q1. Prove that the diagonals of a rectangle are equal. (Outline)
Answer: Let ABCD be a rectangle. In triangles ABC and DCB: AB = DC (opposite sides), ∠B = ∠C = 90°, BC is common. By SAS, the triangles are congruent, so AC = DB.

Q2. Prove that the angles opposite equal sides of a triangle are equal. (Outline)
Answer: In triangle ABC with AB = AC, draw the bisector of ∠A meeting BC at D. Triangles ABD and ACD are congruent by SAS (AB = AC, ∠BAD = ∠CAD, AD common), so ∠B = ∠C.

Multiple Choice Questions — Chapter 10

1. Two triangles with all three sides equal are congruent by:

(A) SAS (B) SSS
(C) ASA (D) RHS

2. In congruent triangles, corresponding angles are:

(A) supplementary (B) equal
(C) complementary (D) unequal

3. The SAS condition needs:

(A) two sides and the included angle (B) two angles and a side
(C) three sides (D) the hypotenuse

4. Which is NOT a congruence condition?

(A) SSS (B) SAS
(C) AAA (D) ASA

5. If $\Delta ABC \cong \Delta DEF$, then AB equals:

(A) DE (B) DF
(C) EF (D) FE

6. RHS applies to:

(A) acute triangles (B) right-angled triangles
(C) obtuse triangles (D) all triangles

7. In an isosceles triangle with AB = AC, which angles are equal?

(A) $\angle B$ and $\angle C$ (B) $\angle A$ and $\angle B$
(C) $\angle A$ and $\angle C$ (D) none

8. Congruent triangles have:

(A) equal area (B) different area
(C) equal perimeter only (D) equal angles only

9. ASA needs:

(A) two angles and the included side (B) two sides and an angle
(C) three angles (D) one side

10. If two angles of a triangle are equal, the opposite sides are:

(A) unequal (B) equal
(C) parallel (D) perpendicular

Answer Key: 1–B, 2–B, 3–A, 4–C, 5–A, 6–B, 7–A, 8–A, 9–A, 10–B

Chapter 11: Parallelograms and Triangles

Study Notes

- A parallelogram is a quadrilateral with both pairs of opposite sides parallel.
- In a parallelogram, opposite sides are equal, opposite angles are equal, and the diagonals bisect each other.
- A quadrilateral is a parallelogram if its opposite sides are equal, or its opposite angles are equal, or its diagonals bisect each other, or one pair of opposite sides is equal and parallel.
- A rectangle has all angles 90°; a rhombus has all sides equal; a square has all sides equal and all angles 90°.
- Triangles on the same base and between the same parallels are equal in area.
- The diagonal of a parallelogram divides it into two congruent triangles.

Key Formulae

Area of a parallelogram = base × height.

Area of a triangle = (1/2) × base × height.

Diagonals of a parallelogram bisect each other.

Worked Example

Prove that the opposite sides of a parallelogram are congruent. In parallelogram ABCD, draw diagonal AC. Since AB ∥ DC, alternate angles ∠BAC = ∠DCA; since AD ∥ BC, ∠DAC = ∠BCA. Triangles ABC and CDA are congruent by ASA (AC common), so AB = CD and AD = CB.

Short Questions with Answers — Chapter 11

Q1. Define a parallelogram.
Answer: A quadrilateral with both pairs of opposite sides parallel.

Q2. What is true about the diagonals of a parallelogram?
Answer: They bisect each other.

Q3. Name a quadrilateral with all sides equal.
Answer: A rhombus (or a square).

Q4. What divides a parallelogram into two congruent triangles?
Answer: Either diagonal.

Q5. How are the opposite angles of a parallelogram related?
Answer: They are equal.

Long Questions with Answers — Chapter 11

Q1. Prove that the diagonals of a parallelogram bisect each other.

Answer: In parallelogram ABCD with diagonals meeting at O: AB ∥ CD gives alternate angles equal; triangles AOB and COD are congruent by ASA, so AO = CO and BO = DO. Hence the diagonals bisect each other.

Q2. Prove that a diagonal divides a parallelogram into two congruent triangles.

Answer: In parallelogram ABCD, diagonal AC: AB = CD, AD = CB (opposite sides), AC is common. By SSS, ΔABC ≅ ΔCDA.

Multiple Choice Questions — Chapter 11

1. In a parallelogram, opposite sides are:

(A) unequal (B) equal
(C) perpendicular (D) parallel only

2. The diagonals of a parallelogram:

(A) are equal (B) bisect each other
(C) are perpendicular (D) are parallel

3. A quadrilateral with one pair of opposite sides equal and parallel is a:

(A) trapezium (B) parallelogram
(C) kite (D) rhombus

4. A rectangle has:

(A) all sides equal (B) all angles 90°
(C) diagonals perpendicular (D) no parallel sides

5. A rhombus has:

(A) all sides equal (B) all angles 90°
(C) diagonals equal (D) no parallel sides

6. The area of a parallelogram with base 6 cm and height 4 cm is:

(A) 10 cm² (B) 24 cm²
(C) 12 cm² (D) 20 cm²

7. A square is a:

(A) rhombus and rectangle (B) trapezium
(C) kite (D) scalene quadrilateral

8. Triangles on the same base and between the same parallels are:

(A) similar (B) equal in area
(C) congruent (D) unequal

9. The diagonal of a parallelogram divides it into:

(A) two equal triangles (B) two unequal triangles
(C) two squares (D) two rectangles

10. Opposite angles of a parallelogram are:

(A) supplementary (B) complementary
(C) equal (D) right angles

Answer Key: 1–B, 2–B, 3–B, 4–B, 5–A, 6–B, 7–A, 8–B, 9–A, 10–C

Chapter 12: Line Bisectors and Angle Bisectors

Study Notes

- The perpendicular bisector of a line segment is the line perpendicular to the segment at its midpoint.
- Every point on the perpendicular bisector of a segment is equidistant from the ends of the segment.
- The angle bisector of an angle is the ray that divides the angle into two equal angles.
- Every point on the bisector of an angle is equidistant from the two sides of the angle.
- The three perpendicular bisectors of the sides of a triangle meet at one point, the circumcentre, which is equidistant from the three vertices.
- The three angle bisectors of a triangle meet at one point, the incentre, which is equidistant from the three sides.

Key Formulae

Perpendicular bisector: perpendicular to the segment at its midpoint.

Circumcentre: intersection of the perpendicular bisectors of the sides.

Incentre: intersection of the angle bisectors.

Worked Example

Prove that the right bisectors of the sides of a triangle are concurrent. Let the perpendicular bisectors of AB and AC meet at O. O is equidistant from A and B (on the bisector of AB) and from A and C (on the bisector of AC), so OB = OC. Hence O is also on the perpendicular bisector of BC. All three bisectors pass through O.

Short Questions with Answers — Chapter 12

Q1. What is a perpendicular bisector?
Answer: A line perpendicular to a segment at its midpoint.

Q2. What is true of a point on the perpendicular bisector of AB?
Answer: It is equidistant from A and B.

Q3. What is an angle bisector?
Answer: A ray that divides an angle into two equal angles.

Q4. Where do the three angle bisectors of a triangle meet?
Answer: At the incentre.

Q5. What is the circumcentre of a triangle?
Answer: The point where the perpendicular bisectors of the sides meet; it is equidistant from the vertices.

Long Questions with Answers — Chapter 12

Q1. Prove that any point on the bisector of an angle is equidistant from the sides of the angle.
Answer: Let P be on the bisector of ∠AOB, with perpendiculars PM to OA and PN to OB. Triangles OMP and ONP are congruent by AAS (∠MOP = ∠NOP, OP common, right angles), so PM = PN.

Q2. Prove that the perpendicular bisectors of the sides of a triangle are concurrent.
Answer: (See the worked example: the meeting point of two bisectors is equidistant from all three vertices, so it lies on the third bisector too.)

Multiple Choice Questions — Chapter 12

1. The perpendicular bisector of a segment is:

(A) parallel to it
(B) perpendicular at its midpoint
(C) at 45° to it
(D) through one end

2. A point on the perpendicular bisector of AB is:

(A) closer to A
(B) closer to B
(C) equidistant from A and B
(D) at the midpoint

3. The angle bisector divides an angle into:

(A) two unequal parts
(B) two equal parts
(C) three parts
(D) four parts

4. The circumcentre is equidistant from the:

(A) sides
(B) vertices
(C) midpoints
(D) angles

5. The incentre is equidistant from the:

(A) vertices
(B) sides
(C) midpoints
(D) bisectors

6. The three perpendicular bisectors of a triangle meet at:

(A) the incentre
(B) the circumcentre
(C) the centroid
(D) a vertex

7. The three angle bisectors of a triangle meet at:

(A) the circumcentre
(B) the incentre
(C) the orthocentre
(D) a vertex

8. A point on an angle bisector is equidistant from the:

(A) vertices
(B) sides of the angle
(C) opposite side
(D) endpoints

9. The perpendicular bisector of a chord of a circle passes through the:

(A) centre
(B) circumference
(C) tangent
(D) radius

10. The bisector of a 60° angle makes two angles of:

(A) 60°
(B) 30°
(C) 90°
(D) 120°

Answer Key: 1–B, 2–C, 3–B, 4–B, 5–B, 6–B, 7–B, 8–B, 9–A, 10–B

Chapter 13: Sides and Angles of a Triangle

Study Notes

- In any triangle, the side opposite the greater angle is longer, and the angle opposite the longer side is greater.
- The sum of the lengths of any two sides of a triangle is greater than the length of the third side (triangle inequality).
- The exterior angle of a triangle equals the sum of the two opposite interior angles.
- The angles of a triangle add up to 180°.
- If two sides of a triangle are equal, the angles opposite them are equal (isosceles triangle).
- In a right-angled triangle, the hypotenuse is the longest side.

Key Formulae

Angles of a triangle sum to 180°.

Exterior angle = sum of the two opposite interior angles.

Longer side $\Leftrightarrow$ greater opposite angle.

Worked Example

Prove that the sum of the lengths of any two sides of a triangle is greater than the third side. In triangle ABC, extend BA to D so that AD = AC. Then ∠ACD = ∠ADC (isosceles triangle). In triangle BCD, ∠BCD > ∠BDC, so the side opposite ∠BCD (BD) is longer than the side opposite ∠BDC (BC): BD > BC. But BD = BA + AC, so BA + AC > BC.

Short Questions with Answers — Chapter 13

Q1. What is the triangle inequality?
Answer: The sum of any two sides of a triangle is greater than the third side.

Q2. Which side is longest in a right-angled triangle?
Answer: The hypotenuse.

Q3. What is the sum of the angles of a triangle?
Answer: 180°.

Q4. If ∠A > ∠B in a triangle, which side is longer?
Answer: The side opposite ∠A (BC) is longer than the side opposite ∠B (AC).

Q5. State the exterior angle theorem.
Answer: An exterior angle of a triangle equals the sum of the two opposite interior angles.

Long Questions with Answers — Chapter 13

Q1. Can a triangle have sides 3 cm, 4 cm and 8 cm? Give a reason.
Answer: No: 3 + 4 = 7 < 8, which breaks the triangle inequality.

Q2. Prove that the angles opposite equal sides of a triangle are equal. (Outline)
Answer: Draw the bisector of the vertex angle; the two triangles formed are congruent by SAS, so the base angles are equal.

Multiple Choice Questions — Chapter 13

1. The sum of two sides of a triangle is:

(A) equal to the third side
(B) greater than the third side
(C) less than the third side
(D) equal to the hypotenuse

2. In a triangle, the greater angle is opposite the:

(A) shorter side
(B) longer side
(C) equal side
(D) hypotenuse

3. An exterior angle of a triangle equals:

(A) the adjacent interior angle
(B) the sum of the two opposite interior angles
(C) 90°
(D) 180°

4. The angles of a triangle add up to:

(A) 90°
(B) 180°
(C) 270°
(D) 360°

5. A triangle with sides 5 cm, 6 cm and 11 cm:

(A) is possible
(B) is impossible
(C) is right-angled
(D) is equilateral

6. The hypotenuse is the:

(A) shortest side
(B) longest side
(C) equal to a leg
(D) opposite the smallest angle

7. In an isosceles triangle, the base angles are:

(A) unequal
(B) equal
(C) right angles
(D) supplementary

8. If two angles of a triangle are 50° and 60°, the third is:

(A) 70°
(B) 60°
(C) 80°
(D) 90°

9. The exterior angle at a vertex and the adjacent interior angle are:

(A) equal
(B) complementary
(C) supplementary
(D) vertical

10. The side opposite the smallest angle is the:

(A) longest
(B) shortest
(C) hypotenuse
(D) median

Answer Key: 1–B, 2–B, 3–B, 4–B, 5–B, 6–B, 7–B, 8–A, 9–C, 10–B

Chapter 14: Ratio and Proportion

Study Notes

- A ratio compares two quantities of the same kind, e.g. a : b. A proportion is an equality of two ratios, a : b = c : d, written a/b = c/d.
- In a proportion a : b = c : d, a and d are the extremes and b and c are the means. The product of the extremes equals the product of the means: ad = bc.
- Two triangles are similar if their corresponding angles are equal and their corresponding sides are in the same ratio.
- The Basic Proportionality Theorem (Thales' theorem): a line parallel to one side of a triangle divides the other two sides proportionally.
- Conversely, if a line divides two sides of a triangle proportionally, it is parallel to the third side.
- Similar triangles have equal corresponding angles; the ratio of their areas equals the square of the ratio of corresponding sides.

Key Formulae

a : b = c : d ⇒ ad = bc.

Basic Proportionality Theorem: DE ∥ BC ⇒ AD/DB = AE/EC.

Similar triangles: corresponding angles equal, corresponding sides proportional.

Worked Example

Prove that the measures of the corresponding sides of two similar triangles are proportional. In triangles ABC and DEF with ∠A = ∠D, ∠B = ∠E, ∠C = ∠F, place DEF on ABC so that D falls on A and DE lies along AB. Then EF is parallel to BC (corresponding angles equal), so by the Basic Proportionality Theorem, AB/DE = AC/DF, and similarly AB/DE = BC/EF. Hence the corresponding sides are proportional.

Short Questions with Answers — Chapter 14

Q1. What is a proportion?
Answer: An equality of two ratios, a : b = c : d.

Q2. In a : b = c : d, which are the means?
Answer: b and c.

Q3. State the Basic Proportionality Theorem.
Answer: A line parallel to one side of a triangle divides the other two sides in the same ratio.

Q4. When are two triangles similar?
Answer: When their corresponding angles are equal and their corresponding sides are proportional.

Q5. If x/3 = 4/6, find x.
Answer: x = 2.

Long Questions with Answers — Chapter 14

Q1. A line DE is drawn parallel to BC in triangle ABC, with AD = 2 cm, DB = 3 cm and AE = 4 cm. Find EC.

Answer: By the Basic Proportionality Theorem, AD/DB = AE/EC ⇒ 2/3 = 4/EC ⇒ EC = 6 cm.

Q2. Prove that if a line divides two sides of a triangle proportionally, it is parallel to the third side. (Outline)

Answer: Suppose DE divides AB and AC so that AD/DB = AE/EC. If DE were not parallel to BC, a line through D parallel to BC would cut AC at a different point E′, giving AD/DB = AE′/EC, so E′ = E. Hence DE ∥ BC.

Multiple Choice Questions — Chapter 14

1. In the proportion 2 : 3 = 4 : 6, the product of the extremes is:

(A) 6 (B) 8
(C) 12 (D) 24

2. If a/b = c/d, then ad equals:

(A) bc (B) bd
(C) ac (D) cd

3. A line parallel to one side of a triangle divides the other two sides:

(A) equally (B) proportionally
(C) unequally (D) at right angles

4. Similar triangles have:

(A) equal corresponding angles (B) equal sides
(C) equal area (D) equal perimeter

5. If x/5 = 6/15, then x equals:

(A) 2 (B) 3
(C) 4 (D) 5

6. The ratio of the areas of two similar triangles with side ratio 2 : 3 is:

(A) 2 : 3 (B) 4 : 9
(C) 3 : 2 (D) 9 : 4

7. Thales' theorem is also called the:

(A) Pythagoras theorem (B) Basic Proportionality Theorem
(C) exterior angle theorem (D) midpoint theorem

8. If DE ∥ BC in triangle ABC, then AD/DB equals:

(A) AE/EC (B) AE/AC
(C) DB/AB (D) EC/AE

9. Two equilateral triangles are always:

(A) congruent (B) similar
(C) equal in area (D) right-angled

10. The ratio 12 : 18 in lowest terms is:

(A) 2 : 3 (B) 6 : 9
(C) 3 : 2 (D) 1 : 6

Answer Key: 1–C, 2–A, 3–B, 4–A, 5–A, 6–B, 7–B, 8–A, 9–B, 10–A

Chapter 15: Pythagoras Theorem

Study Notes

- Pythagoras theorem: in a right-angled triangle, the square of the hypotenuse equals the sum of the squares of the other two sides: $c^2 = a^2 + b^2$.
- The converse is also true: if $c^2 = a^2 + b^2$ in a triangle, the triangle is right-angled, with the right angle opposite the longest side.
- Pythagorean triples are sets of whole numbers that satisfy the theorem, e.g. (3, 4, 5), (5, 12, 13), (8, 15, 17).
- The theorem is used to find an unknown side of a right-angled triangle, to check whether a triangle is right-angled, and to find distances.
- The altitude of an equilateral triangle of side a is $(\sqrt{3}/2)a$, found by applying Pythagoras to half the triangle.
- The diagonal of a rectangle of sides l and b is $\sqrt{(l^2 + b^2)}$.

Key Formulae

$c^2 = a^2 + b^2$ (right-angled triangle, hypotenuse c).

Altitude of equilateral triangle of side a = $(\sqrt{3}/2)a$.

Diagonal of a rectangle = $\sqrt{(\text{length}^2 + \text{breadth}^2)}$.

Worked Example

The length of a side of an equilateral triangle is 10 cm. Find the length of the altitude. The altitude divides the triangle into two right-angled triangles with hypotenuse 10 cm and base 5 cm. Altitude$^2 = 10^2 - 5^2 = 100 - 25 = 75$, so the altitude = $\sqrt{75} = 5\sqrt{3} \approx 8.66$ cm.

Short Questions with Answers — Chapter 15

Q1. State Pythagoras theorem.
Answer: In a right-angled triangle, the square of the hypotenuse equals the sum of the squares of the other two sides.

Q2. Find the hypotenuse of a right triangle with legs 6 cm and 8 cm.
Answer: 10 cm.

Q3. Is a triangle with sides 5, 12, 13 right-angled?
Answer: Yes, because $5^2 + 12^2 = 25 + 144 = 169 = 13^2$.

Q4. Name a Pythagorean triple.
Answer: (3, 4, 5).

Q5. What is the diagonal of a square of side 1 unit?
Answer: $\sqrt{2}$ units.

Long Questions with Answers — Chapter 15

Q1. Prove Pythagoras theorem. (Outline)
Answer: Draw the altitude from the right angle to the hypotenuse. It creates two triangles similar to the whole triangle. From the similarity ratios, $a^2 = c \times$ (projection of a) and $b^2 = c \times$ (projection of b); adding gives $a^2 + b^2 = c \times$ (sum of projections) $= c^2$.

Q2. Find the length of the diagonal of a rectangle 12 cm by 5 cm.
Answer: Diagonal $= \sqrt{(12^2 + 5^2)} = \sqrt{(144 + 25)} = \sqrt{169} = 13$ cm.

Multiple Choice Questions — Chapter 15

1. In a right-angled triangle, the hypotenuse is opposite the:

(A) smallest angle (B) right angle
(C) acute angle (D) longest leg

2. If the legs are 9 cm and 12 cm, the hypotenuse is:

(A) 15 cm (B) 21 cm
(C) 10 cm (D) 18 cm

3. Which triple is Pythagorean?

(A) (2, 3, 4) (B) (3, 4, 5)
(C) (5, 6, 7) (D) (4, 5, 6)

4. The altitude of an equilateral triangle of side 10 cm is:

(A) 5 cm (B) $5\sqrt{3}$ cm
(C) $10\sqrt{3}$ cm (D) $\sqrt{3}$ cm

5. If $c^2 = a^2 + b^2$, the triangle is:

(A) acute (B) obtuse
(C) right-angled (D) equilateral

6. The diagonal of a rectangle 3 cm by 4 cm is:

(A) 5 cm (B) 7 cm
(C) 6 cm (D) 12 cm

7. The square of the hypotenuse equals:

(A) the sum of the squares of the other two sides (B) the difference of the squares
(C) the product of the legs (D) twice the area

8. A ladder 5 m long leans with its foot 3 m from a wall. Its top reaches:

(A) 4 m (B) 2 m
(C) 8 m (D) 3 m

9. In a right triangle with hypotenuse 13 and one leg 5, the other leg is:

(A) 8 (B) 12
(C) 10 (D) 7

10. The converse of Pythagoras theorem is used to:

(A) find an angle (B) test if a triangle is right-angled
(C) find the area (D) draw a circle

Answer Key: 1–B, 2–A, 3–B, 4–B, 5–C, 6–A, 7–A, 8–A, 9–B, 10–B

Chapter 16: Theorems Related with Area

Study Notes

- The area of a parallelogram is base × height, where height is the perpendicular distance between the base and the opposite side.
- The area of a triangle is (1/2) × base × height.
- Parallelograms on the same base and between the same parallels are equal in area.
- Triangles on the same base and between the same parallels are equal in area.
- A triangle on the same base and between the same parallels as a parallelogram has half the area of the parallelogram.
- These theorems let us compare areas without measuring: figures between the same parallels on the same base have equal areas.

Key Formulae

Area of parallelogram = base × height.

Area of triangle = (1/2) × base × height.

Triangle on same base and parallels as a parallelogram = half its area.

Worked Example

Prove that the triangles on the same base and of the same altitude are equal in area. Triangles ABC and DBC stand on the same base BC and between the same parallels BC and AD. Area of ABC = (1/2) × BC × (altitude), and area of DBC = (1/2) × BC × (same altitude). Hence the areas are equal.

Short Questions with Answers — Chapter 16

Q1. What is the area of a triangle with base 8 cm and height 5 cm?
Answer: 20 cm².

Q2. What is the area of a parallelogram with base 7 cm and height 3 cm?
Answer: 21 cm².

Q3. State the theorem about parallelograms on the same base and between the same parallels.
Answer: They are equal in area.

Q4. A triangle and a parallelogram share the same base and height. How do their areas compare?
Answer: The triangle has half the area of the parallelogram.

Q5. What is the height of a parallelogram?
Answer: The perpendicular distance between the base and the opposite side.

Long Questions with Answers — Chapter 16

Q1. Prove that parallelograms on the same base and between the same parallels are equal in area.

Answer: Let ABCD and ABEF be parallelograms on base AB between parallels AB and FC. Triangles ADF and BCE are congruent (AD = BC, AF = BE, ∠DAF = ∠CBE), so area(ABED) + area(ADF) = area(ABED) + area(BCE), giving area(ABCD) = area(ABEF).

Q2. Two triangles stand on the same base 10 cm between the same parallels, one with height 6 cm. Find the area of each.

Answer: Each has area (1/2) × 10 × 6 = 30 cm², and the areas are equal.

Multiple Choice Questions — Chapter 16

1. The area of a triangle with base 10 cm and height 4 cm is:

(A) 40 cm² (B) 20 cm²
(C) 14 cm² (D) 30 cm²

2. Parallelograms on the same base and between the same parallels are:

(A) equal in area (B) equal in perimeter
(C) congruent (D) similar

3. A triangle on the same base and between the same parallels as a parallelogram has:

(A) the same area (B) half the area
(C) twice the area (D) one-third the area

4. The area of a parallelogram is:

(A) base + height (B) base × height
(C) (1/2) × base × height (D) base ÷ height

5. Triangles on the same base and of the same altitude are:

(A) equal in area (B) equal in perimeter
(C) congruent (D) similar

6. If the area of a parallelogram is 48 cm² and its base is 8 cm, its height is:

(A) 6 cm (B) 8 cm
(C) 4 cm (D) 12 cm

7. The area of a triangle is half the area of a parallelogram on the:

(A) same base and same parallels (B) different base
(C) same base only (D) same height only

8. Two triangles on the same base and between the same parallels have:

(A) equal areas (B) equal perimeters
(C) equal heights only (D) equal bases only

9. The area of a triangle with base 6 cm and height 9 cm is:

(A) 27 cm² (B) 54 cm²
(C) 15 cm² (D) 45 cm²

10. The height of a parallelogram is measured:

(A) along the side (B) perpendicular to the base
(C) along the diagonal (D) along the base

Answer Key: 1–B, 2–A, 3–B, 4–B, 5–A, 6–A, 7–A, 8–A, 9–A, 10–B

Chapter 17: Practical Geometry — Triangles

Study Notes

- Practical geometry is the construction of figures accurately with a ruler, compass and protractor.
- To construct a triangle, three independent measurements are needed, e.g. three sides (SSS), two sides and the included angle (SAS), or two angles and the included side (ASA).
- Basic constructions: bisecting a line segment, bisecting an angle, drawing a perpendicular at a point, and copying an angle.
- To construct a triangle with given sides, draw the base, then draw arcs from each end with radii equal to the other two sides; their intersection is the third vertex.
- To construct a triangle given two angles and the included side, draw the side and then draw the two angles at its ends; the rays meet at the third vertex.
- Always label the constructed triangle, write the given measurements, and check the construction by measuring the result.

Key Formulae

SSS: three sides given.

SAS: two sides and the included angle given.

ASA: two angles and the included side given.

Worked Example

Construct a triangle ABC in which mAB = 5 cm, m∠A = 60° and m∠B = 45°. Draw AB = 5 cm. At A, draw a ray making 60° with AB. At B, draw a ray making 45° with BA. The two rays meet at C. Then ∠C = 180° − 60° − 45° = 75°.

Short Questions with Answers — Chapter 17

Q1. Which instruments are used in practical geometry?
Answer: Ruler, compass and protractor.

Q2. What three measurements are needed to construct a triangle by SSS?
Answer: The lengths of all three sides.

Q3. How is a triangle constructed by SAS?
Answer: Draw one side, draw the included angle at one end, mark the second side on the ray, and join the endpoints.

Q4. What is the third angle of a triangle with angles 60° and 45°?
Answer: 75°.

Q5. How do you bisect a line segment?
Answer: Draw arcs of equal radius greater than half the segment from each end; the line joining the arc intersections is the perpendicular bisector.

Long Questions with Answers — Chapter 17

Q1. Describe how to construct a triangle with sides 6 cm, 7 cm and 8 cm.
Answer: Draw the base of 8 cm. From one end draw an arc of radius 6 cm, and from the other end an arc of radius 7 cm. Their intersection is the third vertex; join it to both ends.

Q2. Construct a right-angled triangle with hypotenuse 10 cm and one side 6 cm. Find the third side.
Answer: Draw the side of 6 cm, erect a perpendicular at one end, and draw an arc of radius 10 cm from the other end to meet the perpendicular. The third side = $\sqrt{(10^2 - 6^2)} = 8$ cm.

Multiple Choice Questions — Chapter 17

1. To construct a triangle by SSS you need:

(A) three angles
(B) three sides
(C) two sides and an angle
(D) one side and two angles

2. In the construction with AB = 5 cm, ∠A = 60°, ∠B = 45°, ∠C equals:

(A) 60°
(B) 75°
(C) 45°
(D) 90°

3. A triangle can be constructed from:

(A) two sides only
(B) three sides
(C) one angle only
(D) one side only

4. The perpendicular bisector of a segment is drawn with:

(A) a protractor only
(B) a ruler and compass
(C) a set square only
(D) freehand

5. To copy an angle, you use a:

(A) protractor or compass
(B) ruler only
(C) set square
(D) divider only

6. SAS construction needs:

(A) two sides and the included angle
(B) two angles and a side
(C) three sides
(D) three angles

7. The arcs in an SSS construction are drawn with radii equal to:

(A) the base
(B) the other two sides
(C) half the base
(D) the angles

8. A triangle with angles 90°, 60° and 30° can be constructed from:

(A) SSS only
(B) ASA or AAS
(C) two sides only
(D) one side only

9. In practical geometry, the third vertex of an SSS triangle is the:

(A) midpoint of the base
(B) intersection of two arcs
(C) end of the base
(D) centre of a circle

10. Before constructing, you should first draw a:

(A) rough sketch
(B) circle
(C) square
(D) graph

Answer Key: 1–B, 2–B, 3–B, 4–B, 5–A, 6–A, 7–B, 8–B, 9–B, 10–A

Revision Checklist

- Matrices: order, types, multiplication rule, determinant, adjoint and inverse of a 2 × 2 matrix.
- Real and complex numbers: surds, laws of exponents, scientific notation, conjugate and modulus.
- Logarithms: the three laws, log 1 = 0, and solving simple logarithmic equations.
- Algebra: identities, factorization (including remainder and factor theorems), HCF and LCM of expressions.
- Equations: linear equations, simultaneous equations, inequalities and their number-line graphs.
- Graphs and coordinates: gradient, y = mx + c, distance and midpoint formulas.
- Geometry: congruence conditions, parallelogram properties, bisectors, triangle inequalities, ratio and proportion, Pythagoras theorem, area theorems, and triangle constructions.

Exam Tips

- Attempt the 15 MCQs first and quickly; do not leave any blank.
- In Section B, choose the nine short questions you are most sure about.
- In Section C, show every step of a proof or construction — method marks are given for correct steps.
- Always check a solution by putting it back into the original equation.
- Revise the formulae on the first page of each chapter the night before the paper.

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