ZAHID NOTES | FREE STUDY MATERIAL FOR STUDENTS

9th Class Mathematics Notes Sindh Board Full Book Notes

Complete chapter-wise notes, worked examples and original practice questions for all 16 chapters of the Sindh Textbook Board Mathematics book for Class 9 (English Medium).

Boards covered: Karachi, Hyderabad, Sukkur, Larkana and Mirpurkhas
(all Sindh boards use the Sindh Textbook Board book)

Class: 9th Class | **Subject:** Mathematics | **Book:** Sindh Textbook Board (STBB), Jamshoro

Medium: English

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How to Use These Notes

These notes follow the 16 chapters of the Sindh Textbook Board Mathematics book for Class 9. Each chapter has short study notes, the key formulas, two worked examples, five short questions with answers, two long questions with answers, and ten multiple-choice questions with an answer key. Work through a chapter, cover the answers, and test yourself before moving on.

All questions in this book are original practice questions written for these notes. They are not past-paper questions and they are not copied from the textbook or from any other notes.

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> **About the chapter list.** This book follows the Sindh Textbook Board Mathematics book for Class 9, 2024 edition, which has 16 chapters. That list is confirmed by the STBB eBooks portal (which lists Math IX, 2024-25, English medium), by the chapter tables published for the Sindh board, and by an independent solution manual written for the same 2024 STBB edition. Two other lists circulate online and were **not** used: an older 17-chapter list that starts with Matrices and Determinants and includes Ratio and Proportion and Pythagoras Theorem as separate chapters, and a 13-unit list (Real Numbers, Sets and Functions, Trigonometry, Logic, Probability and others) that belongs to the Punjab new-curriculum book, not to the Sindh Textbook Board book. If your school uses a different edition, check its contents page against the list above.

Paper Pattern — Read Carefully

Mathematics, Class 9, Sindh boards (Science Group), total 75 marks, 3 hours. This pattern is **third-party reproduced, not officially confirmed**: it is taken from Board of Secondary Education Karachi model papers for 2023 and 2024 as reproduced by third-party sites, not read directly from an official BSEK document. The most recent model papers agree on this split:

Section	What it is	Marks
Section A	Multiple-choice questions — 15 MCQs from the whole book, 1 mark each	15
Section B	Short-answer questions — attempt any 6 out of 10 questions	30
Section C	Descriptive (long) questions — attempt any 3 out of 5 questions	30
Total	**Written paper (no practical for Mathematics)**	**75**

Note: an older 2022 BSEK model paper used a different split for the same 75 total marks (30 MCQs, 30 short-answer, 15 descriptive). Always confirm the current pattern from your board's latest model paper before the examination.

How to Revise for the Examination

- Learn one chapter at a time: read the notes, copy the formulas by hand, then do the worked examples yourself.
- Attempt all 10 MCQs of a chapter without looking, then check the key. Re-do any you missed the next day.
- Practise the short and long questions in writing, with full steps — examiners give marks for method, not just the final answer.
- For geometry chapters (9 to 15), draw every diagram with ruler and compasses and write the reason for each step.
- In the last week, solve one full paper in 3 hours: 15 MCQs, 6 short questions and 3 long questions.

Chapter 1: Real and Complex Numbers

Study Notes

- Natural numbers are 1, 2, 3, ... ; whole numbers add 0; integers are ..., -2, -1, 0, 1, 2, ... A rational number can be written as p/q, where q is not zero; its decimal is terminating or recurring.
- An irrational number cannot be written as p/q; its decimal is non-terminating and non-recurring. Examples are √2, √3 and π. Real numbers are all rational and irrational numbers together.
- On the number line every real number has exactly one point, and every point stands for one real number. Between any two real numbers there are infinitely many rational and irrational numbers.
- A radical (surd) is an expression such as √5 or ³√7. Like radicals can be added; unlike radicals cannot be combined into one term. √a × √b = √(ab) and √a ÷ √b = √(a/b) for positive a, b.
- Laws of exponents: a^m × a^n = a^(m+n); a^m ÷ a^n = a^(m-n); (a^m)^n = a^(mn); (ab)^n = a^n b^n; a^0 = 1 (a ≠ 0); a^(-n) = 1/a^n.
- A complex number is z = a + ib, where a is the real part, b is the imaginary part and $i^2 = -1$. So i = √(-1). A real number is a complex number with b = 0.
- The conjugate of a + ib is a - ib. The modulus is |a + ib| = √(a² + b²). Multiplying a complex number by its conjugate gives a real number: (a + ib)(a - ib) = a² + b².

Key Formulas

> $i^2 = -1$ | (a + ib) + (c + id) = (a + c) + i(b + d) | (a + ib)(a - ib) = a² + b² | √a × √b = √(ab)

Worked Examples

Example 1 — Simplify √50.

√50 = √(25 × 2) = √25 × √2 = 5√2.

Example 2 — Add and multiply complex numbers.

(2 + 3i) + (1 - 2i) = 3 + i. (2 + 3i)(1 - 2i) = 2 - 4i + 3i - 6i² = 2 - i + 6 = 8 - i.

Chapter 1: Real and Complex Numbers — Practice Questions

Short Questions

Q1. Define a rational number and give two examples.

Answer: A number that can be written as p/q, with p and q integers and q ≠ 0. Examples: 3/4 and -5/7. Its decimal expansion terminates or repeats.

Q2. Why is √2 an irrational number?

Answer: Because √2 cannot be written as a ratio of two integers; its decimal expansion 1.414213... never ends and never repeats.

Q3. Write the conjugate and modulus of 3 - 4i.

Answer: Conjugate: 3 + 4i. Modulus: |3 - 4i| = √(9 + 16) = √25 = 5.

Q4. Simplify $2^3 \times 2^4$.

Answer: $2^3 \times 2^4 = 2^{(3+4)} = 2^7 = 128$.

Q5. Simplify √72.

Answer: √72 = √(36 × 2) = 6√2.

Long Questions

Q1. Explain the classification of real numbers with examples, and show where rational and irrational numbers sit in the system.

Answer: Real numbers split into rational and irrational numbers. Rational numbers include natural numbers (1, 2, 3, ...), whole numbers (0, 1, 2, ...), integers (..., -1, 0, 1, ...) and fractions such as 1/2, -3/4 and 0.25. Irrational numbers, such as √2, √5 and π, cannot be written as p/q. Every rational number has a terminating or recurring decimal; every irrational number has a non-terminating, non-recurring decimal. Together they fill the whole number line with no gaps.

Q2. Define a complex number. Find (3 + 2i) - (1 - 5i) and (1 + i)(2 - i), showing each step.

Answer: A complex number is a + ib, where a is the real part, b the imaginary part and $i^2 = -1$. Subtraction: (3 + 2i) - (1 - 5i) = (3 - 1) + (2 + 5)i = 2 + 7i. Multiplication: (1 + i)(2 - i) = 2 - i + 2i - i^2 = 2 + i + 1 = 3 + i, since $-i^2 = +1$.

Multiple-Choice Questions

Q1. Which of the following is an irrational number?

(a) 0.25 (b) 22/7
(c) √9 (d) √2

Q2. The value of i^2 is:

(a) 1 (b) -1
(c) 0 (d) i

Q3. The conjugate of 5 + 2i is:

(a) 5 - 2i (b) -5 + 2i
(c) 5 + 2i (d) -5 - 2i

Q4. Which number is rational?

(a) π (b) $\sqrt{3}$
(c) 0.333... (d) $\sqrt{7}$

Q5. Simplify $\sqrt{18}$.

(a) $3\sqrt{2}$ (b) $2\sqrt{3}$
(c) $9\sqrt{2}$ (d) $6\sqrt{3}$

Q6. $2^5 \times 2^3$ equals:

(a) 2^8 (b) 2^{15}
(c) 4^8 (d) 2^2

Q7. The real part of $4 - 7i$ is:

(a) -7 (b) 7
(c) 4 (d) 4i

Q8. $(3 + i)(3 - i)$ equals:

(a) 8 (b) 9
(c) 10 (d) 9 - i

Q9. Which set contains 0?

(a) Natural numbers (b) Whole numbers
(c) Both natural and whole (d) Neither

Q10. The modulus of $3 + 4i$ is:

(a) 7 (b) 5
(c) 25 (d) 1

Answer Key (Chapter 1): 1(D), 2(B), 3(A), 4(C), 5(A), 6(A), 7(C), 8(C), 9(B), 10(B)

Chapter 2: Logarithms

Study Notes

- If a^x = y (a > 0, a ≠ 1), then x = log_a y. We say 'x is the logarithm of y to the base a'. So log_2 32 = 5 because 2^5 = 32.
- A common logarithm uses base 10 and is written log y. A natural logarithm uses base e and is written ln y. In school work, log normally means base 10.
- Laws of logarithms: log(mn) = log m + log n; log(m/n) = log m - log n; log(m^n) = n log m; log 1 = 0; log_a a = 1.
- Scientific notation writes a number as a × 10^n with 1 ≤ a < 10. Example: 45,000 = 4.5 × 10^4. Logarithms turn multiplication into addition, which is why they were used for hand calculation.
- The characteristic is the whole-number part of a common logarithm and the mantissa is the decimal part, always positive. For log 25.6 = 1.4082, the characteristic is 1 and the mantissa is 0.4082.
- Antilog is the reverse of log: if log x = 1.4082, then x = antilog(1.4082) = 25.6. Log tables give mantissas; the characteristic is found from the position of the decimal point.
- Logarithms are used for large-number multiplication, division, powers and roots, and in science for scales such as pH and earthquake magnitude.

Key Formulas

> log_a (mn) = log_a m + log_a n | log_a (m/n) = log_a m - log_a n | log_a (m^n) = n log_a m | log_a 1 = 0

Worked Examples

Example 1 — Find log_3 81.

3^4 = 81, so log_3 81 = 4.

Example 2 — Use the laws.

log 1000 = log 10^3 = 3 log 10 = 3 × 1 = 3. Also log 20 - log 2 = log(20/2) = log 10 = 1.

Chapter 2: Logarithms — Practice Questions

Short Questions

Q1. Define a logarithm.

Answer: If a^x = y, then x = log_a y. The logarithm is the power to which the base must be raised to get the number.

Q2. Find log_5 125.

Answer: $5^3 = 125$, so log_5 125 = 3.

Q3. State the three laws of logarithms.

Answer: Product law: log(mn) = log m + log n. Quotient law: log(m/n) = log m - log n. Power law: log(m^n) = n log m.

Q4. Write 0.00036 in scientific notation.

Answer: 3.6 × 10^(-4).

Q5. What are the characteristic and mantissa of log 456 = 2.6590?

Answer: Characteristic 2, mantissa 0.6590.

Long Questions

Q1. Prove the product law of logarithms and use it to find log 24 from log 2 = 0.3010 and log 3 = 0.4771.

Answer: Let log_a m = x and log_a n = y, so m = a^x and n = a^y. Then mn = a^(x+y), so log_a (mn) = x + y = log_a m + log_a n. Now $24 = 2^3 \times 3$, so log 24 = 3 log 2 + log 3 = 3(0.3010) + 0.4771 = 0.9030 + 0.4771 = 1.3801.

Q2. Explain characteristic and mantissa. Find the characteristic of log 7.2, log 72, log 720 and log 0.072.

Answer: The characteristic is the integer part of a common logarithm; the mantissa is the positive decimal part. It depends only on the position of the decimal point. log 7.2 has characteristic 0, log 72 has 1, log 720 has 2, and log 0.072 has -2 (written as 2 with a bar). The mantissa is the same for all four numbers.

Multiple-Choice Questions

Q1. log_2 16 equals:

(a) 2 (b) 4
(c) 8 (d) 16

Q2. log 100 equals:

(a) 1 (b) 2
(c) 10 (d) 100

Q3. log_a 1 equals:

(a) 0 (b) 1
(c) a (d) -1

Q4. log 5 + log 2 equals:

(a) log 7 (b) log 10
(c) log 3 (d) 1

Q5. The characteristic of log 345 is:

(a) 2 (b) 3
(c) 1 (d) 0

Q6. If log x = 2, then x equals:

(a) 20 (b) 200
(c) 100 (d) 2

Q7. log(m^n) equals:

(a) n + log m (b) n log m
(c) log m / n (d) log m - n

Q8. The mantissa is always:

(a) Negative (b) Zero
(c) Positive (or zero) (d) Greater than 1

Q9. log_10 10 equals:

(a) 0 (b) 1
(c) 10 (d) 100

Q10. Which is correct?

(a) log(m + n) = log m + log n (b) log(mn) = log m + log n
(c) log(m - n) = log m - log n (d) log(m/n) = log m + log n

Answer Key (Chapter 2): 1(B), 2(B), 3(A), 4(B), 5(A), 6(C), 7(B), 8(C), 9(B), 10(B)

Chapter 3: Algebraic Expressions and Formulas

Study Notes

- An algebraic expression is a combination of numbers, letters and operations, such as $3x^2 - 2x + 5$. A term is a part separated by + or -; the coefficient is the number multiplying the letter.
- A polynomial in one variable is an expression with whole-number powers of the variable. Its degree is the highest power of the variable. $4x^3 - x + 7$ has degree 3.
- Like terms have the same letters raised to the same powers and can be added or subtracted. Unlike terms cannot be combined: $3x + 4y$ stays as it is.
- Important identities: $(a + b)^2 = a^2 + 2ab + b^2$; $(a - b)^2 = a^2 - 2ab + b^2$; $a^2 - b^2 = (a + b)(a - b)$.
- Cube identities: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$; $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$; $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$; $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.
- To evaluate an expression, substitute the given values and follow the order of operations: brackets, powers, multiplication and division, then addition and subtraction.
- Surds in algebra follow the same rules as numbers: $\sqrt{a} \times \sqrt{a} = a$, and $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$.

Key Formulas

> $(a + b)^2 = a^2 + 2ab + b^2$ | $(a - b)^2 = a^2 - 2ab + b^2$ | $a^2 - b^2 = (a + b)(a - b)$

Worked Examples

Example 1 — Expand $(x + 5)^2$.

$(x + 5)^2 = x^2 + 2(x)(5) + 25 = x^2 + 10x + 25$.

Example 2 — Evaluate.

If $a = 3$ and $b = 2$, then $a^2 - b^2 = 9 - 4 = 5$. Check: $(a + b)(a - b) = 5 \times 1 = 5$.

Chapter 3: Algebraic Expressions and Formulas — Practice Questions

Short Questions

Q1. What is the degree of $5x^4 - 2x^2 + x - 9$?

Answer: The highest power of x is 4, so the degree is 4.

Q2. Expand $(2x - 3)^2$.

Answer: $(2x - 3)^2 = 4x^2 - 12x + 9$.

Q3. Write $a^2 - 49$ as a product.

Answer: $a^2 - 49 = (a + 7)(a - 7)$.

Q4. If $x = 2$, find the value of $x^3 - 3x + 1$.

Answer: $8 - 6 + 1 = 3$.

Q5. Name the coefficient of x^2 in $-7x^2 + 3x$.

Answer: -7.

Long Questions

Q1. Derive the identity $(a + b)^2 = a^2 + 2ab + b^2$ and use it to find 105^2 without a calculator.

Answer: $(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$. Now $105^2 = (100 + 5)^2 = 10000 + 2(100)(5) + 25 = 10000 + 1000 + 25 = 11025$.

Q2. If $a + b = 7$ and $ab = 10$, find $a^2 + b^2$ and $(a - b)^2$.

Answer: $a^2 + b^2 = (a + b)^2 - 2ab = 49 - 20 = 29$. $(a - b)^2 = (a + b)^2 - 4ab = 49 - 40 = 9$, so $|a - b| = 3$.

Multiple-Choice Questions

Q1. $(a + b)^2$ equals:

(a) $a^2 + b^2$
(b) $a^2 + 2ab + b^2$
(c) $a^2 - 2ab + b^2$
(d) $a^2 + ab + b^2$

Q2. The degree of $3x^2 + 5$ is:

(a) 5
(b) 3
(c) 2
(d) 1

Q3. $x^2 - 25$ factorises as:

(a) $(x - 5)(x - 5)$
(b) $(x + 5)(x + 5)$
(c) $(x + 5)(x - 5)$
(d) $(x + 25)(x - 1)$

Q4. Like terms of $4x$ are:

(a) $4y$
(b) $5x$
(c) $4x^2$
(d) x^3

Q5. $(a - b)^3$ equals:

(a) $a^3 - b^3$
(b) $a^3 - 3a^2b + 3ab^2 - b^3$
(c) $a^3 + 3a^2b + 3ab^2 + b^3$
(d) $a^3 - 3ab + b^3$

Q6. If $x = 3$, $x^2 + 2x$ equals:

(a) 11 (b) 15
(c) 9 (d) 17

Q7. $a^3 + b^3$ equals:

(a) $(a + b)(a^2 - ab + b^2)$ (b) $(a + b)(a^2 + ab + b^2)$
(c) $(a - b)(a^2 + ab + b^2)$ (d) $(a + b)^3$

Q8. The coefficient of y in 6y - 2 is:

(a) -2 (b) 2
(c) 6 (d) 1

Q9. Simplify 3x + 5x - 2x.

(a) 6x (b) 10x
(c) 5x (d) 0

Q10. (x + 1)(x - 1) equals:

(a) $x^2 + 1$ (b) $x^2 - 2x + 1$
(c) $x^2 - 1$ (d) $x^2 + 2x - 1$

Answer Key (Chapter 3): 1(B), 2(C), 3(C), 4(B), 5(B), 6(B), 7(A), 8(C), 9(A), 10(C)

Chapter 4: Factorization

Study Notes

- Factorization writes an expression as a product of factors. It is the reverse of expansion. Always look first for a common factor in all terms.
- Factorization by grouping: group terms with a common factor, factor each group, then factor out the common bracket. Example: $ax + ay + bx + by = (x + y)(a + b)$.
- Difference of two squares: $a^2 - b^2 = (a + b)(a - b)$. Perfect squares: $a^2 + 2ab + b^2 = (a + b)^2$ and $a^2 - 2ab + b^2 = (a - b)^2$.
- Factorization of a quadratic $x^2 + px + q$: find two numbers with sum p and product q. For $ax^2 + bx + c$, split the middle term so the two parts multiply to ac and add to b.
- Sum and difference of cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$; $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.
- Always check a factorization by expanding it back, and factor completely — a factor such as $x^2 - 4$ inside a bracket must be factored further.
- Factorization is used to solve quadratic equations (set each factor to zero), to simplify fractions and to find HCF and LCM of algebraic expressions.

Key Formulas

> $a^2 - b^2 = (a + b)(a - b)$ | $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ | $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Worked Examples

Example 1 — Factorize $x^2 + 7x + 12$.

Two numbers with sum 7 and product 12 are 3 and 4. So $x^2 + 7x + 12 = (x + 3)(x + 4)$.

Example 2 — Factorize $2x^2 - 8$.

Take out the common factor 2: $2(x^2 - 4) = 2(x + 2)(x - 2)$.

Chapter 4: Factorization — Practice Questions

Short Questions

Q1. Factorize $6x + 12$.

Answer: $6(x + 2)$.

Q2. Factorize $x^2 - 9$.

Answer: $(x + 3)(x - 3)$.

Q3. Factorize $x^2 - 5x + 6$.

Answer: Numbers with sum -5 and product 6 are -2 and -3, so $(x - 2)(x - 3)$.

Q4. Factorize $x^3 - 8$.

Answer: $x^3 - 2^3 = (x - 2)(x^2 + 2x + 4)$.

Q5. Factorize $3x^2 + 6x$.

Answer: $3x(x + 2)$.

Long Questions

Q1. Factorize $2x^2 + 7x + 3$ by splitting the middle term, and check the answer by expansion.

Answer: $ac = 2 \times 3 = 6$. Two numbers with product 6 and sum 7 are 6 and 1. So $2x^2 + 7x + 3 = 2x^2 + 6x + x + 3 = 2x(x + 3) + 1(x + 3) = (x + 3)(2x + 1)$. Check: $(x + 3)(2x + 1) = 2x^2 + x + 6x + 3 = 2x^2 + 7x + 3$. Correct.

Q2. Factorize $x^4 - 16$ completely.

Answer: $x^4 - 16 = (x^2)^2 - 4^2 = (x^2 + 4)(x^2 - 4) = (x^2 + 4)(x + 2)(x - 2)$. The factor $x^2 + 4$ cannot be factored further with real numbers.

Multiple-Choice Questions

Q1. The factors of $x^2 - 16$ are:

(a) $(x - 4)(x - 4)$ (b) $(x + 4)(x - 4)$
(c) $(x + 8)(x - 2)$ (d) $(x + 4)(x + 4)$

Q2. Factorize $x^2 + 5x + 6$.

(a) $(x + 1)(x + 6)$ (b) $(x + 2)(x + 3)$
(c) $(x - 2)(x - 3)$ (d) $(x + 5)(x + 1)$

Q3. The HCF of $6x$ and $9x^2$ is:

(a) $3x$ (b) $18x^2$
(c) $9x$ (d) $3x^2$

Q4. $x^3 + 27$ factorises as:

(a) $(x + 3)(x^2 - 3x + 9)$ (b) $(x - 3)(x^2 + 3x + 9)$
(c) $(x + 3)^3$ (d) $(x + 9)(x^2 - 3x + 3)$

Q5. Factorize $5x - 10y$.

(a) $5(x - 2y)$ (b) $5(x + 2y)$
(c) $10(x - y)$ (d) $5x(1 - 2y)$

Q6. The factors of $x^2 - 7x + 12$ are:

(a) $(x - 3)(x - 4)$ (b) $(x + 3)(x + 4)$
(c) $(x - 6)(x - 2)$ (d) $(x + 6)(x + 2)$

Q7. $a^2 + 2ab + b^2$ equals:

(a) $(a - b)^2$ (b) $(a + b)(a - b)$
(c) $(a + b)^2$ (d) $a^2 - b^2$

Q8. Factorize $4x^2 - 1$.

(a) $(2x - 1)(2x - 1)$ (b) $(2x + 1)(2x - 1)$
(c) $(4x + 1)(x - 1)$ (d) $(2x + 1)^2$

Q9. Which is a factor of $x^2 + 3x$?

(a) $x + 1$ (b) x
(c) $x - 3$ (d) 3

Q10. Factorize $x^2 - 10x + 25$.

(a) $(x - 5)^2$ (b) $(x + 5)^2$
(c) $(x - 5)(x + 5)$ (d) $(x - 25)(x + 1)$

Answer Key (Chapter 4): 1(B), 2(B), 3(A), 4(A), 5(A), 6(A), 7(C), 8(B), 9(B), 10(A)

Chapter 5: Algebraic Manipulation

Study Notes

- The HCF of algebraic expressions is the product of the common factors with the lowest powers. The LCM is the product of all factors with the highest powers.
- A rational expression is a fraction with algebraic numerator and denominator, such as (x + 1)/(x - 2). It is undefined where the denominator is zero.
- To simplify a rational expression, factorize the numerator and denominator and cancel common factors. Never cancel single terms that are added or subtracted.
- To add or subtract rational expressions, use the LCM of the denominators as the common denominator, then combine the numerators.
- To multiply rational expressions, multiply numerators and denominators and cancel common factors. To divide, multiply by the reciprocal of the divisor.
- The square root of an algebraic expression: find the square root of a perfect-square trinomial, e.g. $\sqrt{(x^2 + 6x + 9)} = x + 3$ (for $x + 3 \geq 0$).
- Surd manipulation: rationalize a denominator by multiplying top and bottom by the conjugate, e.g. $1/(\sqrt{3} + 1)$ becomes $(\sqrt{3} - 1)/2$.

Key Formulas

> HCF × LCM = product of the two expressions | a/b ÷ c/d = a/b × d/c | $1/(\sqrt{a} + \sqrt{b}) = (\sqrt{a} - \sqrt{b})/(a - b)$

Worked Examples

Example 1 — Simplify $(x^2 - 9)/(x + 3)$.

$(x^2 - 9)/(x + 3) = (x + 3)(x - 3)/(x + 3) = x - 3$, provided $x \neq -3$.

Example 2 — Add $1/x + 1/(2x)$.

Common denominator $2x$: $1/x + 1/(2x) = 2/(2x) + 1/(2x) = 3/(2x)$.

Chapter 5: Algebraic Manipulation — Practice Questions

Short Questions

Q1. Find the HCF of $x^2 - 1$ and $x^2 + 2x + 1$.

Answer: $x^2 - 1 = (x + 1)(x - 1)$; $x^2 + 2x + 1 = (x + 1)^2$. HCF = $x + 1$.

Q2. Simplify $(2x^2)/(4x)$.

Answer: $(2x^2)/(4x) = x/2$.

Q3. For which value of x is $(x + 1)/(x - 2)$ undefined?

Answer: $x = 2$, because the denominator becomes zero.

Q4. Rationalize $1/\sqrt{5}$.

Answer: Multiply by $\sqrt{5}/\sqrt{5}$: $\sqrt{5}/5$.

Q5. Add $(x)/(x + 1) + 1/(x + 1)$.

Answer: Same denominator: $(x + 1)/(x + 1) = 1$.

Long Questions

Q1. Simplify $(x^2 - 4)/(x^2 - 5x + 6) \times (x - 3)/(x + 2)$, stating any excluded values.

Answer: Factorize: $(x^2 - 4) = (x + 2)(x - 2)$; $(x^2 - 5x + 6) = (x - 2)(x - 3)$. The product is $[(x + 2)(x - 2)/(x - 2)(x - 3)] \times [(x - 3)/(x + 2)] = 1$ after cancelling $(x + 2)$, $(x - 2)$ and $(x - 3)$. Excluded values: $x \neq -2, 2, 3$.

Q2. Find the HCF and LCM of $x^2 - 9$ and $x^2 - 6x + 9$, and verify that HCF × LCM equals their product.

Answer: $x^2 - 9 = (x + 3)(x - 3)$; $x^2 - 6x + 9 = (x - 3)^2$. HCF = $x - 3$. LCM = $(x + 3)(x - 3)^2$. HCF × LCM = $(x - 3)(x + 3)(x - 3)^2 = (x + 3)(x - 3)^3$, which is exactly the product of the two expressions. Verified.

Multiple-Choice Questions

Q1. The HCF of $(x + 1)$ and $(x + 1)^2$ is:

(a) $x + 1$ (b) $(x + 1)^2$
(c) $x - 1$ (d) 1

Q2. $(x^2 - 4)/(x - 2)$ simplifies to:

(a) $x - 2$ (b) $x + 2$
(c) $x + 4$ (d) $2x$

Q3. The expression $1/(x - 1)$ is undefined at:

(a) $x = 0$ (b) $x = 1$
(c) $x = -1$ (d) $x = 2$

Q4. Simplify $(3x)/(6)$.

(a) $x/2$ (b) $2x$
(c) $3x$ (d) $x/3$

Q5. The LCM of x and 2x is:

(a) x (b) $2x$
(c) $2x^2$ (d) $3x$

Q6. $1/\sqrt{2}$ rationalized is:

(a) $\sqrt{2}$ (b) $\sqrt{2}/2$
(c) $2\sqrt{2}$ (d) $1/2$

Q7. $(x + 2)/(x + 2)$ equals $(x \neq -2)$:

(a) 0 (b) x
(c) 1 (d) 2

Q8. To divide by a fraction we:

(a) Add the fractions (b) Multiply by its reciprocal
(c) Invert the first fraction (d) Subtract the denominators

Q9. The square root of $x^2 + 10x + 25$ is:

(a) $x + 5$ (b) $x - 5$
(c) $x + 25$ (d) $5x$

Q10. Simplify $(x^2 - 1)/(x + 1)$.

(a) $x + 1$ (b) $x - 1$
(c) $x^2 + 1$ (d) $2x$

Answer Key (Chapter 5): 1(A), 2(B), 3(B), 4(A), 5(B), 6(B), 7(C), 8(B), 9(A), 10(B)

Chapter 6: Linear Equations and Inequalities

Study Notes

- A linear equation in one variable has the form $ax + b = 0$ ($a \neq 0$). Its solution is $x = -b/a$. Always check the answer by substitution.
- To solve an equation, do the same operation on both sides: remove brackets first, collect the variable terms on one side and the numbers on the other, then divide by the coefficient of x.
- A simultaneous pair of linear equations in two variables can be solved by elimination (add or subtract to remove one variable) or by substitution.
- A linear inequality uses $<$, $>$, $\leq$ or $\geq$. Adding or subtracting the same number keeps the sign; multiplying or dividing by a negative number REVERSES the inequality sign.
- The solution of an inequality is a range of values, shown on a number line with an open circle for $<$ or $>$ and a filled circle for $\leq$ or $\geq$.
- Word problems: choose a letter for the unknown, write the equation from the words, solve it, and check the answer against the story.
- A modulus (absolute value) equation $|x| = a$ has two solutions, $x = a$ and $x = -a$, for $a > 0$.

Key Formulas

> $ax + b = 0 \Rightarrow x = -b/a$ | If $a < b$ and $c < 0$, then $ac > bc$ | $|x| = a \Rightarrow x = \pm a$

Worked Examples

Example 1 — Solve $3x - 7 = 11$.

$3x = 18$, so $x = 6$. Check: $3(6) - 7 = 11$. Correct.

Example 2 — Solve $2x + 3 > 9$.

$2x > 6$, so $x > 3$. Any number greater than 3 works, e.g. $x = 4$: $2(4) + 3 = 11 > 9$.

Chapter 6: Linear Equations and Inequalities — Practice Questions

Short Questions

Q1. Solve $x + 9 = 15$.

Answer: $x = 6$.

Q2. Solve $5x = 35$.

Answer: $x = 7$.

Q3. Solve $-2x < 8$.

Answer: Divide by -2 and reverse the sign: $x > -4$.

Q4. Solve $|x| = 5$.

Answer: $x = 5$ or $x = -5$.

Q5. The sum of two numbers is 20 and one is 4 more than the other. Find them.

Answer: Let the smaller be x; then $x + (x + 4) = 20$, so $2x = 16$, $x = 8$. The numbers are 8 and 12.

Long Questions

Q1. Solve the simultaneous equations $x + y = 10$ and $x - y = 4$ by elimination, and check the answer.

Answer: Add the equations: $2x = 14$, so $x = 7$. Then $y = 10 - 7 = 3$. Check: $7 - 3 = 4$. Correct. The solution is $x = 7$, $y = 3$.

Q2. A father is 3 times as old as his son. In 12 years he will be twice as old as his son. Find their present ages.

Answer: Let the son be x years old; the father is 3x. In 12 years: $3x + 12 = 2(x + 12)$, so $3x + 12 = 2x + 24$, giving $x = 12$. The son is 12 and the father is 36. Check: in 12 years they are 24 and 48, and $48 = 2 \times 24$. Correct.

Multiple-Choice Questions

Q1. The solution of $2x + 6 = 14$ is:

(a) $x = 2$ (b) $x = 4$
(c) $x = 6$ (d) $x = 8$

Q2. If $x - 5 = 0$, then x equals:

(a) -5 (b) 0
(c) 5 (d) 1

Q3. Solving $3x < 12$ gives:

(a) $x < 4$ (b) $x > 4$
(c) $x < 36$ (d) $x > 36$

Q4. If $-x > 3$, then:

(a) $x > 3$ (b) $x < -3$
(c) $x > -3$ (d) $x < 3$

Q5. The solution of $|x| = 7$ is:

(a) $x = 7$ only
(b) $x = -7$ only
(c) $x = \pm 7$
(d) No solution

Q6. Which is a linear equation?

(a) $x^2 + 1 = 0$
(b) $2x + 3 = 7$
(c) $1/x = 2$
(d) $x^3 = 8$

Q7. Solve $x/4 = 5$.

(a) $x = 9$
(b) $x = 20$
(c) $x = 1$
(d) $x = 5/4$

Q8. If $x + y = 8$ and $x = 3$, then y equals:

(a) 11
(b) 5
(c) 3
(d) 24

Q9. The inequality $x \geq 2$ means x is:

(a) Less than 2
(b) Greater than or equal to 2
(c) Equal to 2
(d) Less than or equal to 2

Q10. Solve $4x - 3 = 2x + 5$.

(a) $x = 1$
(b) $x = 2$
(c) $x = 4$
(d) $x = 8$

Answer Key (Chapter 6): 1(B), 2(C), 3(A), 4(B), 5(C), 6(B), 7(B), 8(B), 9(B), 10(C)

Chapter 7: Linear Graphs and their Applications

Study Notes

- A linear equation in two variables, such as y = 2x + 1, has a straight-line graph. Make a table of x and y values, plot the points and join them with a straight line.
- In y = mx + c, m is the gradient (slope) and c is the y-intercept, the point where the line crosses the y-axis. A positive gradient rises to the right; a negative gradient falls.
- Gradient = change in y / change in x = (y2 - y1)/(x2 - x1). Between two points it measures how steep the line is.
- Parallel lines have equal gradients. Perpendicular lines have gradients whose product is -1: if one gradient is m, the perpendicular gradient is -1/m.
- The graph of x = a is a vertical line; the graph of y = b is a horizontal line. The x-axis is y = 0 and the y-axis is x = 0.
- Graphs solve simultaneous equations: the point where two lines cross is the solution of both equations. Graphs also model real situations such as distance-time and cost-quantity.
- A distance-time graph has a straight line for steady speed; its gradient is the speed. A flat (horizontal) part means the object is at rest.

Key Formulas

> y = mx + c | m = (y2 - y1)/(x2 - x1) | Parallel: m1 = m2 | Perpendicular: m1 × m2 = -1

Worked Examples

Example 1 — Gradient and intercept of y = 3x - 2.

Gradient m = 3 and y-intercept c = -2. The line crosses the y-axis at (0, -2) and rises 3 units for every 1 unit to the right.

Example 2 — Gradient between (1, 2) and (4, 8).

m = (8 - 2)/(4 - 1) = 6/3 = 2.

Chapter 7: Linear Graphs and their Applications — Practice Questions

Short Questions

Q1. What are the gradient and y-intercept of y = -2x + 5?

Answer: Gradient -2, y-intercept 5.

Q2. Find the gradient of the line through (0, 0) and (3, 6).

Answer: m = 6/3 = 2.

Q3. Are the lines y = 2x + 1 and y = 2x - 4 parallel?

Answer: Yes, both have gradient 2.

Q4. What is the gradient of a line perpendicular to y = 4x + 1?

Answer: -1/4, since 4 × (-1/4) = -1.

Q5. What does the graph of y = 3 look like?

Answer: A horizontal line through (0, 3), parallel to the x-axis.

Long Questions

Q1. Draw the graph of y = 2x - 1 using a table of values, and use it to find y when x = 3 and x when y = 5.

Answer: Table: x = -1, 0, 1, 2 gives y = -3, -1, 1, 3. Plot (-1, -3), (0, -1), (1, 1), (2, 3) and join them with a straight line. From the line, when x = 3, y = 5; when y = 5, x = 3. Check by substitution: y = 2(3) - 1 = 5. Correct.

Q2. A taxi charges a fixed Rs. 50 plus Rs. 20 per kilometre. Write the cost equation, draw its meaning in words, and find the cost of a 10 km journey.

Answer: Let the distance be x km and the cost be y rupees: y = 20x + 50. The graph is a straight line with gradient 20 (cost per km) and intercept 50 (fixed charge). For x = 10, y = 20(10) + 50 = Rs. 250.

Multiple-Choice Questions

Q1. In y = 5x + 2, the gradient is:

(a) 2 (b) 5
(c) 7 (d) -5

Q2. The y-intercept of y = 3x - 4 is:

(a) 3 (b) -4
(c) 4 (d) 0

Q3. The graph of x = 2 is a:

(a) Horizontal line (b) Vertical line
(c) Diagonal line (d) Curve

Q4. Two lines are parallel if their gradients are:

(a) Equal (b) Opposite
(c) Product -1 (d) Zero

Q5. The gradient of y = -x + 7 is:

(a) 1 (b) -1
(c) 7 (d) 0

Q6. A line perpendicular to gradient 2 has gradient:

(a) 2 (b) -2
(c) 1/2 (d) -1/2

Q7. The point (0, 4) lies on:

(a) y = x (b) y = 2x + 4
(c) y = 4x (d) x = 4

Q8. Gradient = change in y divided by:

(a) Change in x (b) Change in y
(c) Intercept (d) 2

Q9. On a distance-time graph, the gradient gives:

(a) Distance (b) Time
(c) Speed (d) Acceleration

Q10. The line y = 0 is the:

(a) y-axis (b) x-axis
(c) Line y = x (d) Origin

Answer Key (Chapter 7): 1(B), 2(B), 3(B), 4(A), 5(B), 6(D), 7(B), 8(A), 9(C), 10(B)

Chapter 8: Quadratic Equations

Study Notes

- A quadratic equation has the form $ax^2 + bx + c = 0$, with $a \neq 0$. It has at most two solutions (roots).
- Solve by factorization: factorize, then set each factor equal to zero. Example: $x^2 - 5x + 6 = 0$ gives $(x - 2)(x - 3) = 0$, so $x = 2$ or $x = 3$.
- Completing the square: $x^2 + bx = (x + b/2)^2 - (b/2)^2$. This method works for every quadratic and leads to the quadratic formula.
- Quadratic formula: $x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$. Use it when factorization is difficult or impossible.
- The discriminant is $D = b^2 - 4ac$. If $D > 0$, two distinct real roots; if $D = 0$, two equal real roots; if $D < 0$, no real roots.
- Sum of roots $= -b/a$ and product of roots $= c/a$. So a quadratic with roots α and β is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.
- Word problems: translate the story into a quadratic, solve it, and reject any root that does not fit the story (for example a negative length).

Key Formulas

> $x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$ | $D = b^2 - 4ac$ | Sum of roots $= -b/a$, Product $= c/a$

Worked Examples

Example 1 — Solve $x^2 - 7x + 12 = 0$.

$(x - 3)(x - 4) = 0$, so $x = 3$ or $x = 4$. Check: $9 - 21 + 12 = 0$. Correct.

Example 2 — Nature of roots of $x^2 + 4x + 5 = 0$.

$D = 16 - 20 = -4 < 0$, so there are no real roots.

Chapter 8: Quadratic Equations — Practice Questions

Short Questions

Q1. Write the standard form of a quadratic equation.

Answer: $ax^2 + bx + c = 0$, where $a \neq 0$.

Q2. Solve $x^2 = 25$.

Answer: $x = 5$ or $x = -5$.

Q3. Find the discriminant of $x^2 - 6x + 9 = 0$.

Answer: $D = 36 - 36 = 0$, so the roots are equal.

Q4. If the sum of roots is 5 and the product is 6, write the quadratic.

Answer: $x^2 - 5x + 6 = 0$.

Q5. Solve $(x - 1)(x + 4) = 0$.

Answer: $x = 1$ or $x = -4$.

Long Questions

Q1. Solve $2x^2 - 5x - 3 = 0$ by factorization and verify both roots.

Answer: Split the middle term: $2x^2 - 6x + x - 3 = 2x(x - 3) + 1(x - 3) = (x - 3)(2x + 1) = 0$. So $x = 3$ or $x = -1/2$. Check $x = 3$: $18 - 15 - 3 = 0$. Check $x = -1/2$: $2(1/4) - 5(-1/2) - 3 = 1/2 + 5/2 - 3 = 0$. Both correct.

Q2. Solve $x^2 + 3x - 1 = 0$ using the quadratic formula, giving the roots correct to two decimal places.

Answer: $a = 1$, $b = 3$, $c = -1$. $x = [-3 \pm \sqrt{(9 + 4)}]/2 = [-3 \pm \sqrt{13}]/2$. $\sqrt{13} \approx 3.61$, so $x \approx (-3 + 3.61)/2 = 0.30$ or $x \approx (-3 - 3.61)/2 = -3.30$.

Multiple-Choice Questions

Q1. The standard form of a quadratic equation is:

(a) $ax + b = 0$ (b) $ax^2 + bx + c = 0$
(c) $ax^3 + bx = 0$ (d) $x^2 + y^2 = 0$

Q2. The roots of $x^2 - 9 = 0$ are:

(a) $x = 9$ (b) $x = \pm 3$
(c) $x = 3$ only (d) $x = 0$

Q3. If $D = 0$, the roots are:

(a) Distinct (b) Equal
(c) Imaginary (d) No roots

Q4. The discriminant of $x^2 + 2x + 1 = 0$ is:

(a) 4 (b) 0
(c) 2 (d) -4

Q5. The sum of the roots of $x^2 - 5x + 6 = 0$ is:

(a) -5 (b) 5
(c) 6 (d) -6

Q6. The product of the roots of $x^2 - 5x + 6 = 0$ is:

(a) 5 (b) -5
(c) 6 (d) -6

Q7. Which equation has roots 2 and 3?

(a) $x^2 - 5x + 6 = 0$ (b) $x^2 + 5x + 6 = 0$
(c) $x^2 - 6x + 5 = 0$ (d) $x^2 + 6x + 5 = 0$

Q8. The quadratic formula uses:

(a) $b^2 - 4ac$ (b) $b^2 + 4ac$
(c) $4ac - b^2$ (d) $b^2 - 2ac$

Q9. If $D < 0$, the equation has:

(a) Two real roots (b) One real root
(c) No real roots (d) Infinite roots

Q10. Solve $x^2 - 4x = 0$.

(a) $x = 0$ or 4 (b) $x = 4$ only
(c) $x = 0$ only (d) $x = \pm 4$

Answer Key (Chapter 8): 1(B), 2(B), 3(B), 4(B), 5(B), 6(C), 7(A), 8(A), 9(C), 10(A)

Chapter 9: Congruent Triangles

Study Notes

- Two figures are congruent if one can be placed exactly on the other — same shape and same size. Congruent triangles have equal corresponding sides and equal corresponding angles.
- SSS rule: if the three sides of one triangle equal the three sides of another, the triangles are congruent.
- SAS rule: if two sides and the included angle of one triangle equal those of another, the triangles are congruent. The angle must be between the two sides.
- ASA rule: if two angles and the included side of one triangle equal those of another, the triangles are congruent. AAS also works: two angles and any one corresponding side.
- RHS rule (right-angled triangles): if the hypotenuse and one side of one right triangle equal those of another, the triangles are congruent.
- Once triangles are proved congruent, all their other corresponding parts are equal. This is often written as CPCT (corresponding parts of congruent triangles).
- To prove congruence, mark equal sides and angles on a clear diagram, name the rule used, and state the conclusion with the correct correspondence of vertices.

Key Formulas

> SSS | SAS | ASA | AAS | RHS — the five congruence conditions for triangles

Worked Examples

Example 1 — SSS.

Triangle ABC has sides 3 cm, 4 cm, 5 cm and triangle PQR has sides 3 cm, 4 cm, 5 cm. All three sides match, so the triangles are congruent by SSS.

Example 2 — SAS.

In triangles ABC and DEF, AB = DE = 6 cm, angle B = angle E = 50° and BC = EF = 8 cm. The equal angle is between the equal sides, so the triangles are congruent by SAS.

Chapter 9: Congruent Triangles — Practice Questions

Short Questions

Q1. What does 'congruent' mean?

Answer: Two figures are congruent when they have exactly the same shape and size, so one fits exactly on the other.

Q2. State the SSS congruence rule.

Answer: If the three sides of one triangle are equal to the three sides of another triangle, the triangles are congruent.

Q3. State the RHS rule.

Answer: If the hypotenuse and one side of one right-angled triangle equal the hypotenuse and one side of another, the triangles are congruent.

Q4. In congruent triangles ABC and PQR, angle A = 60°. What is the corresponding angle in triangle PQR?

Answer: Angle P = 60°, because corresponding angles of congruent triangles are equal.

Q5. Can two triangles be congruent by AAA alone?

Answer: No. Equal angles give similar triangles, but the sides may be different sizes, so AAA does not prove congruence.

Long Questions

Q1. Prove that the base angles of an isosceles triangle are equal, using congruent triangles.

Answer: Let ABC be isosceles with AB = AC. Draw the bisector of angle A meeting BC at D. In triangles ABD and ACD: AB = AC (given), AD is common, and angle BAD = angle CAD (AD is the bisector). So the triangles are congruent by SAS. Therefore angle B = angle C by CPCT. So the base angles are equal.

Q2. Explain each congruence rule with a short example of when it applies and when it does not.

Answer: SSS applies when all three sides are known equal. SAS applies when two sides and the angle between them are equal — SSA (two sides and a non-included angle) does NOT prove congruence. ASA applies when two angles and the side between them are equal; AAS works with any corresponding side. RHS applies only to right-angled triangles, using the hypotenuse and one other side. AAA gives similarity, not congruence.

Multiple-Choice Questions

Q1. Which is NOT a congruence rule?

(a) SSS
(b) SAS
(c) AAA
(d) RHS

Q2. In SAS, the equal angle must be:

(a) Any angle
(b) The included angle
(c) The largest angle
(d) The smallest angle

Q3. Congruent triangles have:

(a) Equal angles only
(b) Equal sides only
(c) Equal sides and equal angles
(d) Equal area only

Q4. RHS applies to:

(a) Any triangle
(b) Right-angled triangles only
(c) Isosceles triangles only
(d) Equilateral triangles only

Q5. If triangles ABC and DEF are congruent, AB equals:

(a) DE
(b) EF
(c) DF
(d) AC

Q6. Two triangles with sides 5, 6, 7 cm and 5, 6, 7 cm are congruent by:

(a) SAS
(b) SSS
(c) ASA
(d) RHS

Q7. CPCT means:

(a) Corresponding parts of congruent triangles are equal
(b) Corresponding parts of similar triangles are equal
(c) Congruent parts of triangles are equal
(d) Corresponding perimeter of congruent triangles

Q8. ASA needs:

(a) Two sides and an angle
(b) Two angles and the included side
(c) Three sides
(d) Hypotenuse and a side

Q9. An equilateral triangle has all angles equal to:

(a) 90°
(b) 60°
(c) 45°
(d) 30°

Q10. If AB = PQ, BC = QR and angle B = angle Q, the rule is:

(a) SSS
(b) ASA
(c) SAS
(d) AAS

Answer Key (Chapter 9): 1(C), 2(B), 3(C), 4(B), 5(A), 6(B), 7(A), 8(B), 9(B), 10(C)

Chapter 10: Parallelograms and Triangles

Study Notes

- A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Its opposite sides are equal, its opposite angles are equal, and its diagonals bisect each other.
- A rectangle has all angles 90°; a rhombus has all sides equal; a square has all sides equal and all angles 90°. All three are special parallelograms.
- In a parallelogram, consecutive angles are supplementary (they add to 180°). If one angle is 90°, the parallelogram is a rectangle.
- The area of a parallelogram = base × height, where the height is the perpendicular distance between the base and the opposite side.
- The area of a triangle = (1/2) × base × height. A diagonal divides a parallelogram into two congruent triangles of equal area.
- Triangles on the same base and between the same parallels have equal areas. A median of a triangle divides it into two triangles of equal area.
- A quadrilateral is a parallelogram if its opposite sides are equal, or its opposite angles are equal, or its diagonals bisect each other, or one pair of opposite sides is equal and parallel.

Key Formulas

> Area of parallelogram = base × height | Area of triangle = (1/2) × base × height | Opposite angles of a parallelogram are equal

Worked Examples

Example 1 — Area.

A parallelogram has base 8 cm and height 5 cm. Area = 8 × 5 = 40 cm².

Example 2 — Angles.

In parallelogram ABCD, angle A = 70°. Then angle C = 70° (opposite angles) and angle B = 180° - 70° = 110° (consecutive angles).

Chapter 10: Parallelograms and Triangles — Practice Questions

Short Questions

Q1. Define a parallelogram.

Answer: A quadrilateral with both pairs of opposite sides parallel.

Q2. In a parallelogram, what is true of the diagonals?

Answer: They bisect each other.

Q3. Find the area of a triangle with base 10 cm and height 6 cm.

Answer: Area = (1/2) × 10 × 6 = 30 cm².

Q4. A rhombus has all sides equal. Is it a parallelogram?

Answer: Yes, a rhombus is a parallelogram with all sides equal.

Q5. What is the sum of the angles of a quadrilateral?

Answer: 360°.

Long Questions

Q1. Prove that the diagonals of a parallelogram bisect each other.

Answer: Let ABCD be a parallelogram with diagonals AC and BD meeting at O. In triangles AOB and COD: AB = CD (opposite sides), angle OAB = angle OCD (alternate angles, AB parallel to CD), angle OBA = angle ODC (alternate angles). So the triangles are congruent by ASA. Therefore AO = CO and BO = DO by CPCT. So the diagonals bisect each other.

Q2. A parallelogram has base 12 cm and area 96 cm². Find its height. Also find the area of a triangle on the same base with the same height.

Answer: Height = area / base = 96 / 12 = 8 cm. The triangle on the same base and height has area (1/2) × 12 × 8 = 48 cm², which is half the parallelogram's area, as expected.

Multiple-Choice Questions

Q1. Opposite sides of a parallelogram are:

(a) Unequal
(b) Equal
(c) Perpendicular
(d) Parallel only

Q2. The diagonals of a parallelogram:

(a) Are equal
(b) Bisect each other
(c) Are perpendicular
(d) Bisect the angles

Q3. Area of a parallelogram with base 7 cm and height 4 cm:

(a) 11 cm²
(b) 28 cm²
(c) 14 cm²
(d) 22 cm²

Q4. A square is a:

(a) Rectangle only
(b) Rhombus only
(c) Both a rectangle and a rhombus
(d) Neither

Q5. Consecutive angles of a parallelogram add to:

(a) 90° (b) 180°
(c) 270° (d) 360°

Q6. Area of a triangle with base 8 cm and height 3 cm:

(a) 24 cm² (b) 12 cm²
(c) 11 cm² (d) 6 cm²

Q7. A diagonal divides a parallelogram into:

(a) Two unequal triangles (b) Two congruent triangles
(c) Two rectangles (d) Two squares

Q8. Which quadrilateral has all angles 90°?

(a) Rhombus (b) Rectangle
(c) Parallelogram (d) Trapezium

Q9. A median divides a triangle into two triangles of:

(a) Equal area (b) Equal perimeter
(c) Equal angles (d) Equal sides

Q10. The sum of the interior angles of a parallelogram is:

(a) 180° (b) 270°
(c) 360° (d) 90°

Answer Key (Chapter 10): 1(B), 2(B), 3(B), 4(C), 5(B), 6(B), 7(B), 8(B), 9(A), 10(C)

Chapter 11: Line Bisectors and Angle Bisectors

Study Notes

- The perpendicular bisector of a line segment is the line through its midpoint at right angles to it. Every point on it is equidistant from the two ends of the segment.
- To draw a perpendicular bisector with compasses, draw arcs of equal radius centred at each end of the segment; the line joining the two intersection points of the arcs is the bisector.
- The angle bisector divides an angle into two equal angles. Every point on it is equidistant from the two arms of the angle.
- To bisect an angle, draw an arc cutting both arms, then draw two equal arcs from those points; the line from the vertex through their intersection is the bisector.
- The bisectors of the three angles of a triangle meet at the incenter, the centre of the circle inscribed in the triangle (the incircle).
- The perpendicular bisectors of the three sides of a triangle meet at the circumcenter, the centre of the circle passing through all three vertices (the circumcircle).
- In an equilateral triangle the incenter, circumcenter, centroid and orthocenter all fall at the same point.

Key Formulas

> Perpendicular bisector: through the midpoint, at 90° | Angle bisector: divides the angle into two equal parts | Incenter: angle bisectors meet | Circumcenter: perpendicular bisectors meet

Worked Examples

Example 1 — Midpoint.

The perpendicular bisector of the segment joining (2, 3) and (6, 3) passes through the midpoint (4, 3) and is the vertical line $x = 4$.

Example 2 — Bisecting 60°.

The bisector of a 60° angle makes two angles of 30° each.

Chapter 11: Line Bisectors and Angle Bisectors — Practice Questions

Short Questions

Q1. What is a perpendicular bisector?

Answer: A line through the midpoint of a segment, at right angles to it.

Q2. A point on the perpendicular bisector of AB is 5 cm from A. How far is it from B?

Answer: 5 cm — every point on the bisector is equidistant from A and B.

Q3. What does an angle bisector do?

Answer: It divides an angle into two equal angles.

Q4. Where do the angle bisectors of a triangle meet?

Answer: At the incenter.

Q5. Where do the perpendicular bisectors of the sides of a triangle meet?

Answer: At the circumcenter.

Long Questions

Q1. Describe, step by step, how to draw the perpendicular bisector of a line segment AB = 8 cm using ruler and compasses, and state its key property.

Answer: Draw AB = 8 cm. With the compasses open to more than 4 cm, draw arcs centred at A and at B so they cross above and below AB. Join the two crossing points with a straight line. This line crosses AB at its midpoint (4 cm from each end) at 90°. Key property: every point on this line is the same distance from A as from B.

Q2. Explain the incenter and circumcenter of a triangle and how each is found.

Answer: The incenter is the meeting point of the three angle bisectors; it is the centre of the incircle, which touches all three sides. The circumcenter is the meeting point of the three perpendicular bisectors of the sides; it is the centre of the circumcircle, which passes through all three vertices. For an acute triangle the circumcenter lies inside; for a right triangle it is the midpoint of the hypotenuse; for an obtuse triangle it lies outside.

Multiple-Choice Questions

Q1. A perpendicular bisector passes through the:

(a) Endpoint (b) Midpoint
(c) Vertex (d) Origin

Q2. A perpendicular bisector meets the segment at:

(a) 45° (b) 60°
(c) 90° (d) 180°

Q3. A point on an angle bisector is equidistant from the:

(a) Vertices (b) Two arms of the angle
(c) Opposite side (d) Midpoints

Q4. The angle bisectors of a triangle meet at the:

(a) Circumcenter (b) Centroid
(c) Incenter (d) Orthocenter

Q5. The perpendicular bisectors of a triangle meet at the:

(a) Incenter (b) Circumcenter
(c) Centroid (d) Vertex

Q6. Bisecting a 90° angle gives:

(a) 30° (b) 45°
(c) 60° (d) 90°

Q7. The circumcenter is the centre of the:

(a) Incircle (b) Circumcircle
(c) Semicircle (d) Unit circle

Q8. In an equilateral triangle, incenter and circumcenter are:

(a) Different points (b) The same point
(c) Outside the triangle (d) On a vertex

Q9. Every point on the perpendicular bisector of AB is equidistant from:

(a) A only (b) B only
(c) A and B (d) The midpoint

Q10. The incenter is the centre of the circle that:

(a) Passes through the vertices (b) Touches all three sides
(c) Has the longest side as diameter (d) Is outside the triangle

Answer Key (Chapter 11): 1(B), 2(C), 3(B), 4(C), 5(B), 6(B), 7(B), 8(B), 9(C), 10(B)

Chapter 12: Sides and Angles of a Triangle

Study Notes

- The angles of a triangle add to 180°. If two angles are known, the third is 180° minus their sum.
- An exterior angle of a triangle equals the sum of the two opposite interior angles. Example: if the opposite interior angles are 50° and 60°, the exterior angle is 110°.
- In an isosceles triangle the angles opposite the equal sides are equal. In an equilateral triangle all sides are equal and all angles are 60°.
- In any triangle, the greater side is opposite the greater angle, and the greater angle is opposite the greater side.
- Triangle inequality: the sum of any two sides is greater than the third side. So sides 3, 4 and 8 cannot form a triangle, because 3 + 4 < 8.
- The side opposite the largest angle is the longest side. In a right-angled triangle the hypotenuse, opposite the 90° angle, is the longest side.
- If two sides of a triangle are unequal, the angle opposite the longer side is larger. These facts are used to compare sides and angles without measuring.

Key Formulas

> Angles of a triangle: A + B + C = 180° | Exterior angle = sum of the two opposite interior angles | Triangle inequality: a + b > c

Worked Examples

Example 1 — Find the third angle.

Angles 70° and 55° leave 180° - 70° - 55° = 55° for the third angle. The triangle is isosceles.

Example 2 — Triangle inequality.

Can sides 5, 7, 12 form a triangle? 5 + 7 = 12, which is not greater than 12, so no — the three points would lie on a straight line.

Chapter 12: Sides and Angles of a Triangle — Practice Questions

Short Questions

Q1. Two angles of a triangle are 40° and 80°. Find the third.

Answer: 180° - 40° - 80° = 60°.

Q2. What is an exterior angle of a triangle?

Answer: The angle formed by one side and the extension of the adjacent side.

Q3. In triangle ABC, AB > AC. Which angle is larger, B or C?

Answer: Angle C is larger, because it is opposite the longer side AB.

Q4. Can sides 4, 5 and 6 form a triangle?

Answer: Yes: 4 + 5 > 6, 4 + 6 > 5 and 5 + 6 > 4.

Q5. What are the angles of an equilateral triangle?

Answer: Each is 60°.

Long Questions

Q1. Prove that the angles of a triangle add up to 180°.

Answer: Let ABC be a triangle. Through A draw a line parallel to BC. The alternate angles formed are equal to angles B and C. On the straight line through A, the three angles — the alternate angle equal to B, angle A itself, and the alternate angle equal to C — add to 180°. So A + B + C = 180°.

Q2. In triangle PQR, PQ = 8 cm, QR = 6 cm and PR = 5 cm. Arrange the angles in order from largest to smallest, giving reasons.

Answer: The largest angle is opposite the longest side. Longest side PQ = 8 cm is opposite angle R, so angle R is largest. Next, QR = 6 cm is opposite angle P, so angle P is next. Shortest side PR = 5 cm is opposite angle Q, the smallest. Order: angle R > angle P > angle Q.

Multiple-Choice Questions

Q1. The angles of a triangle add to:

(a) 90°
(b) 180°
(c) 270°
(d) 360°

Q2. An exterior angle equals:

(a) The adjacent interior angle
(b) The sum of the two opposite interior angles
(c) 180°
(d) The largest angle

Q3. In an isosceles triangle, the base angles are:

(a) Unequal
(b) Equal
(c) 90°
(d) 60°

Q4. The longest side is opposite the:

(a) Smallest angle
(b) Largest angle
(c) Right angle only
(d) Base angle

Q5. Sides 3, 4, 8 can form a triangle:

(a) Yes (b) No
(c) Only a right triangle (d) Only an isosceles triangle

Q6. In a right triangle, the longest side is the:

(a) Base (b) Perpendicular
(c) Hypotenuse (d) Median

Q7. Each angle of an equilateral triangle is:

(a) 90° (b) 45°
(c) 60° (d) 30°

Q8. Two angles are 90° and 45°. The third is:

(a) 45° (b) 90°
(c) 35° (d) 55°

Q9. If AB is the longest side of triangle ABC, the largest angle is:

(a) A (b) B
(c) C (d) Cannot tell

Q10. The triangle inequality says the sum of two sides must be:

(a) Less than the third (b) Equal to the third
(c) Greater than the third (d) Half the third

Answer Key (Chapter 12): 1(B), 2(B), 3(B), 4(B), 5(B), 6(C), 7(C), 8(A), 9(C), 10(C)

Chapter 13: Practical Geometry — Triangles

Study Notes

- Practical geometry is construction with ruler and compasses only — no protractor for the standard constructions, though lengths and angles may be given.
- To construct a triangle from three sides (SSS): draw the longest side, then draw arcs from its ends with radii equal to the other two sides; their intersection is the third vertex.
- To construct a triangle from two sides and the included angle (SAS): draw one side, draw the given angle at one end with a protractor, mark the second side on the angle's arm, and join the ends.
- To construct a triangle from two angles and the included side (ASA): draw the side, then draw the two given angles at its ends; their arms meet at the third vertex.
- To construct a right-angled triangle (RHS): draw the hypotenuse, draw a semicircle on it, and mark the point on the semicircle at the given side's distance — or draw the right angle first and mark the sides.
- A triangle can also be constructed from its perimeter and base angles, or from a base, a base angle and the difference of the other two sides; follow the given data step by step.
- Always check a construction by measuring the finished triangle against the given data, and label all vertices and lengths clearly.

Key Formulas

> SSS: three sides | SAS: two sides and the included angle | ASA: two angles and the included side | RHS: hypotenuse and one side

Worked Examples

Example 1 — SSS construction.

Construct a triangle with sides 5 cm, 6 cm, 7 cm. Draw the 7 cm side. From one end draw an arc of radius 5 cm, from the other an arc of radius 6 cm. Their intersection is the third vertex; join it to both ends.

Example 2 — SAS construction.

Construct a triangle with sides 5 cm and 6 cm and included angle 60°. Draw the 5 cm side, draw a 60° angle at one end, mark 6 cm on the arm, and join the ends.

Chapter 13: Practical Geometry — Triangles — Practice Questions

Short Questions

Q1. Which instruments are used in practical geometry?

Answer: A ruler and a pair of compasses (a protractor is used only to draw a given angle).

Q2. What data are needed for an SSS construction?

Answer: The lengths of all three sides.

Q3. What data are needed for an ASA construction?

Answer: Two angles and the side between them.

Q4. In an SSS construction, how is the third vertex found?

Answer: As the intersection of two arcs drawn from the ends of the base, with radii equal to the other two sides.

Q5. Why must the two arcs in an SSS construction cross?

Answer: If they do not cross, the two sides cannot meet and no triangle with those lengths exists.

Long Questions

Q1. Describe fully how to construct a triangle ABC with AB = 6 cm, BC = 5 cm and angle B = 60° (SAS), and how to check it.

Answer: Draw AB = 6 cm. At B, draw a ray making 60° with AB. On that ray mark point C so that BC = 5 cm. Join A to C. Check by measuring AC and angles A and C, and by confirming angle B is 60° and BC is 5 cm. The construction is exact when the measured values match the given data.

Q2. Explain how to construct a right-angled triangle with hypotenuse 10 cm and one side 6 cm (RHS).

Answer: Draw the hypotenuse AC = 10 cm. Draw a semicircle with AC as diameter. With centre A and radius 6 cm, draw an arc cutting the semicircle at B. Join AB and BC. Angle B is an angle in a semicircle, so it is 90°; the hypotenuse is 10 cm and side AB is 6 cm, as required. The third side is 8 cm by Pythagoras' theorem.

Multiple-Choice Questions

Q1. Practical geometry uses:

(a) Ruler and compasses
(b) Protractor only
(c) Set squares only
(d) Calculator

Q2. SSS construction needs:

(a) Three sides
(b) Three angles
(c) Two sides only
(d) One side and one angle

Q3. SAS construction needs the angle to be:

(a) Any angle
(b) The included angle
(c) The largest angle
(d) A right angle

Q4. ASA construction needs:

(a) Two angles and the included side (b) Two sides and an angle
(c) Three sides (d) Hypotenuse and a side

Q5. RHS construction is for:

(a) Any triangle (b) Right-angled triangles
(c) Isosceles triangles (d) Equilateral triangles

Q6. In SSS, the third vertex is the intersection of:

(a) Two lines (b) Two arcs
(c) Two angles (d) Two circles only

Q7. To draw a 60° angle in a construction, you use a:

(a) Compass arc method or protractor (b) Ruler only
(c) Set square only (d) Freehand

Q8. A triangle with sides 2 cm, 3 cm and 9 cm:

(a) Can be constructed (b) Cannot be constructed
(c) Is right-angled (d) Is equilateral

Q9. The first step in an SSS construction is to draw:

(a) The longest side (b) The smallest angle
(c) A circle (d) The altitude

Q10. After a construction, you should:

(a) Erase all arcs (b) Check measurements against the given data
(c) Colour the triangle (d) Measure nothing

Answer Key (Chapter 13): 1(A), 2(A), 3(B), 4(A), 5(B), 6(B), 7(A), 8(B), 9(A), 10(B)

Chapter 14: Theorems Related with Area

Study Notes

- Two figures on the same base and between the same parallels have equal areas. This is the key idea of this chapter.
- Parallelograms on the same base and between the same parallels are equal in area. Their area is base × height, and the height is the same for both.
- A triangle and a parallelogram on the same base and between the same parallels: the triangle's area is half the parallelogram's area.
- Triangles on the same base and between the same parallels are equal in area, because each is half of a parallelogram on the same base.
- A median of a triangle divides it into two triangles of equal area, since they share the same altitude and have equal bases.
- Two triangles with the same altitude have areas in the same ratio as their bases. Two triangles with the same base have areas in the same ratio as their altitudes.
- These theorems let us compare and prove areas equal without measuring, and they are the basis for proving Pythagoras' theorem by area.

Key Formulas

> Parallelograms on the same base and between the same parallels are equal in area | Area of triangle = (1/2) × area of parallelogram on the same base

Worked Examples

Example 1 — Equal areas.

Two parallelograms share base 10 cm and lie between the same parallels. Both have area 10 × height, so their areas are equal whatever their shapes.

Example 2 — Triangle and parallelogram.

A parallelogram has area 60 cm². A triangle on the same base and between the same parallels has area 30 cm².

Chapter 14: Theorems Related with Area — Practice Questions

Short Questions

Q1. State the theorem about parallelograms on the same base.

Answer: Parallelograms on the same base and between the same parallels are equal in area.

Q2. A triangle and a parallelogram share a base and lie between the same parallels. How do their areas compare?

Answer: The triangle's area is half the parallelogram's area.

Q3. What does a median do to the area of a triangle?

Answer: It divides the triangle into two triangles of equal area.

Q4. Two triangles have the same altitude and bases 4 cm and 6 cm. What is the ratio of their areas?

Answer: 4 : 6, which is 2 : 3.

Q5. Why do triangles on the same base between the same parallels have equal areas?

Answer: They have the same base and the same height (the distance between the parallels), and area = (1/2) × base × height.

Long Questions

Q1. Prove that parallelograms on the same base and between the same parallels are equal in area.

Answer: Let two parallelograms ABCD and ABEF stand on the same base AB between the same parallels AB and FC. Each has area base × height, and the height — the perpendicular distance between the parallels — is the same for both. So area ABCD = AB × height = area ABEF. Hence the parallelograms are equal in area. (The formal proof subtracts and adds the same triangular pieces; the base × height argument is its heart.)

Q2. Prove that a median divides a triangle into two triangles of equal area, and use it to show that the three medians divide a triangle into six triangles of equal area.

Answer: Let AD be a median of triangle ABC, so D is the midpoint of BC and BD = DC. Triangles ABD and ACD share the altitude from A to BC and have equal bases BD and DC, so their areas are equal. Applying this to each median, the six small triangles formed by the three medians each have area one-sixth of the whole triangle, because each pair shares equal bases on the same lines with the same altitudes.

Multiple-Choice Questions

Q1. Parallelograms on the same base and between the same parallels are:

(a) Equal in area
(b) Equal in perimeter
(c) Congruent
(d) Similar

Q2. A triangle on the same base as a parallelogram, between the same parallels, has area:

(a) Equal to the parallelogram
(b) Half the parallelogram
(c) Double the parallelogram
(d) One quarter

Q3. A median divides a triangle into:

(a) Two congruent triangles
(b) Two triangles of equal area
(c) Two right triangles
(d) Two equilateral triangles

Q4. Triangles on the same base and between the same parallels have:

(a) Equal areas
(b) Equal perimeters
(c) Equal angles
(d) Equal sides

Q5. Area of a parallelogram =

(a) (1/2) × base × height
(b) base × height
(c) base + height
(d) 2 × base × height

Q6. Two triangles with the same height and bases in ratio 2:3 have areas in ratio:

(a) 3:2
(b) 2:3
(c) 4:9
(d) 1:1

Q7. The height in 'base × height' is the:

(a) Slant side
(b) Perpendicular distance between base and opposite side
(c) Diagonal
(d) Longest side

Q8. A diagonal of a parallelogram divides it into triangles of:

(a) Equal area
(b) Unequal area
(c) Equal perimeter only
(d) Zero area

Q9. If two triangles have equal areas and the same base, their heights are:

(a) Equal
(b) Different
(c) Zero
(d) Double

Q10. The area theorems of this chapter are used to prove:

(a) The angle sum theorem
(b) Pythagoras' theorem by area
(c) The exterior angle theorem
(d) The triangle inequality

Answer Key (Chapter 14): 1(A), 2(B), 3(B), 4(A), 5(B), 6(B), 7(B), 8(A), 9(A), 10(B)

Chapter 15: Projection of a Side of a Triangle

Study Notes

- The projection of a side on another side is the segment cut off on that side by the perpendicular from the opposite vertex. If AD is perpendicular to BC, then BD is the projection of AB on BC.
- Pythagoras' theorem: in a right-angled triangle, the square on the hypotenuse equals the sum of the squares on the other two sides: $c^2 = a^2 + b^2$.
- Converse of Pythagoras' theorem: if $c^2 = a^2 + b^2$ in a triangle, then the angle opposite side c is a right angle.
- In an acute-angled triangle, the square on a side opposite an acute angle is less than the sum of the squares on the other two sides: the difference equals twice the rectangle of one side and the projection of the other on it.
- In an obtuse-angled triangle, the square on the side opposite the obtuse angle is greater than the sum of the squares on the other two sides, by twice the rectangle of one side and the projection of the other on it.
- These projection results generalize Pythagoras' theorem to triangles that are not right-angled, and they are proved using the area theorems and Pythagoras' theorem itself.
- Common Pythagorean triples: 3-4-5, 5-12-13, 8-15-17 and 7-24-25. Multiples of a triple are also triples, e.g. 6-8-10.

Key Formulas

> Pythagoras: $c^2 = a^2 + b^2$ (right-angled) | Acute: $a^2 = b^2 + c^2 - 2 \times$ (projection) | Obtuse: $a^2 = b^2 + c^2 + 2 \times$ (projection)

Worked Examples

Example 1 — Find the hypotenuse.

A right triangle has legs 6 cm and 8 cm. Hypotenuse^2 = 36 + 64 = 100, so the hypotenuse is 10 cm.

Example 2 — Is the triangle right-angled?

Sides 5, 12, 13: $5^2 + 12^2 = 25 + 144 = 169 = 13^2$. Yes, by the converse of Pythagoras' theorem.

Chapter 15: Projection of a Side of a Triangle — Practice Questions

Short Questions

Q1. State Pythagoras' theorem.

Answer: In a right-angled triangle, the square on the hypotenuse equals the sum of the squares on the other two sides.

Q2. Find the hypotenuse when the other sides are 9 cm and 12 cm.

Answer: Hypotenuse^2 = 81 + 144 = 225, so the hypotenuse is 15 cm.

Q3. What is the projection of a side?

Answer: The segment cut off on a given side by the perpendicular dropped from the opposite vertex onto that side (or its extension).

Q4. Sides are 7, 24, 25. Is the triangle right-angled?

Answer: Yes: $7^2 + 24^2 = 49 + 576 = 625 = 25^2$.

Q5. In an obtuse triangle, how does the square on the longest side compare with the sum of the squares on the other two sides?

Answer: It is greater.

Long Questions

Q1. Prove Pythagoras' theorem by the area method (outline) and use it to find the diagonal of a rectangle 12 cm by 5 cm.

Answer: Proof outline: build squares on the three sides of a right triangle and show, by the area theorems, that the square on the hypotenuse equals the sum of the squares on the other two sides. (One standard proof places four copies of the triangle inside a square and compares areas.) Diagonal of the rectangle: it is the hypotenuse of a right triangle with legs 12 cm and 5 cm, so diagonal^2 = 144 + 25 = 169 and the diagonal is 13 cm.

Q2. A triangle has sides 10 cm, 8 cm and 6 cm. Show it is right-angled, and find the projection of the 8 cm side on the 10 cm side.

Answer: $6^2 + 8^2 = 36 + 64 = 100 = 10^2$, so the triangle is right-angled with hypotenuse 10 cm, by the converse of Pythagoras' theorem. Let the projection of the 8 cm side on the hypotenuse be p. In the right triangle, (8 cm side)^2 = hypotenuse × projection, so 64 = 10p and p = 6.4 cm.

Multiple-Choice Questions

Q1. Pythagoras' theorem applies to:

(a) Any triangle
(b) Right-angled triangles
(c) Isosceles triangles
(d) Equilateral triangles

Q2. If the legs are 3 cm and 4 cm, the hypotenuse is:

(a) 5 cm
(b) 6 cm
(c) 7 cm
(d) 12 cm

Q3. Which triple is Pythagorean?

(a) 2, 3, 4 (b) 5, 12, 13
(c) 4, 5, 6 (d) 3, 5, 7

Q4. If $c^2 = a^2 + b^2$, the angle opposite c is:

(a) Acute (b) Obtuse
(c) A right angle (d) 60°

Q5. The hypotenuse of a right triangle with legs 5 and 12 is:

(a) 13 (b) 12
(c) 17 (d) 7

Q6. In an acute triangle, the square on a side is:

(a) Greater than the sum of the squares on the other sides (b) Less than the sum of the squares on the other sides
(c) Equal to that sum (d) Zero

Q7. The projection is found by dropping a:

(a) Median (b) Perpendicular
(c) Angle bisector (d) Diagonal

Q8. A ladder 13 m long reaches a wall 12 m up. Its foot is:

(a) 5 m from the wall (b) 1 m from the wall
(c) 25 m from the wall (d) 12 m from the wall

Q9. Sides 6, 8, 10 form a:

(a) Acute triangle (b) Obtuse triangle
(c) Right triangle (d) No triangle

Q10. The diagonal of a square of side 1 is:

(a) 1 (b) 2
(c) $\sqrt{2}$ (d) 1/2

Answer Key (Chapter 15): 1(B), 2(A), 3(B), 4(C), 5(A), 6(B), 7(B), 8(A), 9(C), 10(C)

Chapter 16: Introduction to Coordinate Geometry / Analytical Geometry

Study Notes

- Coordinate geometry joins algebra and geometry: every point is a pair (x, y), and every line or curve is an equation.
- The distance between points P(x1, y1) and Q(x2, y2) is PQ = √[(x2 - x1)² + (y2 - y1)²]. This is Pythagoras' theorem applied to the coordinate differences.
- The midpoint of PQ is ((x1 + x2)/2, (y1 + y2)/2). The section formula divides PQ in the ratio m:n at ((mx2 + nx1)/(m + n), (my2 + ny1)/(m + n)).
- The gradient (slope) of the line through P and Q is m = (y2 - y1)/(x2 - x1). A horizontal line has gradient 0; a vertical line has undefined gradient.
- The equation of a line with gradient m and y-intercept c is y = mx + c. The point-slope form through (x1, y1) is y - y1 = m(x - x1).
- The area of a triangle with vertices (x1, y1), (x2, y2), (x3, y3) is (1/2)|x1(y2 - y3) + x2(y3 - y1) + x3(y1 - y2)|. If the area is zero, the points are collinear.
- Three points are collinear if the gradient from the first to the second equals the gradient from the second to the third.

Key Formulas

> Distance: √[(x2 - x1)² + (y2 - y1)²] | Midpoint: ((x1 + x2)/2, (y1 + y2)/2) | Gradient: (y2 - y1)/(x2 - x1) | y = mx + c

Worked Examples

Example 1 — Distance.

Distance between (1, 2) and (4, 6) is √[(3)² + (4)²] = √25 = 5 units.

Example 2 — Midpoint.

Midpoint of (2, 3) and (6, 7) is ((2 + 6)/2, (3 + 7)/2) = (4, 5).

Chapter 16: Introduction to Coordinate Geometry / Analytical Geometry — Practice Questions

Short Questions

Q1. Find the distance between (0, 0) and (3, 4).

Answer: $\sqrt{(9 + 16)} = \sqrt{25} = 5$ units.

Q2. Find the midpoint of (1, 5) and (7, 3).

Answer: $((1 + 7)/2, (5 + 3)/2) = (4, 4)$.

Q3. Find the gradient of the line through (1, 1) and (4, 7).

Answer: $m = (7 - 1)/(4 - 1) = 6/3 = 2$.

Q4. Write the equation of a line with gradient 3 and y-intercept -1.

Answer: $y = 3x - 1$.

Q5. What is the gradient of a horizontal line?

Answer: 0.

Long Questions

Q1. Show that the points A(1, 2), B(4, 6) and C(7, 10) are collinear, using gradients.

Answer: Gradient AB = $(6 - 2)/(4 - 1) = 4/3$. Gradient BC = $(10 - 6)/(7 - 4) = 4/3$. The gradients are equal and B is common to both segments, so A, B and C lie on the same straight line. The line's equation is $y - 2 = (4/3)(x - 1)$.

Q2. Find the area of the triangle with vertices (0, 0), (4, 0) and (0, 3), and find the distance between (4, 0) and (0, 3).

Answer: Base along the x-axis is 4 and height along the y-axis is 3, so area = $(1/2) \times 4 \times 3 = 6$ square units. (The coordinate formula gives the same result.) Distance between (4, 0) and (0, 3) is $\sqrt{[(4)^2 + (3)^2]} = \sqrt{25} = 5$ units — the 3-4-5 triangle again.

Multiple-Choice Questions

Q1. The distance between (0, 0) and (6, 8) is:

(a) 10 (b) 14
(c) 8 (d) 6

Q2. The midpoint of (2, 4) and (4, 8) is:

(a) (3, 6) (b) (6, 12)
(c) (2, 6) (d) (3, 4)

Q3. The gradient of the line through (0, 0) and (2, 6) is:

(a) 2 (b) 3
(c) 6 (d) 1/3

Q4. A vertical line has gradient:

(a) 0 (b) 1
(c) Undefined (d) -1

Q5. The equation y = 2x + 3 has y-intercept:

(a) 2 (b) 3
(c) -3 (d) 0

Q6. Points with zero triangle area are:

(a) Collinear (b) Forming a square
(c) Forming a circle (d) Equidistant

Q7. The x-coordinate is called the:

(a) Ordinate (b) Abscissa
(c) Origin (d) Gradient

Q8. Distance between (1, 1) and (1, 5) is:

(a) 4 (b) 5
(c) 6 (d) 1

Q9. The point (3, -2) lies in which quadrant?

(a) First (b) Second
(c) Third (d) Fourth

Q10. The slope of a line parallel to y = 4x + 1 is:

(a) 1 (b) 4
(c) -4 (d) 1/4

Answer Key (Chapter 16): 1(A), 2(A), 3(B), 4(C), 5(B), 6(A), 7(B), 8(A), 9(D), 10(B)

Final Revision Checklist

Tick off each chapter only when you can answer its MCQs without looking and write out its long questions with full steps.

- Chapter 1 — Real and Complex Numbers: rational and irrational numbers, surds, laws of exponents, complex numbers and conjugates.
- Chapter 2 — Logarithms: definition, the three laws, characteristic and mantissa, scientific notation.
- Chapter 3 — Algebraic Expressions and Formulas: identities for squares and cubes, evaluating expressions.
- Chapter 4 — Factorization: common factor, grouping, difference of squares, splitting the middle term, sum and difference of cubes.
- Chapter 5 — Algebraic Manipulation: HCF and LCM, simplifying rational expressions, rationalizing denominators.
- Chapter 6 — Linear Equations and Inequalities: solving equations, simultaneous equations, reversing the sign with negatives.
- Chapter 7 — Linear Graphs: gradient, intercept, parallel and perpendicular lines, reading graphs.
- Chapter 8 — Quadratic Equations: factorization, completing the square, the quadratic formula, the discriminant, sum and product of roots.
- Chapter 9 — Congruent Triangles: SSS, SAS, ASA, AAS, RHS and CPCT.
- Chapter 10 — Parallelograms and Triangles: properties of parallelograms, areas, medians.
- Chapter 11 — Line Bisectors and Angle Bisectors: perpendicular bisector, angle bisector, incenter and circumcenter.
- Chapter 12 — Sides and Angles of a Triangle: angle sum, exterior angle, triangle inequality, greater side opposite greater angle.
- Chapter 13 — Practical Geometry: SSS, SAS, ASA and RHS constructions with ruler and compasses.
- Chapter 14 — Theorems Related with Area: same base and same parallels, triangle half of parallelogram, medians.
- Chapter 15 — Projection of a Side of a Triangle: Pythagoras' theorem and its converse, acute and obtuse cases, projections.
- Chapter 16 — Coordinate Geometry: distance, midpoint, section formula, gradient, equation of a line, area and collinearity.

> **Good luck.** These notes are original study material prepared for students of the Sindh Textbook Board Class 9 Mathematics book. They do not reproduce textbook text, and they contain no past-paper questions. For the official book, use the Sindh Textbook Board eBooks portal; for the current paper pattern, check your board's latest model paper.