CBSE Class 10 Mathematics Complete Notes 2026-27

NCERT-Based Full Book Notes, MCQs and Solved Questions

All 14 chapters of the NCERT Mathematics textbook for Class X - notes, formulas, worked examples, 140 MCQs with answer keys, 70 solved short questions and 28 solved long questions.

Every chapter gets the same treatment: short revision notes first, then 10 MCQs, then 5 short questions, then 2 long questions - all solved in our own words, step by step.

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What is inside:

- 14 chapters - notes, formulas and worked examples for every chapter.
- 140 MCQs (10 per chapter) with an answer key at the end of each chapter.
- 70 solved short questions (5 per chapter) and 28 solved long questions (2 per chapter).
- Paper pattern, unit-wise weightage and a 3-round study plan.
- Revision checklist at the end of the book.

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How to Use This Book

Round 1 - Learn: read the notes of one chapter, then answer its 10 MCQs without looking at the key. Read the short and long answers and compare them with your own working. One chapter a day finishes the whole book in 14 days.

Round 2 - Revise: re-do only the MCQs and questions you got wrong the first time, using the answer keys in the book. Copy the formulas you keep forgetting into a small notebook.

Round 3 - Practise with papers: in the last weeks, solve official CBSE sample papers and previous year papers in 3 hours, no book, no phone. CBSE publishes its sample papers with marking schemes free at cbseacademic.nic.in. Use those first: they show exactly how CBSE frames questions and how many marks each step earns.

> **Board tip:** in Mathematics, method matters as much as the answer. Write every step clearly, show the formula you are using, and never skip the final statement of the answer. A correct answer with no working earns fewer marks than a full solution.

CBSE Class 10 Mathematics Paper Pattern

Theory paper: 80 marks, 3 hours. Internal assessment: 20 marks. Total: 100 marks.

Passing marks: 33% in theory and 33% in internal assessment separately.

Theory paper structure (80 marks): Section A - 20 MCQs, 20 marks; Section B - 5 very short answer questions of 2 marks each, 10 marks; Section C - 6 short answer questions of 3 marks each, 18 marks; Section D - 4 long answer questions of 5 marks each, 20 marks; Section E - 3 case-based questions of 4 marks each, 12 marks.

Unit-wise weightage (theory, 80 marks): Number Systems 6; Algebra 20; Coordinate Geometry 6; Geometry 15; Trigonometry 12; Mensuration 10; Statistics and Probability 11. This weightage is third-party reproduced from the CBSE curriculum and sample papers, not an official CBSE statement - confirm it on cbseacademic.nic.in.

Internal assessment (20 marks): periodic tests, multiple assessment, portfolio and subject enrichment activities, as set by your school under CBSE guidelines.

Two-attempt exam: from 2026, CBSE holds the Class 10 exam twice a year - a compulsory first exam in February-March and an optional second exam in May. Class 10 students get two chances; the better result counts as per CBSE rules. No 2027 date is printed here - the date sheet had not been released at the time of writing. Check cbseacademic.nic.in when it is published.

About this book: the 14 chapters follow the rationalised NCERT Mathematics textbook for Class X (2026-27). The old chapter "Constructions" was removed in the 2022 rationalisation and is not included. All solutions are written in our own words. Where a question is taken from an NCERT exercise, the answer is our own original solution. This is practice material - not a leaked paper, not a confirmed question list, and not an official CBSE publication. CBSE issues no official guess paper; the questions in this book are each chapter's important questions, chosen from NCERT exercises, previous year papers and chapter weightage. Use them the honest way: if you can solve every question in a chapter without the book, that chapter is ready.

Official sources: free NCERT textbook PDFs are available at ncert.nic.in; sample papers, marking schemes and the official syllabus are at cbseacademic.nic.in.

Marking scheme / blueprint / unit-wise weightage is the Indian term for what the paper carries.

Chapter 1 - Real Numbers

Unit: Number Systems (6 marks) | Euclid's Division Lemma, Fundamental Theorem of Arithmetic, HCF and LCM, Irrational Numbers, Decimal Expansions

Notes

Real numbers are all the numbers on the number line - natural numbers, whole numbers, integers, rational numbers and irrational numbers. This chapter gives you the tools to work with them: Euclid's division algorithm for HCF, prime factorisation, and the test for whether a decimal terminates.

1. Euclid's Division Lemma

For any two positive integers a and b, there exist unique integers q and r such that a = bq + r, where 0 ≤ r < b. Here a is the dividend, b the divisor, q the quotient and r the remainder.

> **Euclid's division algorithm (for HCF):** apply the lemma again and again - divide the larger number by the smaller, then divide the divisor by the remainder, and keep going until the remainder is 0. The last divisor is the HCF.
>
> Example: HCF of 135 and 225. 225 = 135 x 1 + 90; 135 = 90 x 1 + 45; 90 = 45 x 2 + 0. The last divisor is 45, so HCF(135, 225) = 45.

2. Fundamental Theorem of Arithmetic

Every composite number can be expressed as a product of primes, and this factorisation is unique (apart from the order of the factors). For example, 140 = 2 x 2 x 5 x 7 = 2^2 x 5 x 7.

> **HCF and LCM by prime factorisation:** HCF is the product of the smallest power of each common prime factor. LCM is the product of the greatest power of each prime factor in the numbers.
>
> Example: 12 = 2^2 x 3 and 18 = 2 x 3^2. HCF = 2 x 3 = 6. LCM = 2^2 x 3^2 = 36.
>
> **For any two positive integers a and b: HCF(a, b) x LCM(a, b) = a x b.** Check: 6 x 36 = 216 = 12 x 18.

Chapter 1 - Real Numbers (continued)

Irrational numbers and decimal expansions

3. Irrational Numbers

A number that cannot be written as p/q (q not equal to 0, p and q integers) is irrational. Its decimal expansion is non-terminating and non-recurring. Examples: √2, √3, √5 and π are irrational.

Key results: √2, √3 and √5 are irrational. The sum or difference of a rational and an irrational number is irrational (for example, 5 - √3 is irrational). The product of a non-zero rational and an irrational number is irrational. The product of two irrationals can be rational - for example, √3 x √3 = 3.

4. Decimal Expansions of Rational Numbers

Let x = p/q in lowest terms. The decimal expansion of x terminates if and only if the prime factorisation of q is of the form 2^m x 5^n (m, n are whole numbers). Otherwise the expansion is non-terminating and recurring.

> Examples: 17/8 = 2.125 (terminating, since 8 = 2^3). 13/3125 = 0.00416 (terminating, since 3125 = 5^5). 1/7 = 0.142857142857... (non-terminating recurring, since 7 is neither 2 nor 5).

> **Remember for the exam:** 1 is neither prime nor composite. The smallest prime is 2, and it is the only even prime. The smallest composite number is 4. Two numbers are co-prime when their HCF is 1. √2 is irrational, but √4 = 2 is rational - a square root is irrational only when the number is not a perfect square.

Chapter 1 - MCQs (1 to 5)

1. The HCF of 96 and 404 is:

(a) 2 (b) 4 (c) 8 (d) 12

2. The LCM of 12 and 18 is:

(a) 36 (b) 72 (c) 18 (d) 6

3. Which of the following is an irrational number?

(a) √4 (b) √9 (c) √2 (d) 0.5

4. The decimal expansion of 17/8 is:

(a) terminating (b) non-terminating recurring (c) non-terminating non-recurring (d) none of these

5. If HCF(a, b) = 2 and LCM(a, b) = 60, then a x b is:

(a) 120 (b) 60 (c) 12 (d) 30

Chapter 1 - MCQs (6 to 10)

6. The prime factorisation of 140 is:

(a) $2^2 \times 5 \times 7$ (b) $2 \times 5^2 \times 7$ (c) $2^2 \times 5^2 \times 7$ (d) $2 \times 5 \times 7^2$

7. The decimal expansion of 1/7 is:

(a) terminating (b) non-terminating recurring (c) irrational (d) none of these

8. The smallest composite number is:

(a) 1 (b) 2 (c) 3 (d) 4

9. The HCF of 18 and 24 is:

(a) 3 (b) 6 (c) 12 (d) 18

10. Which of the following is a rational number?

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{3} \times \sqrt{3}$ (d) π

Answer Key - Chapter 1

Q	1	2	3	4	5	6	7	8	9	10
Ans	b	a	c	a	a	a	b	d	b	c

Chapter 1 - Solved Short Questions (1 to 3)

1. Find the HCF and LCM of 12 and 18 by prime factorisation.

Solution: 12 = 2 x 2 x 3 = 2^2 x 3. 18 = 2 x 3 x 3 = 2 x 3^2. HCF = product of the smallest powers of common primes = 2 x 3 = 6. LCM = product of the greatest powers of all primes = 2^2 x 3^2 = 4 x 9 = 36. Check: HCF x LCM = 6 x 36 = 216 = 12 x 18. Correct.

2. Use Euclid's division algorithm to find the HCF of 135 and 225.

Solution: 225 = 135 x 1 + 90. Then 135 = 90 x 1 + 45. Then 90 = 45 x 2 + 0. The remainder is now 0, so the last divisor, 45, is the HCF. Hence HCF(135, 225) = 45.

3. Show that 5 - √3 is irrational.

Solution: Suppose 5 - √3 is rational. Then √3 = 5 - (5 - √3). Since 5 is rational and (5 - √3) is assumed rational, their difference √3 would be rational (difference of two rationals is rational). But √3 is irrational. This is a contradiction. So our assumption is wrong and 5 - √3 is irrational.

Chapter 1 - Solved Short Questions (4 to 5)

4. Find the decimal expansion of 13/3125 and state whether it is terminating or not.

Solution: 3125 = 5 x 5 x 5 x 5 x 5 = 5^5, which is of the form 2^m x 5^n (with m = 0, n = 5). So the decimal expansion terminates. 13/3125 = 13 x 32 / (3125 x 32) = 416/100000 = 0.00416. Hence the expansion is terminating, 0.00416.

5. Prove that √2 is irrational.

Solution: Suppose √2 is rational. Write √2 = p/q, where p and q are co-prime positive integers. Then 2 = p^2/q^2, so $p^2 = 2q^2$. Thus p^2 is even, so p is even. Let p = 2r. Then $4r^2 = 2q^2$, so $q^2 = 2r^2$. Thus q^2 is even, so q is even. But then p and q have the common factor 2, contradicting that they are co-prime. Hence our assumption is wrong and √2 is irrational.

Chapter 1 - Solved Long Questions

1. Prove that √5 is irrational. Hence show that 3 + 2√5 is irrational.

Solution: Suppose √5 is rational, √5 = p/q with p, q co-prime positive integers. Then 5 = p^2/q^2, so $p^2 = 5q^2$. Thus 5 divides p^2, so 5 divides p. Let p = 5r. Then $25r^2 = 5q^2$, so $q^2 = 5r^2$. Thus 5 divides q^2, so 5 divides q. But p and q are co-prime, a contradiction. Hence √5 is irrational.

Now suppose 3 + 2√5 is rational, equal to r. Then 2√5 = r - 3, so √5 = (r - 3)/2, a rational number (since r - 3 and 2 are rational). This contradicts the irrationality of √5. Hence 3 + 2√5 is irrational.

2. Three bells toll at intervals of 12, 15 and 18 minutes respectively. If they toll together at 8:00 a.m., at what time will they toll together again?

Solution: The bells toll together again after a time equal to the LCM of 12, 15 and 18 minutes. 12 = 2^2 x 3; 15 = 3 x 5; 18 = 2 x 3^2. LCM = 2^2 x 3^2 x 5 = 4 x 9 x 5 = 180 minutes = 3 hours. They toll together at 8:00 a.m., so they will toll together again at 8:00 + 3 hours = 11:00 a.m.

Chapter 2 - Polynomials

Unit: Algebra (20 marks) | Degree, Zeroes, Relationship Between Zeroes and Coefficients, Division Algorithm

Notes

A **polynomial** in x is an expression of the form a(n)x^n + a(n-1)x^(n-1) + ... + a(1)x + a(0), where the coefficients are real numbers and n is a whole number. The highest power of x is the **degree** of the polynomial. A polynomial of degree 1 is linear, degree 2 is quadratic, degree 3 is cubic.

1. Zeroes of a Polynomial

A real number a is a **zero** of p(x) if p(a) = 0. Geometrically, the zeroes are the x-coordinates of the points where the graph of y = p(x) cuts the x-axis. A linear polynomial has at most 1 zero, a quadratic at most 2, and a cubic at most 3.

> **Quadratic polynomial $ax^2 + bx + c$:** if α and β are the zeroes, then sum of zeroes $\alpha + \beta = -b/a$ and product of zeroes $\alpha\beta = c/a$.
>
> **Cubic polynomial $ax^3 + bx^2 + cx + d$:** $\alpha + \beta + \gamma = -b/a$; $\alpha\beta + \beta\gamma + \gamma\alpha = c/a$; $\alpha\beta\gamma = -d/a$.

2. Division Algorithm

For polynomials p(x) and g(x) (g(x) not zero), there exist q(x) and r(x) such that p(x) = g(x) x q(x) + r(x), where either r(x) = 0 or degree of r(x) is less than degree of g(x). If r(x) = 0, then g(x) is a factor of p(x).

> **How to form a polynomial from its zeroes:** if the zeroes are α and β, the polynomial is $x^2 - (\alpha + \beta)x + \alpha\beta$. Example: zeroes 2 and -3 give $x^2 - (2 - 3)x + (2 \times -3) = x^2 + x - 6$.
>
> **Remainder theorem:** if p(x) is divided by (x - a), the remainder is p(a). **Factor theorem:** (x - a) is a factor of p(x) if and only if p(a) = 0.

Chapter 2 - Polynomials (continued)

Graphs and worked division

3. Graphs of Polynomials

The graph of a linear polynomial is a straight line. The graph of a quadratic polynomial is a parabola - it opens upwards if a > 0 and downwards if a < 0. The number of zeroes equals the number of times the graph cuts the x-axis: a parabola can cut the x-axis at 0, 1 or 2 points, so a quadratic has 0, 1 or 2 real zeroes.

4. Worked Division Example

Divide $2x^4 - 3x^3 - 3x^2 + 6x - 2$ by $x^2 - 2$.

Step 1: $2x^4 / x^2 = 2x^2$. Multiply: $2x^2(x^2 - 2) = 2x^4 - 4x^2$. Subtract from the dividend: $-3x^3 + x^2 + 6x - 2$.

Step 2: $-3x^3 / x^2 = -3x$. Multiply: $-3x(x^2 - 2) = -3x^3 + 6x$. Subtract: $x^2 - 2$.

Step 3: $x^2 / x^2 = 1$. Multiply: $1(x^2 - 2) = x^2 - 2$. Subtract: remainder 0.

> Quotient = $2x^2 - 3x + 1$, Remainder = 0. Check: $(x^2 - 2)(2x^2 - 3x + 1) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$. Verified.

> **Exam tip:** always verify a division by multiplying quotient and divisor and adding the remainder - it must equal the dividend. In factorisation questions, first find one zero by trial (try x = 1, -1, 2, -2), then divide to get the remaining quadratic factor.

Chapter 2 - MCQs (1 to 5)

1. The zeroes of $x^2 - 2x - 8$ are:

(a) 4, -2 (b) -4, 2 (c) 4, 2 (d) -4, -2

2. The sum of the zeroes of $x^2 - 5x + 6$ is:

(a) 5 (b) -5 (c) 6 (d) -6

3. The product of the zeroes of $x^2 - 5x + 6$ is:

(a) 5 (b) -5 (c) 6 (d) -6

4. A quadratic polynomial with zeroes 2 and -3 is:

(a) $x^2 + x - 6$ (b) $x^2 - x - 6$ (c) $x^2 + x + 6$ (d) $x^2 - x + 6$

5. The degree of the polynomial $5x^3 - 2x^2 + 7$ is:

(a) 2 (b) 3 (c) 5 (d) 7

Chapter 2 - MCQs (6 to 10)

6. The zeroes of $x^2 - 9$ are:

(a) 3, 3 (b) -3, -3 (c) 3, -3 (d) 0, 3

7. If α and β are the zeroes of $x^2 - 4x + 3$, then:

(a) $\alpha + \beta = 4, \alpha\beta = 3$ (b) $\alpha + \beta = -4, \alpha\beta = 3$ (c) $\alpha + \beta = 4, \alpha\beta = -3$ (d) $\alpha + \beta = -4, \alpha\beta = -3$

8. The maximum number of zeroes a cubic polynomial can have is:

(a) 1 (b) 2 (c) 3 (d) 4

9. A quadratic polynomial with zeroes 0 and 5 is:

(a) $x^2 - 5x$ (b) $x^2 + 5x$ (c) $x^2 - 5$ (d) $x - 5$

10. If one zero of $x^2 + px + 12$ is 3, then p is:

(a) 7 (b) -7 (c) 4 (d) -4

Answer Key - Chapter 2

Q	1	2	3	4	5	6	7	8	9	10
Ans	a	a	c	a	b	c	a	c	a	b

Chapter 2 - Solved Short Questions (1 to 3)

1. Find the zeroes of $x^2 + 7x + 12$ and verify the relationship between the zeroes and the coefficients.

Solution: $x^2 + 7x + 12 = x^2 + 3x + 4x + 12 = x(x + 3) + 4(x + 3) = (x + 3)(x + 4)$. Zeroes are -3 and -4. Sum = -3 + (-4) = -7 = -b/a = -7/1. Product = (-3)(-4) = 12 = c/a = 12/1. Verified.

2. Find a quadratic polynomial whose zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

Solution: Sum of zeroes = $\sqrt{2} + (-\sqrt{2}) = 0$. Product = $\sqrt{2} \times (-\sqrt{2}) = -2$. Required polynomial = x^2 - (sum)x + product = $x^2 - 0x - 2 = x^2 - 2$.

3. Divide $x^3 - 3x^2 + x + 2$ by $x - 2$ and find the quotient and remainder.

Solution: $x^3/(x) = x^2$; $x^2(x - 2) = x^3 - 2x^2$; subtract: $-x^2 + x$. $-x^2/x = -x$; $-x(x - 2) = -x^2 + 2x$; subtract: $-x + 2$. $-x/x = -1$; $-1(x - 2) = -x + 2$; subtract: remainder 0. Quotient = $x^2 - x - 1$, Remainder = 0.

Chapter 2 - Solved Short Questions (4 to 5)

4. If α and β are the zeroes of $2x^2 - 5x + 3$, find $1/\alpha + 1/\beta$.

Solution: $\alpha + \beta = -b/a = 5/2$ and $\alpha\beta = c/a = 3/2$. Now $1/\alpha + 1/\beta = (\alpha + \beta)/\alpha\beta = (5/2) / (3/2) = 5/3$.

5. Check whether $x + 2$ is a factor of $x^3 + 3x^2 + 5x + 6$.

Solution: Let $p(x) = x^3 + 3x^2 + 5x + 6$. By the factor theorem, $(x + 2)$ is a factor if $p(-2) = 0$. $p(-2) = (-2)^3 + 3(-2)^2 + 5(-2) + 6 = -8 + 12 - 10 + 6 = 0$. Hence $x + 2$ is a factor of $p(x)$.

Chapter 2 - Solved Long Questions

1. Divide $2x^4 - 3x^3 - 3x^2 + 6x - 2$ by $x^2 - 2$ and verify the division algorithm.

Solution: Step 1: $2x^4 / x^2 = 2x^2$. $2x^2(x^2 - 2) = 2x^4 - 4x^2$. Subtracting gives $-3x^3 + x^2 + 6x - 2$. Step 2: $-3x^3 / x^2 = -3x$. $-3x(x^2 - 2) = -3x^3 + 6x$. Subtracting gives $x^2 - 2$. Step 3: $x^2 / x^2 = 1$. $1(x^2 - 2) = x^2 - 2$. Subtracting gives remainder 0. So quotient $q(x) = 2x^2 - 3x + 1$ and remainder $r(x) = 0$.

Verification: $g(x) \times q(x) + r(x) = (x^2 - 2)(2x^2 - 3x + 1) + 0 = 2x^4 - 3x^3 + x^2 - 4x^2 + 6x - 2 = 2x^4 - 3x^3 - 3x^2 + 6x - 2 = p(x)$. Verified.

2. Find all the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

Solution: Since $\sqrt{2}$ and $-\sqrt{2}$ are zeroes, $(x - \sqrt{2})(x + \sqrt{2}) = x^2 - 2$ is a factor of the polynomial. Dividing by $x^2 - 2$ (from Q1) gives quotient $2x^2 - 3x + 1$. Factor it: $2x^2 - 3x + 1 = 2x^2 - 2x - x + 1 = 2x(x - 1) - 1(x - 1) = (x - 1)(2x - 1)$. Its zeroes are 1 and 1/2. Hence all four zeroes are $\sqrt{2}$, $-\sqrt{2}$, 1 and 1/2.

Chapter 3 - Pair of Linear Equations in Two Variables

Unit: Algebra (20 marks) | Graphical Method, Substitution, Elimination, Cross-Multiplication, Word Problems

Notes

A pair of linear equations in two variables is written as a1x + b1y + c1 = 0 and a2x + b2y + c2 = 0. A solution is a pair (x, y) that satisfies both equations. Graphically, each equation is a straight line, and the solution is the point where the lines meet.

1. Graphical Method and Consistency

Compare the ratios a1/a2, b1/b2 and c1/c2:

> **Intersecting lines:** a1/a2 is not equal to b1/b2. One unique solution. The pair is consistent.
>
> **Coincident lines:** a1/a2 = b1/b2 = c1/c2. Infinitely many solutions. The pair is consistent (dependent).
>
> **Parallel lines:** a1/a2 = b1/b2 is not equal to c1/c2. No solution. The pair is inconsistent.

2. Algebraic Methods

Substitution: express one variable in terms of the other from one equation and substitute it in the second equation.

Elimination: multiply the equations so that one variable has the same (or opposite) coefficient in both, then add or subtract to remove it.

Cross-multiplication: for a1x + b1y + c1 = 0 and a2x + b2y + c2 = 0, x/(b1c2 - b2c1) = y/(c1a2 - c2a1) = 1/(a1b2 - a2b1). Use it when the coefficients are large or fractions appear.

> **Exam tip:** before solving, always check the ratios first - in a 1-mark or 2-mark question you may only be asked whether the pair is consistent or how many solutions it has, and the ratios give the answer without any solving.

Chapter 3 - Pair of Linear Equations (continued)

Word problems and cross-multiplication

3. Word Problems - The Method

Step 1: read the problem and choose the unknowns (usually x and y). Step 2: write two equations from the two conditions given. Step 3: solve the pair by the easiest method. Step 4: check the answer against the problem statement and write the final answer in words.

4. Worked Example - Cross-Multiplication

Solve 2x + y = 7 and x - y = 2 by cross-multiplication. Write as 2x + y - 7 = 0 and x - y - 2 = 0.

x / [(1)(-2) - (-1)(-7)] = y / [(-7)(1) - (-2)(2)] = 1 / [(2)(-1) - (1)(1)]

x / (-2 - 7) = y / (-7 + 4) = 1 / (-2 - 1), so x/(-9) = y/(-3) = 1/(-3).

> From x/(-9) = 1/(-3): x = 3. From y/(-3) = 1/(-3): y = 1. Solution: x = 3, y = 1. Check: 2(3) + 1 = 7 and 3 - 1 = 2. Correct.

> **Common mistake:** when you multiply an equation to eliminate a variable, multiply EVERY term, including the constant. Forgetting the constant is the most common error in elimination questions.

Chapter 3 - MCQs (1 to 5)

1. The pair 2x + 3y = 7 and 4x + 6y = 14 has:

(a) a unique solution (b) infinitely many solutions (c) no solution (d) two solutions

2. The pair 2x + y = 5 and 3x + 2y = 8 has:

(a) a unique solution (b) infinitely many solutions (c) no solution (d) none of these

3. The pair x + 2y = 3 and 2x + 4y = 6 has:

(a) a unique solution (b) infinitely many solutions (c) no solution (d) none of these

4. The solution of x + y = 5 and x - y = 1 is:

(a) x = 3, y = 2 (b) x = 2, y = 3 (c) x = 4, y = 1 (d) x = 1, y = 4

5. The lines x = 2 and y = 3 intersect at:

(a) (2, 3) (b) (3, 2) (c) (0, 0) (d) (2, 0)

Chapter 3 - MCQs (6 to 10)

6. Graphically, a pair of parallel lines has:

(a) one solution (b) no solution (c) infinitely many solutions (d) two solutions

7. The lines 3x + 2y = 12 and 6x + 4y = 24 are:

(a) intersecting (b) parallel (c) coincident (d) none of these

8. If a1/a2 = b1/b2 is not equal to c1/c2, the lines are:

(a) intersecting (b) parallel (c) coincident (d) perpendicular

9. The solution of x + y = 10 and x - y = 4 is:

(a) x = 7, y = 3 (b) x = 3, y = 7 (c) x = 6, y = 4 (d) x = 4, y = 6

10. The pair 2x - 3y = 5 and 4x - 6y = 7 is:

(a) consistent (b) inconsistent (c) dependent (d) none of these

Answer Key - Chapter 3

Q	1	2	3	4	5	6	7	8	9	10
Ans	b	a	b	a	a	b	c	b	a	b

Chapter 3 - Solved Short Questions (1 to 3)

1. Solve x + y = 14 and x - y = 4.

Solution: Adding the two equations: (x + y) + (x - y) = 14 + 4, so 2x = 18 and x = 9. From x + y = 14: 9 + y = 14, so y = 5. Solution: x = 9, y = 5.

2. Check whether the pair 3x + 2y = 5 and 6x + 4y = 10 is consistent.

Solution: a1/a2 = 3/6 = 1/2, b1/b2 = 2/4 = 1/2, c1/c2 = 5/10 = 1/2. Since a1/a2 = b1/b2 = c1/c2, the lines are coincident. The pair is consistent and has infinitely many solutions.

3. Find the value of k for which 2x + 3y = 7 and (k + 1)x + 6y = 14 have infinitely many solutions.

Solution: For infinitely many solutions, a1/a2 = b1/b2 = c1/c2. b1/b2 = 3/6 = 1/2 and c1/c2 = 7/14 = 1/2. So we need a1/a2 = 2/(k + 1) = 1/2, giving k + 1 = 4, so k = 3.

Chapter 3 - Solved Short Questions (4 to 5)

4. Solve 2x + y = 7 and x - y = 2 by substitution.

Solution: From the second equation, x = y + 2. Substitute in the first: 2(y + 2) + y = 7, so 2y + 4 + y = 7, 3y = 3, y = 1. Then x = 1 + 2 = 3. Solution: x = 3, y = 1.

5. Find the value of k for which 3x + ky = 5 and 6x + 4y = 12 have no solution.

Solution: For no solution, a1/a2 = b1/b2 is not equal to c1/c2. a1/a2 = 3/6 = 1/2 and c1/c2 = 5/12. So we need b1/b2 = k/4 = 1/2, giving k = 2. With k = 2, a1/a2 = b1/b2 = 1/2, which is not equal to c1/c2 = 5/12, so the pair has no solution.

Chapter 3 - Solved Long Questions

1. 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs 46. Find the cost of one pencil and that of one pen.

Solution: Let the cost of one pencil be Rs x and of one pen Rs y. Then 5x + 7y = 50 and 7x + 5y = 46. Multiply the first by 7 and the second by 5: 35x + 49y = 350 and 35x + 25y = 230. Subtracting: 24y = 120, so y = 5. From 5x + 7y = 50: 5x + 35 = 50, so 5x = 15 and x = 3. Hence one pencil costs Rs 3 and one pen costs Rs 5.

2. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

Solution: Let the speed of the boat in still water be x km/h and of the stream y km/h. Upstream speed = x - y, downstream speed = x + y. Then 30/(x - y) + 44/(x + y) = 10 and 40/(x - y) + 55/(x + y) = 13. Put 1/(x - y) = u and 1/(x + y) = v: 30u + 44v = 10 and 40u + 55v = 13. Multiply the first by 4 and the second by 3: 120u + 176v = 40 and 120u + 165v = 39. Subtracting: 11v = 1, so v = 1/11. From 30u + 44(1/11) = 10: 30u + 4 = 10, so u = 1/5. Hence x - y = 5 and x + y = 11. Adding: 2x = 16, x = 8; then y = 3. Speed of the boat in still water is 8 km/h and speed of the stream is 3 km/h.

Chapter 4 - Quadratic Equations

Unit: Algebra (20 marks) | Standard Form, Factorisation, Completing the Square, Quadratic Formula, Discriminant, Nature of Roots

Notes

A **quadratic equation** in x is an equation of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and a is not equal to 0. A real number a is a root of the equation if it satisfies it. A quadratic equation has at most two roots.

1. Solving by Factorisation

Write the quadratic as a product of two linear factors and set each factor equal to zero. Example: $x^2 - 3x - 10 = 0$ gives $(x - 5)(x + 2) = 0$, so $x = 5$ or $x = -2$.

2. Completing the Square

Write $x^2 + bx + c = 0$ as $(x + b/2)^2 = (b/2)^2 - c$, then take square roots. Example: $x^2 + x - 6 = 0$ becomes $(x + 1/2)^2 = 6 + 1/4 = 25/4$, so $x + 1/2 = \pm 5/2$, giving $x = 2$ or $x = -3$.

3. Quadratic Formula

> For $ax^2 + bx + c = 0$: **$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$**
>
> Example: $3x^2 - 5x + 2 = 0$. Here $a = 3$, $b = -5$, $c = 2$. $x = [5 \pm \sqrt{(25 - 24)}] / 6 = [5 \pm 1] / 6$. So $x = 1$ or $x = 2/3$.

4. Discriminant and Nature of Roots

The expression $D = b^2 - 4ac$ is called the **discriminant**.

> If $D > 0$: two distinct real roots. If $D = 0$: two equal real roots. If $D < 0$: no real roots.
>
> Example: $2x^2 - 4x + 3 = 0$ has $D = 16 - 24 = -8 < 0$, so it has no real roots.

Chapter 4 - Quadratic Equations (continued)

Sum and product of roots, word problems

5. Sum and Product of Roots

If α and β are the roots of $ax^2 + bx + c = 0$, then $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$. Use these to check your answers: for $x^2 - 5x + 6 = 0$, the roots are 2 and 3; sum = 5 = -(-5)/1 and product = 6 = 6/1. Correct.

6. Word Problems - The Method

Step 1: let the unknown be x. Step 2: translate the words into a quadratic equation. Step 3: solve it by factorisation or the formula. Step 4: reject any root that does not make sense in the problem (for example, a negative speed or a negative length), and write the answer in words.

> Worked example: the sum of the squares of two consecutive natural numbers is 365. Let the numbers be n and n + 1. Then $n^2 + (n + 1)^2 = 365$, so $2n^2 + 2n - 364 = 0$, or $n^2 + n - 182 = 0$. Factorising: $(n + 14)(n - 13) = 0$. Since n is a natural number, n = 13. The numbers are 13 and 14. Check: $13^2 + 14^2 = 169 + 196 = 365$. Correct.

> **Exam tip:** always find the discriminant first when a question asks about the nature of roots - do not solve the equation. And in word problems, always reject the negative root when the quantity (speed, length, time, number of articles) cannot be negative.

Chapter 4 - MCQs (1 to 5)

1. The discriminant of $2x^2 - 4x + 3 = 0$ is:

(a) -8 (b) 8 (c) 40 (d) -40

2. The roots of $x^2 - 9 = 0$ are:

(a) 3, 3 (b) -3, -3 (c) 3, -3 (d) 0, 9

3. The nature of the roots of $x^2 - 6x + 9 = 0$ is:

(a) real and equal (b) real and distinct (c) no real roots (d) none of these

4. The roots of $x^2 + 4x + 4 = 0$ are:

(a) -2, -2 (b) 2, 2 (c) 2, -2 (d) 0, -2

5. The sum of the roots of $3x^2 - 6x + 1 = 0$ is:

(a) 2 (b) -2 (c) 1/3 (d) 6

Chapter 4 - MCQs (6 to 10)

6. The product of the roots of $2x^2 - 3x - 5 = 0$ is:

(a) 5/2 (b) -5/2 (c) 3/2 (d) -3/2

7. Which equation has no real roots?

(a) $2x^2 - 4x + 3 = 0$ (b) $x^2 - 4x + 3 = 0$ (c) $x^2 - 2x - 3 = 0$ (d) $3x^2 - 6x + 3 = 0$

8. The roots of $x^2 - 5x + 6 = 0$ are:

(a) 2, 3 (b) -2, -3 (c) 2, -3 (d) -2, 3

9. If one root of $x^2 - 7x + 12 = 0$ is 3, the other root is:

(a) 4 (b) 3 (c) -4 (d) 12

10. The quadratic formula is:

(a) $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$ (b) $x = [-b \pm \sqrt{(b^2 + 4ac)}] / 2a$ (c) $x = [b \pm \sqrt{(b^2 - 4ac)}] / 2a$ (d) $x = [-b \pm \sqrt{(b^2 - 4ac)}] / a$

Answer Key - Chapter 4

Q	1	2	3	4	5	6	7	8	9	10
Ans	a	c	a	a	a	b	a	a	a	a

Chapter 4 - Solved Short Questions (1 to 3)

1. Solve $x^2 - 3x - 10 = 0$ by factorisation.

Solution: $x^2 - 3x - 10 = x^2 - 5x + 2x - 10 = x(x - 5) + 2(x - 5) = (x - 5)(x + 2) = 0$. So $x = 5$ or $x = -2$.

2. Find the value of k for which $2x^2 + kx + 3 = 0$ has equal roots.

Solution: For equal roots, $D = 0$. $D = b^2 - 4ac = k^2 - 4(2)(3) = k^2 - 24$. Setting $D = 0$ gives $k^2 = 24$, so $k = \pm\sqrt{24} = \pm 2\sqrt{6}$.

3. Solve $3x^2 - 5x + 2 = 0$ using the quadratic formula.

Solution: $a = 3$, $b = -5$, $c = 2$. $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a = [5 \pm \sqrt{(25 - 24)}] / 6 = [5 \pm 1] / 6$. So $x = 6/6 = 1$ or $x = 4/6 = 2/3$.

Chapter 4 - Solved Short Questions (4 to 5)

4. Find the nature of the roots of $3x^2 - 4\sqrt{3}x + 4 = 0$.

Solution: $D = b^2 - 4ac = (4\sqrt{3})^2 - 4(3)(4) = 48 - 48 = 0$. Since $D = 0$, the roots are real and equal.

5. Solve $x^2 + x - 6 = 0$ by completing the square.

Solution: $x^2 + x = 6$. Add $(1/2)^2 = 1/4$ to both sides: $x^2 + x + 1/4 = 6 + 1/4 = 25/4$. So $(x + 1/2)^2 = 25/4$, giving $x + 1/2 = \pm 5/2$. Hence $x = 5/2 - 1/2 = 2$ or $x = -5/2 - 1/2 = -3$.

Chapter 4 - Solved Long Questions

1. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Solution: Let the speed be x km/h. Time taken = 360/x hours. At speed (x + 5) km/h, time = 360/(x + 5) hours. The difference is 1 hour: 360/x - 360/(x + 5) = 1. Multiply by x(x + 5): 360(x + 5) - 360x = x(x + 5), so 1800 = x^2 + 5x, or x^2 + 5x - 1800 = 0. Factorising: (x + 45)(x - 40) = 0. So x = -45 or x = 40. Speed cannot be negative, so x = 40. The speed of the train is 40 km/h.

2. The sum of the squares of two consecutive natural numbers is 365. Find the numbers.

Solution: Let the numbers be n and n + 1. Then n^2 + $(n + 1)^2$ = 365. Expanding: n^2 + n^2 + 2n + 1 = 365, so $2n^2$ + 2n - 364 = 0, or n^2 + n - 182 = 0. Factorising: (n + 14)(n - 13) = 0. Since n is a natural number, n = 13. The numbers are 13 and 14. Check: 13^2 + 14^2 = 169 + 196 = 365. Correct.

Chapter 5 - Arithmetic Progressions

Unit: Algebra (20 marks) | nth Term, Sum of First n Terms, Word Problems

Notes

An **arithmetic progression (AP)** is a list of numbers in which each term is obtained by adding a fixed number to the preceding term. The fixed number is the **common difference** d. The first term is denoted by a.

Examples: 2, 7, 12, 17, ... is an AP with a = 2 and d = 5. 10, 7, 4, 1, ... is an AP with a = 10 and d = -3.

1. nth Term of an AP

a(n) = a + (n - 1)d

Example: the 10th term of 2, 7, 12, ... is a(10) = 2 + (10 - 1) x 5 = 2 + 45 = 47.

To find which term a number is: set a + (n - 1)d equal to the number and solve for n. If n comes out a whole number, the number is a term of the AP.

2. Sum of the First n Terms

S(n) = n/2 x [2a + (n - 1)d] or **S(n) = n/2 x (a + l)**, where l is the last term.

Example: sum of the first 10 natural numbers = 10/2 x (1 + 10) = 5 x 11 = 55.

Also useful: a(n) = S(n) - S(n - 1). If you know the sum formula, you can get any term from it.

Exam tip: in word problems, first identify a and d from the problem, then decide whether the question asks for a term (use a(n)) or a total (use S(n)). "How many terms?" means find n. "Which term is 77?" means solve a(n) = 77 for n.

Chapter 5 - Arithmetic Progressions (continued)

Worked examples

3. Worked Example - Number of Terms

How many terms of the AP 9, 17, 25, ... must be taken to give a sum of 636?

Here a = 9 and d = 8. S(n) = n/2 x [2a + (n - 1)d] = 636. So n/2 x [18 + (n - 1) x 8] = 636. Simplifying: n(4n + 5) = 636, or $4n^2 + 5n - 636 = 0$. Discriminant = 25 + 4 x 4 x 636 = 10201 = 101^2. So n = (-5 + 101)/8 = 12 (the negative root is rejected). Hence 12 terms are needed.

4. Worked Example - Sum with Negative Difference

Find the sum of the first 22 terms of the AP 8, 3, -2, ...

Here a = 8 and d = 3 - 8 = -5. S(22) = 22/2 x [2 x 8 + (22 - 1) x (-5)] = 11 x [16 - 105] = 11 x (-89) = -979.

> Note: a sum can be negative when the terms become negative - that is normal. Always check the sign of d first; a negative d means the terms decrease.

> **Remember:** the sum of the first n natural numbers is n(n + 1)/2. The sum of the first n odd numbers is n^2. The sum of the first n even numbers is n(n + 1). These three results are asked again and again - learn them by heart.

Chapter 5 - MCQs (1 to 5)

1. The 10th term of the AP 2, 7, 12, ... is:

(a) 47 (b) 45 (c) 52 (d) 42

2. The sum of the first 10 natural numbers is:

(a) 45 (b) 55 (c) 50 (d) 100

3. The common difference of the AP 7, 13, 19, ... is:

(a) 6 (b) 7 (c) 5 (d) 13

4. The 5th term of an AP with a = 3 and d = 4 is:

(a) 19 (b) 15 (c) 23 (d) 11

5. Which of the following is an AP?

(a) 1, 3, 9, 27 (b) 2, 4, 6, 8 (c) 1, 4, 9, 16 (d) 3, 6, 12, 24

Chapter 5 - MCQs (6 to 10)

6. The 20th term of the AP 10, 7, 4, ... is:

(a) -47 (b) -37 (c) 47 (d) -57

7. The sum of the first n terms of an AP is given by:

(a) n/2 x [2a + (n - 1)d] (b) n/2 x [a + (n - 1)d] (c) n x [2a + (n - 1)d] (d) a + (n - 1)d

8. If a = 5 and d = 3, then the 7th term is:

(a) 20 (b) 23 (c) 26 (d) 17

9. The number of terms in the AP 3, 7, 11, ..., 47 is:

(a) 12 (b) 11 (c) 13 (d) 10

10. The sum of the first 5 terms of 2, 4, 6, 8, 10 is:

(a) 20 (b) 25 (c) 30 (d) 15

Answer Key - Chapter 5

Q	1	2	3	4	5	6	7	8	9	10
Ans	a	b	a	a	b	a	a	b	a	c

Chapter 5 - Solved Short Questions (1 to 3)

1. Find the 11th term of the AP 7, 13, 19, ...

Solution: a = 7, d = 13 - 7 = 6. a(11) = a + 10d = 7 + 10 x 6 = 7 + 60 = 67.

2. Find the sum of the first 22 terms of the AP 8, 3, -2, ...

Solution: a = 8, d = -5. S(22) = 22/2 x [2 x 8 + 21 x (-5)] = 11 x [16 - 105] = 11 x (-89) = -979.

3. Which term of the AP 5, 9, 13, ... is 77?

Solution: a = 5, d = 4. Let 77 be the nth term: a(n) = a + (n - 1)d = 77. So 5 + (n - 1) x 4 = 77, giving (n - 1) x 4 = 72, n - 1 = 18, n = 19. Hence 77 is the 19th term.

Chapter 5 - Solved Short Questions (4 to 5)

4. Find the sum of the first 15 multiples of 8.

Solution: The multiples of 8 are 8, 16, 24, ..., an AP with a = 8 and d = 8. S(15) = 15/2 x [2 x 8 + 14 x 8] = 15/2 x [16 + 112] = 15/2 x 128 = 15 x 64 = 960.

5. How many terms of the AP 9, 17, 25, ... must be taken to give a sum of 636?

Solution: a = 9, d = 8. S(n) = n/2 x [18 + (n - 1) x 8] = 636. So n(4n + 5) = 636, or $4n^2 + 5n - 636 = 0$. Discriminant = 25 + 10176 = 10201 = 101^2. n = (-5 + 101)/8 = 12 (the negative root is rejected). Hence 12 terms.

Chapter 5 - Solved Long Questions

1. Find the sum of the first 1000 positive integers.

Solution: The first 1000 positive integers are 1, 2, 3, ..., 1000, an AP with a = 1, d = 1 and l = 1000. S(1000) = n/2 x (a + l) = 1000/2 x (1 + 1000) = 500 x 1001 = 500500. In general, the sum of the first n natural numbers is n(n + 1)/2.

2. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Solution: The prizes form an AP with n = 7, d = -20 and sum 700. Let the first prize be a. S(7) = 7/2 x [2a + 6 x (-20)] = 700. So 7/2 x (2a - 120) = 700, giving 2a - 120 = 200, so 2a = 320 and a = 160. The prizes are 160, 140, 120, 100, 80, 60 and 40 rupees. Check: 160 + 140 + 120 + 100 + 80 + 60 + 40 = 700. Correct.

Chapter 6 - Triangles

Unit: Geometry (15 marks) | Similarity, Basic Proportionality Theorem, Pythagoras Theorem

Notes

Two figures are **similar** if they have the same shape but not necessarily the same size. All congruent figures are similar, but similar figures need not be congruent. For triangles, similarity is decided by angles and by the ratios of sides.

1. Similarity Criteria

> **AAA (Angle-Angle-Angle):** if two angles of one triangle equal two angles of another, the triangles are similar. (AA is enough - the third angles are automatically equal.)
>
> **SSS (Side-Side-Side):** if the sides of one triangle are in the same ratio as the sides of another, the triangles are similar.
>
> **SAS (Side-Angle-Side):** if one angle of a triangle equals one angle of another and the sides around those angles are in the same ratio, the triangles are similar.

2. Basic Proportionality Theorem (Thales Theorem)

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. In triangle ABC, if DE is parallel to BC with D on AB and E on AC, then AD/DB = AE/EC.

Converse: if a line divides any two sides of a triangle in the same ratio, the line is parallel to the third side.

> **Ratio of areas:** the ratio of the areas of two similar triangles equals the ratio of the squares of their corresponding sides. If $\Delta ABC \sim \Delta DEF$ with AB/DE = 2/3, then area(ABC)/area(DEF) = 4/9.

Chapter 6 - Triangles (continued)

Pythagoras theorem

3. Pythagoras Theorem

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. If ABC is right-angled at B, then $AC^2 = AB^2 + BC^2$.

Converse: if in a triangle the square of the longest side equals the sum of the squares of the other two sides, the triangle is right-angled.

4. Worked Example - BPT

In triangle ABC, DE is parallel to BC. If AD = 3 cm, DB = 6 cm and AE = 4 cm, find EC.

By the Basic Proportionality Theorem, AD/DB = AE/EC. So 3/6 = 4/EC, giving EC = 8 cm.

5. Worked Example - Pythagoras

A ladder 5 m long reaches a window 4 m above the ground. How far is the foot of the ladder from the wall?

Let the distance be x m. By Pythagoras: $x^2 + 4^2 = 5^2$, so $x^2 = 25 - 16 = 9$, giving x = 3 m. The foot of the ladder is 3 m from the wall.

> **Common triples to remember:** (3, 4, 5), (5, 12, 13), (8, 15, 17), (7, 24, 25) and their multiples. If you spot a triple in a Pythagoras question, the answer is instant.

Chapter 6 - MCQs (1 to 5)

1. In ΔABC, DE || BC, AD = 3, DB = 6 and AE = 4. Then EC is:

(a) 8 (b) 6 (c) 4 (d) 12

2. In a right triangle with legs 3 cm and 4 cm, the hypotenuse is:

(a) 6 cm (b) 5 cm (c) 7 cm (d) 4 cm

3. If ΔABC ~ ΔDEF with AB : DE = 2 : 3, the ratio of their areas is:

(a) 4 : 9 (b) 2 : 3 (c) 9 : 4 (d) 3 : 2

4. The Basic Proportionality Theorem is also known as:

(a) Pythagoras theorem (b) Thales theorem (c) Converse of Pythagoras (d) Mid-point theorem

5. Two triangles are similar if:

(a) AAA (b) SSS (c) SAS (d) all of these

Chapter 6 - MCQs (6 to 10)

6. In a right triangle, the side opposite the 90-degree angle is the:

(a) base (b) hypotenuse (c) perpendicular (d) none of these

7. If two triangles have their corresponding angles equal, they are:

(a) congruent (b) similar (c) equal in area (d) none of these

8. ΔABC ~ ΔPQR. If AB = 6, PQ = 9 and BC = 8, then QR is:

(a) 12 (b) 10 (c) 14 (d) 8

9. In a right triangle with hypotenuse 13 cm and one side 5 cm, the other side is:

(a) 8 cm (b) 10 cm (c) 12 cm (d) 7 cm

10. If DE || BC in ΔABC with AD/DB = 2/3 and AE = 6, then EC is:

(a) 4 (b) 9 (c) 6 (d) 12

Answer Key - Chapter 6

Q	1	2	3	4	5	6	7	8	9	10
Ans	a	b	a	b	d	b	b	a	c	b

Chapter 6 - Solved Short Questions (1 to 3)

1. In ΔABC, D and E are points on AB and AC such that DE || BC. If AD = 2, DB = 3 and AE = 4, find EC.

Solution: By BPT, AD/DB = AE/EC. So 2/3 = 4/EC, giving EC = 4 x 3/2 = 6 cm.

2. A ladder 5 m long reaches a window 4 m above the ground. How far is the foot of the ladder from the wall?

Solution: Let the distance be x m. By Pythagoras: $x^2 + 4^2 = 5^2$. So $x^2 = 25 - 16 = 9$, giving x = 3 m. The foot is 3 m from the wall.

3. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Solution: The difference in heights is 11 - 6 = 5 m and the horizontal distance is 12 m. By Pythagoras, the distance between the tops = $\sqrt{(12^2 + 5^2)} = \sqrt{(144 + 25)} = \sqrt{169} = 13$ m.

Chapter 6 - Solved Short Questions (4 to 5)

4. ΔABC ~ ΔDEF. If AB = 4, DE = 6 and BC = 6, find EF.

Solution: Since the triangles are similar, AB/DE = BC/EF. So 4/6 = 6/EF, giving EF = 6 x 6/4 = 9 cm.

5. In a right triangle, the hypotenuse is 25 cm and one of the other sides is 7 cm. Find the third side.

Solution: Let the third side be x cm. By Pythagoras: $x^2 + 7^2 = 25^2$. So $x^2 = 625 - 49 = 576$, giving x = 24 cm.

Chapter 6 - Solved Long Questions

1. Prove the Pythagoras theorem: in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution: Let ABC be right-angled at B. Draw BD perpendicular to the hypotenuse AC. In triangles ADB and ABC: angle A is common and angle ADB = angle ABC = 90 degrees, so ΔADB ~ ΔABC by AA. Hence AD/AB = AB/AC, so AB^2 = AD x AC. Similarly, in triangles BDC and ABC: angle C is common and angle BDC = angle ABC = 90 degrees, so ΔBDC ~ ΔABC. Hence CD/BC = BC/AC, so BC^2 = CD x AC. Adding: $AB^2 + BC^2$ = AD x AC + CD x AC = AC x (AD + CD) = AC x AC = AC^2. Hence the square of the hypotenuse equals the sum of the squares of the other two sides.

2. Prove the Basic Proportionality Theorem: if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Solution: Let DE be parallel to BC in triangle ABC, with D on AB and E on AC. Join BE and CD, and draw DM perpendicular to AC and EN perpendicular to AB. Triangles ADE and BDE share the vertex E and their bases AD and DB lie on the same line AB, so ar(ADE)/ar(BDE) = AD/DB. Similarly, triangles ADE and CDE share the vertex D and their bases AE and EC lie on the same line AC, so ar(ADE)/ar(CDE) = AE/EC. But ar(BDE) = ar(CDE), because the two triangles stand on the same base DE and lie between the same parallels DE and BC. Hence AD/DB = AE/EC. Proved.

Chapter 7 - Coordinate Geometry

Unit: Coordinate Geometry (6 marks) | Distance Formula, Section Formula, Midpoint, Area of a Triangle

Notes

Coordinate geometry connects algebra with geometry: every point is an ordered pair (x, y), and distances, divisions and areas become calculations.

1. Distance Formula

> Distance between P(x1, y1) and Q(x2, y2) = $\sqrt{[(x2 - x1)^2 + (y2 - y1)^2]}$.
>
> Distance of a point P(x, y) from the origin = $\sqrt{(x^2 + y^2)}$.
>
> Example: distance between (5, -2) and (1, 1) = $\sqrt{[(1 - 5)^2 + (1 + 2)^2]} = \sqrt{(16 + 9)} = 5$.

2. Section Formula

The point dividing the join of P(x1, y1) and Q(x2, y2) in the ratio m : n is ((mx2 + nx1)/(m + n), (my2 + ny1)/(m + n)).

Midpoint formula (ratio 1 : 1): ((x1 + x2)/2, (y1 + y2)/2).

3. Area of a Triangle

> Area of triangle with vertices (x1, y1), (x2, y2), (x3, y3) = 1/2 x |x1(y2 - y3) + x2(y3 - y1) + x3(y1 - y2)|.
>
> If the area is 0, the three points are **collinear** (they lie on one straight line).

> **Exam tip:** to prove a figure is a square, rhombus or parallelogram, find all four side lengths (and diagonals if needed) with the distance formula. For a square: all sides equal and diagonals equal. For a parallelogram: opposite sides equal, or the diagonals bisect each other (midpoints equal).

Chapter 7 - Coordinate Geometry (continued)

Worked examples

4. Worked Example - Point on the x-axis

Find the point on the x-axis which is equidistant from (2, -5) and (-2, 9).

Let the point be (x, 0). Equidistant means the distances are equal: $\sqrt{[(x - 2)^2 + (0 + 5)^2]} = \sqrt{[(x + 2)^2 + (0 - 9)^2]}$. Squaring: $(x - 2)^2 + 25 = (x + 2)^2 + 81$. Expanding: $x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$. Simplifying: $-4x + 29 = 4x + 85$, so $-8x = 56$ and $x = -7$. The point is (-7, 0).

5. Worked Example - Section Formula

Find the coordinates of the point which divides the join of (4, -3) and (8, 5) in the ratio 3 : 1.

By the section formula: $x = (3 \times 8 + 1 \times 4)/(3 + 1) = 28/4 = 7$; $y = (3 \times 5 + 1 \times (-3))/(3 + 1) = 12/4 = 3$. The point is (7, 3).

> **Remember:** in the section formula, m multiplies the SECOND point and n multiplies the FIRST point. Swapping them gives the division in the ratio n : m instead - the most common error in this chapter.

> **Axis facts:** a point on the x-axis has y = 0; a point on the y-axis has x = 0. The origin is (0, 0). The point (0, -5) lies on the y-axis, not in a quadrant.

Chapter 7 - MCQs (1 to 5)

1. The distance between (2, 3) and (4, 1) is:

(a) $2\sqrt{2}$ (b) 4 (c) 8 (d) $\sqrt{2}$

2. The midpoint of the join of (2, 3) and (4, 7) is:

(a) (3, 5) (b) (5, 3) (c) (3, 4) (d) (6, 10)

3. The point dividing (1, 2) and (3, 4) in the ratio 1 : 1 is:

(a) (3, 2) (b) (2, 3) (c) (1, 3) (d) (2, 2)

4. The distance of the point (3, 4) from the origin is:

(a) 7 (b) 5 (c) 3 (d) 4

5. The area of the triangle with vertices (0, 0), (4, 0) and (0, 3) is:

(a) 12 (b) 6 (c) 7 (d) 3

Chapter 7 - MCQs (6 to 10)

6. If the points (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram in order, then x is:

(a) 6 (b) 3 (c) 4 (d) 5

7. The point (0, -5) lies on the:

(a) x-axis (b) y-axis (c) origin (d) fourth quadrant

8. The distance between (0, 0) and (3, 4) is:

(a) 7 (b) 25 (c) 5 (d) 1

9. The point dividing (x1, y1) and (x2, y2) in the ratio m : n is:

(a) ((mx2 + nx1)/(m + n), (my2 + ny1)/(m + n)) (b) ((mx1 + nx2)/(m + n), (my1 + ny2)/(m + n)) (c) ((x1 + x2)/2, (y1 + y2)/2) (d) none of these

10. The points (2, 3), (4, 5) and (6, 7) are:

(a) collinear (b) vertices of a triangle (c) on a circle (d) none of these

Answer Key - Chapter 7

Q	1	2	3	4	5	6	7	8	9	10
Ans	a	a	b	b	b	a	b	c	a	a

Chapter 7 - Solved Short Questions (1 to 3)

1. Find the distance between the points (5, -2) and (1, 1).

Solution: Distance = $\sqrt{[(1 - 5)^2 + (1 - (-2))^2]} = \sqrt{[16 + 9]} = \sqrt{25} = 5$ units.

2. Find the point on the x-axis which is equidistant from (2, -5) and (-2, 9).

Solution: Let the point be (x, 0). Then $(x - 2)^2 + 25 = (x + 2)^2 + 81$. Expanding: $x^2 - 4x + 29 = x^2 + 4x + 85$. So $-8x = 56$ and $x = -7$. The point is (-7, 0).

3. Find the area of the triangle with vertices (2, 3), (-1, 0) and (2, -4).

Solution: Area = 1/2 x |2(0 - (-4)) + (-1)(-4 - 3) + 2(3 - 0)| = 1/2 x |8 + 7 + 6| = 1/2 x 21 = 10.5 square units.

Chapter 7 - Solved Short Questions (4 to 5)

4. Find the coordinates of the point which divides the join of (4, -3) and (8, 5) in the ratio 3 : 1.

Solution: x = (3 x 8 + 1 x 4)/4 = 28/4 = 7. y = (3 x 5 + 1 x (-3))/4 = 12/4 = 3. The point is (7, 3).

5. Check whether the points (1, 5), (2, 3) and (3, 1) are collinear.

Solution: Area = 1/2 x |1(3 - 1) + 2(1 - 5) + 3(5 - 3)| = 1/2 x |2 - 8 + 6| = 1/2 x 0 = 0. Since the area is 0, the points are collinear.

Chapter 7 - Solved Long Questions

1. Show that the points A(1, 7), B(4, 2), C(-1, -1) and D(-4, 4) form a square.

Solution: AB = √[(4 - 1)² + (2 - 7)²] = √(9 + 25) = √34. BC = √[(-1 - 4)² + (-1 - 2)²] = √(25 + 9) = √34. CD = √[(-4 + 1)² + (4 + 1)²] = √(9 + 25) = √34. DA = √[(1 + 4)² + (7 - 4)²] = √(25 + 9) = √34. All four sides are equal. Diagonals: AC = √[(-1 - 1)² + (-1 - 7)²] = √(4 + 64) = √68. BD = √[(-4 - 4)² + (4 - 2)²] = √(64 + 4) = √68. The diagonals are equal. A quadrilateral with all sides equal and diagonals equal is a square. Hence ABCD is a square.

2. Find the ratio in which the line segment joining A(1, -5) and B(-4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

Solution: Let the ratio be k : 1 and the point of division be P. The y-coordinate of P is 0 (it lies on the x-axis). By the section formula: 0 = (k x 5 + 1 x (-5))/(k + 1). So 5k - 5 = 0, giving k = 1. The x-coordinate is (1 x (-4) + 1 x 1)/(1 + 1) = -3/2. Hence the x-axis divides AB in the ratio 1 : 1 at the point (-3/2, 0).

Chapter 8 - Introduction to Trigonometry

Unit: Trigonometry (12 marks) | Trigonometric Ratios, Standard Angles, Identities, Complementary Angles

Notes

In a right-angled triangle, the **trigonometric ratios** of an acute angle A are defined from the sides: hypotenuse (opposite the 90-degree angle), perpendicular (opposite angle A) and base (adjacent to angle A).

> **sin A = perpendicular / hypotenuse; cos A = base / hypotenuse; tan A = perpendicular / base.**
>
> **cosec A = 1 / sin A; sec A = 1 / cos A; cot A = 1 / tan A.**
>
> Also: tan A = sin A / cos A.

1. Standard Angles

Angle	0°	30°	45°	60°	90°
sin	0	1/2	1/√2	√3/2	1
cos	1	√3/2	1/√2	1/2	0
tan	0	1/√3	1	√3	not defined

2. Trigonometric Identities

> **$\sin^2\theta + \cos^2\theta = 1$**
>
> **$1 + \tan^2\theta = \sec^2\theta$**
>
> **$1 + \cot^2\theta = \operatorname{cosec}^2\theta$**

> **Complementary angles:** $\sin(90° - \theta) = \cos\theta$; $\cos(90° - \theta) = \sin\theta$; $\tan(90° - \theta) = \cot\theta$. These are used when a question mixes angles like 30° and 60°.
>
> **Memory trick:** for sin, the values at 0, 30, 45, 60, 90 are √0/2, √1/2, √2/2, √3/2, √4/2. For cos, the same list runs backwards.

Chapter 8 - Introduction to Trigonometry (continued)

Finding ratios from a given ratio

3. Worked Example - From tan to sin and cos

If tan A = 3/4, find sin A and cos A (A is acute).

tan A = perpendicular / base = 3/4. Take perpendicular = 3k and base = 4k. By Pythagoras, hypotenuse = $\sqrt{[(3k)^2 + (4k)^2]} = \sqrt{(25k^2)} = 5k$. So sin A = perpendicular / hypotenuse = 3k/5k = 3/5, and cos A = base / hypotenuse = 4k/5k = 4/5.

4. Worked Example - Evaluating an Expression

Evaluate sin 60° cos 30° + sin 30° cos 60°.

sin 60° = √3/2, cos 30° = √3/2, sin 30° = 1/2, cos 60° = 1/2. So the value = (√3/2)(√3/2) + (1/2)(1/2) = 3/4 + 1/4 = 1.

Identity method: to prove an identity, start from the more complicated side and use $\sin^2 + \cos^2 = 1$, $1 + \tan^2 = \sec^2$ and $1 + \cot^2 = \text{cosec}^2$ until both sides match. Never move terms across the equals sign while proving - work on one side only.

Common mistake: sin 0° = 0 but cos 0° = 1. tan 90° and sec 90° are not defined (division by zero). cosec 0° and cot 0° are also not defined. If an MCQ asks for tan 90°, the answer is "not defined".

Chapter 8 - MCQs (1 to 5)

1. The value of sin 45° is:

(a) 1/√2 (b) √3/2 (c) 1/2 (d) 1

2. The value of tan 60° is:

(a) 1 (b) √3 (c) 1/√3 (d) 2

3. $\sin^2 \theta + \cos^2 \theta$ equals:

(a) 0 (b) 1 (c) 2 (d) $\tan^2 \theta$

4. The value of cos 0° is:

(a) 0 (b) 1 (c) 1/2 (d) √3/2

5. If sin A = 3/5, then cos A is:

(a) 4/5 (b) 3/5 (c) 5/4 (d) 5/3

Chapter 8 - MCQs (6 to 10)

6. $\sec^2 \theta - \tan^2 \theta$ equals:

(a) 0 (b) 1 (c) -1 (d) 2

7. The value of sin 30° is:

(a) 1/2 (b) √3/2 (c) 1 (d) 0

8. The value of cosec 90° is:

(a) 0 (b) 1 (c) 2 (d) not defined

9. The value of tan 45° is:

(a) 0 (b) 1 (c) √3 (d) 1/√3

10. If cos A = 12/13, then sin A is:

(a) 5/13 (b) 12/13 (c) 13/5 (d) 13/12

Answer Key - Chapter 8

Q	1	2	3	4	5	6	7	8	9	10
Ans	a	b	b	b	a	b	a	b	b	a

Chapter 8 - Solved Short Questions (1 to 3)

1. Evaluate sin 60° cos 30° + sin 30° cos 60°.

Solution: sin 60° = √3/2, cos 30° = √3/2, sin 30° = 1/2, cos 60° = 1/2. Value = (√3/2)(√3/2) + (1/2)(1/2) = 3/4 + 1/4 = 1.

2. If tan A = 3/4, find sin A and cos A.

Solution: tan A = perpendicular/base = 3/4. Take perpendicular = 3k, base = 4k. Hypotenuse = √(9k² + 16k²) = 5k. So sin A = 3/5 and cos A = 4/5.

3. Evaluate 2 tan² 45° + cos² 30° - sin² 60°.

Solution: tan 45° = 1, cos 30° = √3/2, sin 60° = √3/2. Value = 2(1)² + 3/4 - 3/4 = 2.

Chapter 8 - Solved Short Questions (4 to 5)

4. If $\sin\theta = 4/5$, find $\cos\theta$ (θ acute).

Solution: $\sin^2\theta + \cos^2\theta = 1$, so $\cos^2\theta = 1 - 16/25 = 9/25$. Since θ is acute, $\cos\theta = 3/5$.

5. Prove that $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$.

Solution: LHS = $\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1 + 2\sin\theta\cos\theta$ = RHS. Proved.

Chapter 8 - Solved Long Questions

1. Prove that (1 + tan² A) / (1 + cot² A) = tan² A.

Solution: LHS = (1 + tan² A) / (1 + cot² A). Using 1 + tan² A = sec² A and 1 + cot² A = cosec² A, LHS = sec² A / cosec² A = (1/cos² A) / (1/sin² A) = sin² A / cos² A = tan² A = RHS. Proved.

2. Prove that (sin A + cosec A)² + (cos A + sec A)² = 7 + tan² A + cot² A.

Solution: LHS = sin² A + cosec² A + 2 sin A cosec A + cos² A + sec² A + 2 cos A sec A. Since sin A cosec A = 1 and cos A sec A = 1, LHS = (sin² A + cos² A) + cosec² A + sec² A + 4 = 1 + (1 + cot² A) + (1 + tan² A) + 4 = 7 + tan² A + cot² A = RHS. Proved.

Chapter 9 - Some Applications of Trigonometry

Unit: Trigonometry (12 marks) | Angle of Elevation, Angle of Depression, Heights and Distances

Notes

This chapter uses the trigonometric ratios from Chapter 8 to find heights and distances that cannot be measured directly - the height of a tower, the width of a river, the height of a kite.

1. Angle of Elevation and Angle of Depression

The **angle of elevation** is the angle between the horizontal line from the observer's eye and the line of sight to an object ABOVE the eye level. The **angle of depression** is the angle between the horizontal line and the line of sight to an object BELOW the eye level.

> **Key fact:** the angle of depression from the top of a tower to a point on the ground equals the angle of elevation from that point to the top of the tower (alternate angles between parallel horizontal lines).

2. The Method

Step 1: draw a diagram and mark the right triangle. Step 2: mark the known side and the angle. Step 3: choose the ratio that connects them - tan is used most often, since height and horizontal distance are the opposite and adjacent sides. Step 4: solve and write the answer with units.

> **tan θ = height / horizontal distance** so **height = distance x tan θ**.
> Example: from a point 30 m from a tower, the angle of elevation of the top is 30°. Height = 30 x tan 30° = 30 x (1/√3) = 10√3 m.

> **Observer's eye level:** if the observer's eye is 1.5 m above the ground, add 1.5 m to the height found by trigonometry. Many textbook problems state the eye level - do not forget it.

Chapter 9 - Some Applications of Trigonometry (continued)

Complementary angles and two-point problems

3. Complementary Angles Trick

If the angles of elevation from two points are complementary (they add to 90°), the height of the tower is the square root of the product of the two distances. Example: from points 4 m and 9 m from the base, the angles are complementary. Height = $\sqrt{(4 \times 9)} = 6$ m. Proof: $\tan \theta = h/4$ and $\tan(90° - \theta) = \cot \theta = h/9$, so $(h/4)(h/9) = \tan \theta \times \cot \theta = 1$, giving $h^2 = 36$ and $h = 6$ m.

4. Worked Example - Broken Tree

A tree breaks due to a storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree and the point where the top touches the ground is 8 m. Find the height of the tree.

Let the standing part be x m and the broken part (hypotenuse) be y m. $\cos 30° = 8/y$, so $y = 8 / (\sqrt{3}/2) = 16/\sqrt{3}$ m. $\tan 30° = x/8$, so $x = 8/\sqrt{3}$ m. Total height = $x + y = 24/\sqrt{3} = 8\sqrt{3}$ m.

> **Standard values to use:** $\tan 30° = 1/\sqrt{3}$, $\tan 45° = 1$, $\tan 60° = \sqrt{3}$. If $\tan \theta = 1$, the angle is 45° and height equals distance - a very common exam result.

Chapter 9 - MCQs (1 to 5)

1. The angle of elevation is measured from the:

(a) horizontal line (b) vertical line (c) ground (d) none of these

2. A tower casts a shadow 30 m long when the angle of elevation is 30°. Its height is:

(a) $10\sqrt{3}$ m (b) $30\sqrt{3}$ m (c) 10 m (d) 30 m

3. If the angle of elevation is 45°, the height of the tower is ___ the length of its shadow.

(a) double (b) equal to (c) half (d) none of these

4. The angle of depression from the top of a tower to a point on the ground equals:

(a) the angle of elevation from the point to the top (b) 90° (c) 0° (d) none of these

5. A kite is flying at a height of 60 m with a string 120 m long. The angle of elevation is:

(a) 45° (b) 30° (c) 60° (d) 90°

Chapter 9 - MCQs (6 to 10)

6. The value of tan 45° is:

(a) 1 (b) √3 (c) 1/√3 (d) 0

7. A pole 10 m high casts a shadow 10√3 m long. The angle of elevation of the sun is:

(a) 45° (b) 60° (c) 30° (d) 90°

8. An angle of depression is always:

(a) acute (b) obtuse (c) right (d) reflex

9. A 20 m high tower stands on the ground. The distance of a point from its foot when the angle of elevation is 60° is:

(a) 20√3 m (b) 20 m (c) 20√3/3 m (d) 10 m

10. In heights and distances problems, the observer's eye level:

(a) is ignored (b) is added to the height found (c) is subtracted from the height (d) none of these

Answer Key - Chapter 9

Q	1	2	3	4	5	6	7	8	9	10
Ans	a	a	b	a	b	a	c	a	c	b

Chapter 9 - Solved Short Questions (1 to 3)

1. A tower is 20 m high. Find the length of its shadow when the angle of elevation of the sun is 60°.

Solution: Let the shadow be x m. tan 60° = 20/x, so √3 = 20/x, giving x = 20/√3 = 20√3/3 m.

2. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming there is no slack in it.

Solution: sin 60° = 60/length, so length = 60 / (√3/2) = 120/√3 = 40√3 m.

3. From a point 30 m away from the foot of a tower, the angle of elevation of the top is 30°. Find the height of the tower.

Solution: Height = 30 x tan 30° = 30 x (1/√3) = 30/√3 = 10√3 m.

Chapter 9 - Solved Short Questions (4 to 5)

4. A tree breaks due to a storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree and the point where the top touches the ground is 8 m. Find the height of the tree.

Solution: Let the standing part be x and the broken part (hypotenuse) be y. cos 30° = 8/y, so y = 8 / (√3/2) = 16/√3 m. tan 30° = x/8, so x = 8/√3 m. Total height = x + y = 24/√3 = 8√3 m.

5. The angle of elevation of the top of a 20 m high building from a point 20 m away from its foot is:

Solution: tan θ = 20/20 = 1, so θ = 45°. The angle of elevation is 45°.

Chapter 9 - Solved Long Questions

1. The angles of elevation of the top of a tower from two points at distances of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Solution: Let the height of the tower be h m and the angles of elevation be θ and (90° - θ). From the nearer point: tan θ = h/4. From the farther point: tan(90° - θ) = h/9, so cot θ = h/9. Multiplying: tan θ x cot θ = (h/4)(h/9). Since tan θ x cot θ = 1, we get $h^2/36 = 1$, so $h^2 = 36$ and h = 6 m. Hence the height of the tower is 6 m. Proved.

2. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Solution: Let AB be the building (7 m) and CD the tower. Let the horizontal distance between them be x m. From the top of the building, the angle of depression of the foot of the tower is 45°, so tan 45° = 7/x, giving x = 7 m. Let the tower rise h m above the level of the top of the building. Then tan 60° = h/x, so h = 7√3 m. Total height of the tower = 7 + 7√3 = 7(√3 + 1) m.

Chapter 10 - Circles

Unit: Geometry (15 marks) | Tangent to a Circle, Number of Tangents, Length of a Tangent

Notes

A **tangent** to a circle is a line that touches the circle at exactly one point. That point is the **point of contact**. A line that cuts the circle at two points is a **secant**.

1. Number of Tangents from a Point

From a point inside the circle: no tangent can be drawn. From a point on the circle: exactly one tangent. From a point outside the circle: exactly two tangents. Infinitely many tangents can be drawn to a circle in total (one at each point of the circle).

2. Tangent and Radius

> **The tangent at any point of a circle is perpendicular to the radius through the point of contact.** If PT is tangent at P and O is the centre, then OP is perpendicular to PT, so angle OPT = 90°.
>
> **Length of a tangent:** if a point T is at distance d from the centre O and the radius is r, the length of the tangent from T is $\sqrt{(d^2 - r^2)}$. Example: d = 13 cm, r = 5 cm gives $\sqrt{(169 - 25)} = \sqrt{144} = 12$ cm.

3. Two Tangents from an External Point

The lengths of tangents drawn from an external point to a circle are equal. If TP and TQ are tangents from T, then TP = TQ. Also, the line OT bisects the angle between the tangents, and the centre lies on the bisector of the angle between the two tangents.

> **Exam tip:** every tangent problem is a right-triangle problem. The radius to the point of contact, the tangent and the line from the centre to the external point form a right triangle - use Pythagoras on it.

Chapter 10 - Circles (continued)

Proofs and worked examples

4. Worked Example - Tangent Length

A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Find the length PQ.

Since the radius OP is perpendicular to the tangent PQ, triangle OPQ is right-angled at P. By Pythagoras: $PQ^2 = OQ^2 - OP^2 = 144 - 25 = 119$. So PQ = $\sqrt{119}$ cm.

5. Worked Example - Chord of the Larger Circle

Two concentric circles have radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Let the chord be AB, touching the smaller circle at P. Then OP is perpendicular to AB and OP = 3 cm (radius of the smaller circle). OA = 5 cm. In right triangle OPA: AP = $\sqrt{(OA^2 - OP^2)} = \sqrt{(25 - 9)}$ = 4 cm. Since the perpendicular from the centre bisects the chord, AB = 2 x AP = 8 cm.

> **Remember:** the perpendicular from the centre of a circle to a chord bisects the chord. This result is used in almost every chord problem in this chapter.

Chapter 10 - MCQs (1 to 5)

1. The number of tangents that can be drawn from an external point to a circle is:

(a) 1 (b) 2 (c) 0 (d) infinite

2. A tangent to a circle is perpendicular to the radius at the:

(a) centre (b) point of contact (c) chord (d) diameter

3. The length of the tangent from a point 13 cm from the centre of a circle of radius 5 cm is:

(a) 8 cm (b) 12 cm (c) 5 cm (d) 13 cm

4. The two tangents drawn from an external point to a circle are:

(a) equal in length (b) unequal in length (c) parallel (d) none of these

5. A line intersecting a circle in two points is called a:

(a) tangent (b) secant (c) chord (d) radius

Chapter 10 - MCQs (6 to 10)

6. The angle between a tangent and the radius at the point of contact is:

(a) 45° (b) 90° (c) 60° (d) 180°

7. The maximum number of tangents that can be drawn to a circle is:

(a) 1 (b) 2 (c) infinite (d) 0

8. If two tangents PA and PB are drawn from an external point P and PA = 8 cm, then PB is:

(a) 8 cm (b) 4 cm (c) 16 cm (d) 6 cm

9. A tangent touches a circle at:

(a) exactly one point (b) two points (c) no point (d) three points

10. The length of the tangent from a point 10 cm from the centre of a circle of radius 6 cm is:

(a) 4 cm (b) 6 cm (c) 8 cm (d) 10 cm

Answer Key - Chapter 10

Q	1	2	3	4	5	6	7	8	9	10
Ans	b	b	b	a	b	b	c	a	a	c

Chapter 10 - Solved Short Questions (1 to 3)

1. Find the length of the tangent from a point 13 cm from the centre of a circle of radius 5 cm.

Solution: Length = $\sqrt{(d^2 - r^2)} = \sqrt{(169 - 25)} = \sqrt{144} = 12$ cm.

2. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that angle PTQ = 2 x angle OPQ.

Solution: Let angle PTQ = θ. TP = TQ, OP = OQ and OT is common, so triangles OPT and OQT are congruent, giving angle PTO = angle QTO = θ/2. In triangle OPT, angle OPT = 90°, so angle POT = 90° - θ/2. In triangle OPQ, OP = OQ, so angle OPQ = angle OQP. Angle POQ = 2 x angle POT = 180° - θ. In triangle OPQ: 2 x angle OPQ + (180° - θ) = 180°, so 2 x angle OPQ = θ = angle PTQ. Proved.

3. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Find the length PQ.

Solution: OP is perpendicular to PQ, so triangle OPQ is right-angled at P. $PQ^2 = OQ^2 - OP^2 = 144 - 25 = 119$. So PQ = $\sqrt{119}$ cm.

Chapter 10 - Solved Short Questions (4 to 5)

4. The length of a tangent from a point A at a distance of 5 cm from the centre of a circle is 4 cm. Find the radius of the circle.

Solution: Let the radius be r. Then $4^2 = 5^2 - r^2$ (by Pythagoras in the right triangle formed by the radius, the tangent and the line from the centre). So $r^2 = 25 - 16 = 9$, giving r = 3 cm.

5. Two concentric circles have radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Solution: Let the chord be AB touching the smaller circle at P. OP = 3 cm is perpendicular to AB and OA = 5 cm. AP = $\sqrt{(25 - 9)} = 4$ cm. The perpendicular from the centre bisects the chord, so AB = 2 x 4 = 8 cm.

Chapter 10 - Solved Long Questions

1. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Solution: Let TP and TQ be two tangents from an external point T to a circle with centre O. Join OP, OQ and OT. OP and OQ are radii, and the tangent is perpendicular to the radius at the point of contact, so angle OPT = angle OQT = 90°. In right triangles OPT and OQT: OT is common, OP = OQ (radii). So the triangles are congruent by RHS. Hence PT = QT (corresponding parts of congruent triangles). The lengths of the two tangents are equal. Proved.

2. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Solution: Let PT be the tangent at point P of a circle with centre O, and join OP. Suppose OP is not perpendicular to PT. Draw OQ perpendicular to PT, meeting PT at Q (Q different from P). Then OQ is the shortest distance from O to the line PT, so OQ < OP. But Q lies on the tangent line and Q is not P, so Q lies outside the circle, which means OQ > OP (radius). This contradicts OQ < OP. Hence our assumption is wrong, and OP is perpendicular to PT. Proved.

Chapter 11 - Areas Related to Circles

Unit: Mensuration (10 marks) | Circumference, Area, Sector, Segment, Length of an Arc

Notes

For a circle of radius r (using $\pi = 22/7$ unless stated otherwise):

> **Circumference = $2\pi r$** and **Area = πr^2**
> Example: r = 7 cm gives circumference 2 x (22/7) x 7 = 44 cm and area (22/7) x 49 = 154 cm².

1. Sector of a Circle

A **sector** is the region between two radii and the arc joining them. If the angle at the centre is θ degrees:

> **Area of sector = $(\theta/360) \times \pi r^2$**
> **Length of the arc = $(\theta/360) \times 2\pi r$**
> Example: sector of angle 90° and radius 7 cm has area (90/360) x 154 = 38.5 cm².

2. Segment of a Circle

A **segment** is the region between a chord and its arc. Area of a segment = area of the sector - area of the triangle formed by the two radii and the chord.

For a chord subtending angle θ at the centre: area of triangle = (1/2) x r^2 x sin θ. For θ = 90°, sin θ = 1, so the triangle area is $r^2/2$.

> **Exam tip:** always use $\pi = 22/7$ when the radius is a multiple of 7 (7, 14, 21, 28...). The numbers are chosen to cancel nicely. If the radius is not a multiple of 7, the question will say to use $\pi = 3.14$.

Chapter 11 - Areas Related to Circles (continued)

Worked examples

3. Worked Example - Segment Area

A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding minor segment.

Area of the sector = (90/360) x π x 100 = 25π = 25 x 3.14 = 78.5 cm². Area of the triangle = (1/2) x 10 x 10 = 50 cm². Area of the minor segment = 78.5 - 50 = 28.5 cm².

4. Worked Example - Arc Length

In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the length of the arc.

Length = (60/360) x 2 x (22/7) x 21 = (1/6) x 132 = 22 cm.

Scaling: if the radius is doubled, the circumference doubles but the area becomes 4 times. If the radius is halved, the area becomes one-fourth. Area scales with the square of the radius - a favourite MCQ fact.

Remember: the major sector is the larger part (angle > 180°) and the minor sector is the smaller part. Major segment area = area of the circle - area of the minor segment.

Chapter 11 - MCQs (1 to 5)

1. The area of a circle of radius 7 cm is:

(a) 154 cm² (b) 44 cm² (c) 49 cm² (d) 22 cm²

2. The circumference of a circle of radius 14 cm is:

(a) 44 cm (b) 88 cm (c) 154 cm (d) 616 cm

3. The area of a sector of angle 90° in a circle of radius 7 cm is:

(a) 38.5 cm² (b) 77 cm² (c) 154 cm² (d) 19.25 cm²

4. The length of an arc of a sector of angle 60° in a circle of radius 6 cm is:

(a) 44/7 cm (b) 22/7 cm (c) 88/7 cm (d) 6 cm

5. The area of a quadrant of a circle of radius 14 cm is:

(a) 77 cm² (b) 154 cm² (c) 308 cm² (d) 44 cm²

Chapter 11 - MCQs (6 to 10)

6. If the radius of a circle is doubled, its area becomes:

(a) 2 times (b) 4 times (c) 8 times (d) the same

7. The area of a sector of angle θ in a circle of radius r is:

(a) $(\theta/360) \times \pi r^2$ (b) $(\theta/360) \times 2\pi r$ (c) πr^2 (d) $2\pi r$

8. The circumference of a circle of diameter 14 cm is:

(a) 88 cm (b) 44 cm (c) 154 cm (d) 22 cm

9. The area of a circle whose circumference is 44 cm is:

(a) 44 cm^2 (b) 88 cm^2 (c) 154 cm^2 (d) 616 cm^2

10. The area of the minor segment of a circle is:

(a) area of sector - area of triangle (b) area of sector + area of triangle (c) area of circle - area of sector (d) none of these

Answer Key - Chapter 11

Q	1	2	3	4	5	6	7	8	9	10
Ans	a	b	a	a	b	b	a	b	c	a

Chapter 11 - Solved Short Questions (1 to 3)

1. Find the area of a sector of a circle with radius 6 cm and angle 60°.

Solution: Area = (60/360) x (22/7) x 36 = (1/6) x (792/7) = 132/7 = 18.86 cm^2 (approx).

2. Find the length of the arc of a sector of angle 90° in a circle of radius 14 cm.

Solution: Length = (90/360) x 2 x (22/7) x 14 = (1/4) x 88 = 22 cm.

3. Find the area of a quadrant of a circle of radius 7 cm.

Solution: Area = (1/4) x (22/7) x 49 = 154/4 = 38.5 cm^2.

Chapter 11 - Solved Short Questions (4 to 5)

4. The circumference of a circle is 88 cm. Find its area.

Solution: $2\pi r = 88$, so $r = 88 / (2 \times 22/7) = 88 \times 7/44 = 14$ cm. Area $= (22/7) \times 196 = 616$ cm².

5. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding minor segment. (Use $\pi = 3.14$)

Solution: Area of sector $= (90/360) \times 3.14 \times 100 = 78.5$ cm². Area of triangle $= (1/2) \times 10 \times 10 = 50$ cm². Area of minor segment $= 78.5 - 50 = 28.5$ cm².

Chapter 11 - Solved Long Questions

1. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments. (Use π = 3.14 and √3 = 1.73)

Solution: Area of the sector = (60/360) x 3.14 x 225 = 117.75 cm². Area of the triangle = (1/2) x 15 x 15 x sin 60° = (1/2) x 225 x (√3/2) = 56.25 x 1.73 = 97.31 cm². Area of the minor segment = 117.75 - 97.31 = 20.44 cm². Area of the circle = 3.14 x 225 = 706.5 cm². Area of the major segment = 706.5 - 20.44 = 686.06 cm².

2. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find (i) the length of the arc, (ii) the area of the sector formed by the arc.

Solution: (i) Length of arc = (60/360) x 2 x (22/7) x 21 = (1/6) x 132 = 22 cm. (ii) Area of sector = (60/360) x (22/7) x 441 = (1/6) x 1386 = 231 cm².

Chapter 12 - Surface Areas and Volumes

Unit: Mensuration (10 marks) | Cube, Cuboid, Cylinder, Cone, Sphere, Hemisphere, Frustum, Combinations

Notes

This chapter is a formula chapter. Learn the table below by heart, then practise combinations: a solid made of two shapes joined together (a cone on a cylinder, a hemisphere on a cube) is solved by adding the volumes and adjusting the surface area for the joined face.

1. Formula Table

Solid	Curved / Lateral Surface Area	Total Surface Area	Volume
Cube (edge a)	$4a^2$	$6a^2$	a^3
Cuboid (l, b, h)	$2h(l + b)$	$2(lb + bh + hl)$	l x b x h
Cylinder (r, h)	$2\pi rh$	$2\pi r(r + h)$	πr^2h
Cone (r, h, l)	πrl	$\pi r(l + r)$	$(1/3)\pi r^2h$
Sphere (r)	$4\pi r^2$	$4\pi r^2$	$(4/3)\pi r^3$
Hemisphere (r)	$2\pi r^2$	$3\pi r^2$	$(2/3)\pi r^3$

For a cone, the slant height is $l = \sqrt{(r^2 + h^2)}$. For a frustum of a cone with radii r1 and r2 and height h: curved surface area $= \pi(r1 + r2)l$, volume $= (1/3)\pi h(r1^2 + r2^2 + r1r2)$, where $l = \sqrt{(h^2 + (r1 - r2)^2)}$.

> **Combination rule:** volume of a combined solid = sum of the volumes of its parts. Surface area = sum of the curved surface areas of the parts, PLUS the exposed flat faces, MINUS the area of any face that is hidden where two parts join.

Chapter 12 - Surface Areas and Volumes (continued)

Worked examples

2. Worked Example - Cone on a Hemisphere

A solid is in the shape of a cone standing on a hemisphere with both their radii equal to 1 cm and the height of the cone equal to its radius. Find the volume of the solid in terms of π.

Volume of the cone = $(1/3)\pi r^2 h = (1/3)\pi \times 1 \times 1 = \pi/3$. Volume of the hemisphere = $(2/3)\pi r^3 = (2/3)\pi \times 1 = 2\pi/3$. Total volume = $\pi/3 + 2\pi/3 = \pi$ cm³.

3. Worked Example - Cube Surmounted by a Hemisphere

A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

The greatest diameter is the side of the cube, 7 cm, so the radius is 3.5 cm. Surface area = TSA of the cube - area of the circular base + CSA of the hemisphere = $6 \times 49 - (22/7)(3.5)^2 + 2(22/7)(3.5)^2 = 294 - 38.5 + 77 = 332.5$ cm².

Scaling: if the radius of a sphere is doubled, its surface area becomes 4 times and its volume 8 times. Surface area scales with the square of the length, volume with the cube.

Exam tip: read the question twice - "total surface area" includes all flat faces; "curved surface area" does not. A hemisphere on a table has TSA $3\pi r^2$ (including the flat circular base); its CSA alone is $2\pi r^2$.

Chapter 12 - MCQs (1 to 5)

1. The volume of a cylinder of radius 7 cm and height 10 cm is:

(a) 1540 cm^3 (b) 440 cm^3 (c) 154 cm^3 (d) 3080 cm^3

2. The curved surface area of a cone of radius 3 cm and slant height 5 cm is: (Use $\pi = 3.14$)

(a) 37.7 cm^2 (b) 47.1 cm^2 (c) 94.2 cm^2 (d) 15.7 cm^2

3. The volume of a sphere of radius 7 cm is:

(a) 1437.33 cm^3 (b) 1078 cm^3 (c) 154 cm^3 (d) 4312 cm^3

4. The total surface area of a cube of edge 5 cm is:

(a) 125 cm^2 (b) 150 cm^2 (c) 100 cm^2 (d) 25 cm^2

5. The volume of a cuboid of length 4 cm, breadth 3 cm and height 2 cm is:

(a) 24 cm^3 (b) 18 cm^3 (c) 12 cm^3 (d) 9 cm^3

Chapter 12 - MCQs (6 to 10)

6. The curved surface area of a cylinder of radius 7 cm and height 10 cm is:

(a) 1540 cm² (b) 220 cm² (c) 440 cm² (d) 880 cm²

7. The surface area of a sphere of radius 7 cm is:

(a) 616 cm² (b) 154 cm² (c) 308 cm² (d) 1232 cm²

8. The volume of a cone of radius 3 cm and height 4 cm is: (Use π = 3.14)

(a) 113.04 cm³ (b) 37.68 cm³ (c) 12.56 cm³ (d) 75.36 cm³

9. The total surface area of a hemisphere of radius 7 cm is:

(a) 308 cm² (b) 462 cm² (c) 154 cm² (d) 616 cm²

10. If the radius of a sphere is doubled, its volume becomes:

(a) 2 times (b) 4 times (c) 8 times (d) 16 times

Answer Key - Chapter 12

Q	1	2	3	4	5	6	7	8	9	10
Ans	a	b	a	b	a	c	a	b	b	c

Chapter 12 - Solved Short Questions (1 to 3)

1. Find the volume of a sphere of radius 7 cm.

Solution: Volume = $(4/3)\pi r^3 = (4/3) \times (22/7) \times 343 = (4/3) \times 1078 = 1437.33\ cm^3$.

2. Find the total surface area of a hemisphere of radius 7 cm.

Solution: TSA = $3\pi r^2 = 3 \times (22/7) \times 49 = 3 \times 154 = 462\ cm^2$.

3. Find the volume of a cylinder of radius 3 cm and height 7 cm.

Solution: Volume = $\pi r^2 h = (22/7) \times 9 \times 7 = 198\ cm^3$.

Chapter 12 - Solved Short Questions (4 to 5)

4. Find the curved surface area of a cone of radius 5 cm and slant height 13 cm. (Use π = 3.14)

Solution: CSA = πrl = 3.14 x 5 x 13 = 204.1 cm^2.

5. Find the volume of a cube of edge 6 cm.

Solution: Volume = $a^3 = 6^3 = 216$ cm^3.

Chapter 12 - Solved Long Questions

1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Solution: Radius r = 1 cm, height of cone h = 1 cm. Volume of cone = $(1/3)\pi r^2 h = (1/3)\pi \times 1 \times 1 = \pi/3$ cm³. Volume of hemisphere = $(2/3)\pi r^3 = (2/3)\pi \times 1 = 2\pi/3$ cm³. Total volume = $\pi/3 + 2\pi/3 = \pi$ cm³.

2. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Solution: The greatest diameter the hemisphere can have is the side of the cube, 7 cm, so its radius is 3.5 cm. Surface area = TSA of cube - area of the circular base of the hemisphere + CSA of the hemisphere = $6 \times 49 - (22/7) \times (3.5)^2 + 2 \times (22/7) \times (3.5)^2 = 294 - 38.5 + 77 = 332.5$ cm².

Chapter 13 - Statistics

Unit: Statistics and Probability (11 marks) | Mean, Median and Mode of Grouped Data

Notes

Statistics is the study of data. For grouped data (data in class intervals), the three averages are the mean, the median and the mode.

1. Mean of Grouped Data

Let x(i) be the class mark (mid-value) of class i and f(i) its frequency. Then mean = Σf(i)x(i) / Σf(i).

> **Direct method:** mean = Σfx / Σf.
>
> **Assumed mean method:** take an assumed mean a, compute d(i) = x(i) - a, then mean = a + Σfd / Σf.
>
> **Step-deviation method:** with class size h, put u(i) = (x(i) - a)/h. Then mean = a + (Σfu / Σf) x h. Use this when the numbers are large.

2. Mode of Grouped Data

The **modal class** is the class with the highest frequency. Mode = l + [(f1 - f0) / (2f1 - f0 - f2)] x h, where l is the lower limit of the modal class, h the class size, f1 the frequency of the modal class, f0 the frequency of the preceding class and f2 the frequency of the following class.

3. Median of Grouped Data

Find n/2, where n = Σf. The **median class** is the class whose cumulative frequency first exceeds n/2. Median = l + [(n/2 - cf) / f] x h, where l is the lower limit of the median class, cf the cumulative frequency of the preceding class, f the frequency of the median class and h the class size.

> **Empirical relationship:** 3 x Median = Mode + 2 x Mean. If you know any two of the three averages, you can find the third. **Class mark** = (upper limit + lower limit) / 2. For class 10-20, the class mark is 15.

Chapter 13 - Statistics (continued)

Worked examples

4. Worked Example - Mean by Step Deviation

Find the mean of: classes 0-10, 10-20, 20-30, 30-40, 40-50 with frequencies 5, 8, 12, 10, 5.

Class marks: 5, 15, 25, 35, 45. Take a = 25 and h = 10. Then u = (x - 25)/10 = -2, -1, 0, 1, 2. Σfu = 5(-2) + 8(-1) + 12(0) + 10(1) + 5(2) = -10 - 8 + 0 + 10 + 10 = 2. Σf = 40. Mean = a + (Σfu/Σf) x h = 25 + (2/40) x 10 = 25 + 0.5 = 25.5.

5. Worked Example - Median

For the same data: n = 40, so n/2 = 20. Cumulative frequencies: 5, 13, 25, 35, 40. The median class is 20-30 (its cumulative frequency 25 first exceeds 20). l = 20, cf = 13, f = 12, h = 10. Median = 20 + [(20 - 13)/12] x 10 = 20 + 70/12 = 20 + 5.83 = 25.83.

Ogive: the "less than" ogive is drawn by plotting cumulative frequencies against upper class limits; the "more than" ogive uses lower limits. The median is the x-coordinate of the point where the two ogives cross.

Exam tip: in the median formula, cf is the cumulative frequency of the class BEFORE the median class - not the cumulative frequency of the median class itself. Mixing these up is the most common error in this chapter.

Chapter 13 - MCQs (1 to 5)

1. The mean of the first 10 natural numbers is:

(a) 5 (b) 5.5 (c) 6 (d) 10

2. The mode of 2, 3, 4, 4, 5 is:

(a) 4 (b) 3 (c) 5 (d) 2

3. The median of 3, 5, 7, 9, 11 is:

(a) 7 (b) 5 (c) 9 (d) 6

4. For grouped data, the modal class is the class with the:

(a) highest frequency (b) lowest frequency (c) highest upper limit (d) none of these

5. The mean of 10, 20 and 30 is:

(a) 10 (b) 30 (c) 20 (d) 60

Chapter 13 - MCQs (6 to 10)

6. The median of 2, 4, 6, 8 is:

(a) 5 (b) 4 (c) 6 (d) 8

7. The empirical relationship between mean, median and mode is:

(a) 3 Median = Mode + 2 Mean (b) 2 Median = Mode + Mean (c) Mode = 3 Median - 2 Mean (d) both (a) and (c)

8. The class mark of the class 10-20 is:

(a) 10 (b) 20 (c) 15 (d) 5

9. If the mean of 5 observations is 20, their sum is:

(a) 20 (b) 25 (c) 100 (d) 4

10. The assumed mean method is used when the values are:

(a) large (b) small (c) equal in frequency (d) none of these

Answer Key - Chapter 13

Q	1	2	3	4	5	6	7	8	9	10
Ans	b	a	a	a	c	a	d	c	c	a

Chapter 13 - Solved Short Questions (1 to 3)

1. Find the mean of the first 10 natural numbers.

Solution: Sum = 1 + 2 + ... + 10 = 55. Mean = 55/10 = 5.5.

2. Find the mode of 3, 5, 7, 7, 9, 11.

Solution: The value 7 occurs twice, more often than any other value. Mode = 7.

3. Find the median of 12, 15, 18, 21, 24.

Solution: The data is already in order and has 5 values. The middle value (3rd) is 18. Median = 18.

Chapter 13 - Solved Short Questions (4 to 5)

4. Find the mean of the following data: x = 5, 10, 15, 20, 25 with frequencies f = 2, 3, 4, 3, 2.

Solution: Σfx = 5x2 + 10x3 + 15x4 + 20x3 + 25x2 = 10 + 30 + 60 + 60 + 50 = 210. Σf = 2 + 3 + 4 + 3 + 2 = 14. Mean = 210/14 = 15.

5. Find the class mark of the class 20-30.

Solution: Class mark = (20 + 30)/2 = 25.

Chapter 13 - Solved Long Questions

1. Find the mean of the following data by the step-deviation method: classes 0-10, 10-20, 20-30, 30-40, 40-50 with frequencies 5, 8, 12, 10, 5.

Solution: Class marks are 5, 15, 25, 35, 45. Take a = 25 and h = 10. Then u = -2, -1, 0, 1, 2. Σfu = 5(-2) + 8(-1) + 12(0) + 10(1) + 5(2) = -10 - 8 + 0 + 10 + 10 = 2. Σf = 40. Mean = a + (Σfu/Σf) x h = 25 + (2/40) x 10 = 25 + 0.5 = 25.5.

2. Find the median of the following data: classes 0-10, 10-20, 20-30, 30-40, 40-50 with frequencies 5, 8, 12, 10, 5.

Solution: n = 40, so n/2 = 20. Cumulative frequencies: 5, 13, 25, 35, 40. The median class is 20-30. Here l = 20, cf = 13, f = 12, h = 10. Median = l + [(n/2 - cf)/f] x h = 20 + [(20 - 13)/12] x 10 = 20 + 70/12 = 20 + 5.83 = 25.83.

Chapter 14 - Probability

Unit: Statistics and Probability (11 marks) | Classical Probability, Complementary Events, Dice, Coins, Cards and Balls

Notes

An **experiment** is an action with an uncertain result. Each possible result is an **outcome**. The set of all outcomes is the **sample space**. An **event** is a collection of outcomes.

1. Classical Definition of Probability

If all outcomes are equally likely, **P(E) = number of outcomes favourable to E / total number of outcomes**.

Example: on throwing a die, P(getting 6) = 1/6. P(getting an even number) = 3/6 = 1/2.

2. Rules to Remember

0 ≤ P(E) ≤ 1 for every event E. The probability of a sure event is 1 and of an impossible event is 0.

P(E) + P(not E) = 1. If P(E) = 0.05, then P(not E) = 0.95.

Two coins: sample space is {HH, HT, TH, TT} - 4 outcomes. **Two dice:** 36 outcomes. **One die:** 6 outcomes. **Cards:** 52 cards - 26 red, 26 black, 4 kings, 4 queens, 4 jacks, 4 aces, 13 of each suit.

3. Standard Counts

Prime numbers on a die: 2, 3, 5 (3 outcomes). Even numbers: 2, 4, 6 (3 outcomes). Numbers greater than 6 on a die: none (probability 0). A face card is a king, queen or jack - 12 face cards in a deck.

Chapter 14 - Probability (continued)

Worked examples

4. Worked Example - Two Dice

Two dice are thrown simultaneously. Find the probability of getting a sum of 7.

Total outcomes = 36. Favourable outcomes: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) - 6 outcomes. P(sum 7) = 6/36 = 1/6.

5. Worked Example - Balls in a Bag

A bag contains 5 red balls, 8 white balls, 4 green balls and 7 black balls. One ball is drawn at random. Find the probability that it is (i) red, (ii) not green, (iii) neither white nor black.

Total balls = 5 + 8 + 4 + 7 = 24. (i) P(red) = 5/24. (ii) P(not green) = 1 - P(green) = 1 - 4/24 = 20/24 = 5/6. (iii) Neither white nor black means red or green: 5 + 4 = 9 balls, so P = 9/24 = 3/8.

Complementary counting: "not green" is easier as 1 - P(green) than by counting every other colour. Use complements whenever the question says "not", "at least one" or "neither".

Exam tip: always write the total number of outcomes first, then count the favourable ones, then simplify the fraction. A probability is always written as a fraction (or decimal) between 0 and 1 - never as a percentage unless asked.

Chapter 14 - MCQs (1 to 5)

1. The probability of getting a 6 when a die is thrown once is:

(a) 1/6 (b) 1/3 (c) 1/2 (d) 1

2. The probability of a sure event is:

(a) 0 (b) 1 (c) 1/2 (d) 2

3. The probability of an impossible event is:

(a) 1 (b) 0 (c) 1/2 (d) -1

4. When a coin is tossed once, the probability of getting heads is:

(a) 1 (b) 1/2 (c) 0 (d) 1/4

5. The probability of drawing a red card from a well-shuffled deck of 52 cards is:

(a) 1/2 (b) 1/4 (c) 1/13 (d) 1/26

Chapter 14 - MCQs (6 to 10)

6. If P(E) = 0.3, then P(not E) is:

(a) 0.7 (b) 0.3 (c) 1.3 (d) 0

7. The probability of getting an even number when a die is thrown is:

(a) 1/3 (b) 1/2 (c) 1/6 (d) 2/3

8. The probability of drawing a king from a deck of 52 cards is:

(a) 1/13 (b) 1/4 (c) 1/52 (d) 4/13

9. A bag contains 5 red balls and 3 blue balls. The probability of drawing a red ball is:

(a) 3/8 (b) 5/8 (c) 1/2 (d) 5/3

10. The probability of getting a number greater than 6 when a die is thrown is:

(a) 0 (b) 1/6 (c) 1 (d) 1/2

Answer Key - Chapter 14

Q	1	2	3	4	5	6	7	8	9	10
Ans	a	b	b	b	a	a	b	a	b	a

Chapter 14 - Solved Short Questions (1 to 3)

1. A die is thrown once. Find the probability of getting a prime number.

Solution: Total outcomes = 6. Prime numbers on a die are 2, 3 and 5 - 3 favourable outcomes. P(prime) = 3/6 = 1/2.

2. Two coins are tossed together. Find the probability of getting exactly one head.

Solution: Sample space = {HH, HT, TH, TT} - 4 outcomes. Exactly one head: {HT, TH} - 2 outcomes. P = 2/4 = 1/2.

3. A bag contains 5 red balls and 3 blue balls. One ball is drawn at random. Find the probability that it is blue.

Solution: Total balls = 8. Blue balls = 3. P(blue) = 3/8.

Chapter 14 - Solved Short Questions (4 to 5)

4. A card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting a queen.

Solution: There are 4 queens in a deck of 52 cards. P(queen) = 4/52 = 1/13.

5. If the probability of an event E is 0.05, find the probability of "not E".

Solution: P(not E) = 1 - P(E) = 1 - 0.05 = 0.95.

Chapter 14 - Solved Long Questions

1. Two dice are thrown simultaneously. Find the probability of getting (i) a sum of 7, (ii) a doublet.

Solution: Total outcomes = 6 x 6 = 36. (i) Sum 7: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) - 6 outcomes. P(sum 7) = 6/36 = 1/6. (ii) Doublet: (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) - 6 outcomes. P(doublet) = 6/36 = 1/6.

2. A bag contains 5 red balls, 8 white balls, 4 green balls and 7 black balls. If one ball is drawn at random, find the probability that it is (i) red, (ii) not green, (iii) neither white nor black.

Solution: Total balls = 5 + 8 + 4 + 7 = 24. (i) P(red) = 5/24. (ii) P(not green) = 1 - P(green) = 1 - 4/24 = 20/24 = 5/6. (iii) Neither white nor black means red or green: 5 + 4 = 9 balls. P = 9/24 = 3/8.

Revision Checklist - Before the Exam

1. I can state Euclid's division lemma and find HCF by Euclid's algorithm and by prime factorisation.
2. I know the Fundamental Theorem of Arithmetic and the rule HCF x LCM = product of the numbers.
3. I can prove √2, √3 and √5 are irrational and test whether a decimal terminates.
4. I know the zero-coefficient relationships for quadratic and cubic polynomials and the division algorithm.
5. I can decide consistency of a pair of linear equations from the ratios and solve pairs by substitution, elimination and cross-multiplication.
6. I can solve quadratic equations by factorisation, completing the square and the quadratic formula, and use the discriminant for the nature of roots.
7. I know the nth term and sum formulas for an AP and can solve word problems with them.
8. I can apply BPT, similarity criteria and Pythagoras theorem, including the area ratio of similar triangles.
9. I know the distance, section and midpoint formulas and the area of a triangle in coordinates.

Revision Checklist - Before the Exam (continued)

10. I have memorised the trigonometric ratios for 0, 30, 45, 60 and 90 degrees and the three identities.
11. I can solve heights and distances problems with tan, including complementary-angle and two-point problems.
12. I know the tangent theorems: tangent perpendicular to radius, tangent length by Pythagoras, equal tangents from an external point.
13. I can find the area of a sector, the length of an arc and the area of a segment.
14. I know the surface area and volume formulas for cube, cuboid, cylinder, cone, sphere and hemisphere, and how to handle combinations and frustums.
15. I can find the mean, median and mode of grouped data and use the empirical relationship.
16. I can solve probability problems with dice, coins, cards and balls, including complements.

Final note: this book is practice material written in our own words from the NCERT textbook. It is not an official CBSE or NCERT publication. Check the chapter list against your issued book, confirm the paper pattern and dates on cbseacademic.nic.in, and practise with the official CBSE sample papers before the exam. Good luck!